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8725 John J. Kingman Road, MS
6201 Fort Belvoir, VA 22060-6201

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TECHNICAL REPORT

Theoretical and Experimental Investigation of Opinion Dynamics in Small Social Networks

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Michael Gabbay

Prepared by:
University of Washington
1013 NE 40th Street
Seattle, WA 98105

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6. AUTHOR(S)
Michael Gabbay

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
University of Washington – Applied Physics Laboratory
4333 Brooklyn Avenue NE
Seattle, WA 98105-6613

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14. ABSTRACT
This report describes research conducted under the subject grant on opinion dynamics in small networks and applications to terrorist networks and WMD decision making. Mathematical analysis and computational simulation were used to study the effects of network structure and nonlinearity on group discussion and decision making outcomes. Theoretical findings included a new route to extreme decisions based on a symmetry-breaking bifurcation; the capability of lower density networks to better reduce discord than higher density ones at high disagreement; and a novel nonlinear-based explanation of the primacy-recency effect. Online discussion experiments involving manipulations of network structure, disagreement level, and other variables were performed in order to investigate effects on consensus, group efficacy, and groupthink-related behavior. Chain networks were found to be capable of greater decision-making performance and less susceptible to group polarization toward extreme decisions than complete networks. A new theoretical explanation for the group polarization effect was developed to account for experimental results counter to the expectations of standard theories. Empirical modeling of terrorist groups and nuclear decision making by a foreign regime were conducted. An analytical framework for assessing the role of small group dynamics in disrupting terrorist groups was proposed.

15. SUBJECT TERMS
Opinion Change, Group Decision Making, Small Groups, Social Networks, Group Polarization, Terrorism, Insurgency

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UNIT CONVERSION TABLE

U.S. customary units to and from international units of measurement*

U.S. Customary Units	Multiply by → / ← Divide by†		International Units
Length/Area/Volume			
inch (in)	2.54	$\times 10^{-2}$	meter (m)
foot (ft)	3.048	$\times 10^{-1}$	meter (m)
yard (yd)	9.144	$\times 10^{-1}$	meter (m)
mile (mi, international)	1.609 344	$\times 10^{3}$	meter (m)
mile (nmi, nautical, U.S.)	1.852	$\times 10^{3}$	meter (m)
barn (b)	1	$\times 10^{-28}$	square meter (m^2)
gallon (gal, U.S. liquid)	3.785 412	$\times 10^{-3}$	cubic meter (m^3)
cubic foot (ft^3)	2.831 685	$\times 10^{-2}$	cubic meter (m^3)
Mass/Density			
pound (lb)	4.535 924	$\times 10^{-1}$	kilogram (kg)
unified atomic mass unit (amu)	1.660 539	$\times 10^{-27}$	kilogram (kg)
pound-mass per cubic foot (lb ft^{-3})	1.601 846	$\times 10^{1}$	kilogram per cubic meter (kg m^{-3})
pound-force (lbf avoirdupois)	4.448 222		newton (N)
Energy/Work/Power			
electron volt (eV)	1.602 177	$\times 10^{-19}$	joule (J)
erg	1	$\times 10^{-7}$	joule (J)
kiloton (kt) (TNT equivalent)	4.184	$\times 10^{12}$	joule (J)
British thermal unit (Btu) (thermochemical)	1.054 350	$\times 10^{3}$	joule (J)
foot-pound-force (ft lbf)	1.355 818		joule (J)
calorie (cal) (thermochemical)	4.184		joule (J)
Pressure			
atmosphere (atm)	1.013 250	$\times 10^{5}$	pascal (Pa)
pound force per square inch (psi)	6.984 757	$\times 10^{3}$	pascal (Pa)
Temperature			
degree Fahrenheit (°F)	[T(°F) − 32]/1.8		degree Celsius (°C)
degree Fahrenheit (°F)	[T(°F) + 459.67]/1.8		kelvin (K)
Radiation			
curie (Ci) [activity of radionuclides]	3.7	$\times 10^{10}$	per second (s^{-1}) [becquerel (Bq)]
roentgen (R) [air exposure]	2.579 760	$\times 10^{-4}$	coulomb per kilogram (C kg^{-1})
rad [absorbed dose]	1	$\times 10^{-2}$	joule per kilogram (J kg^{-1}) [gray (Gy)]
rem [equivalent and effective dose]	1	$\times 10^{-2}$	joule per kilogram (J kg^{-1}) [sievert (Sv)]

*Specific details regarding the implementation of SI units may be viewed at http://www.bipm.org/en/si/.
†Multiply the U.S. customary unit by the factor to get the international unit. Divide the international unit by the factor to get the U.S. customary unit.

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Appendices

1 Research Objectives

The following were the research objectives of our program:

1. Formulation of the terrorist/WMD decision making problem in relation to the opinion modeling formalism.

2. Analysis of the effects of network structure and nonlinearity on the convergence to specified end states, in particular consensus around a given opinion or a divisive factional deadlock.

3. Analysis of the effects of the order in which group members interact with each other along the network.

4. Analysis of the interaction between network structure and groupthink-type behaviors.

5. Use of optimization methods for the design of network structure to achieve desired end states.

6. Conduct of laboratory-based experiments on attitude change in small groups of human subjects, manipulating factors such as network structure, initial opinion disagreement, communication rate, and group cohesion.

We employed theoretical, mathematical, computational, and experimental methods to address these objectives (#5 was not addressed due to personnel changes). Additional objectives involving empirical application to terrorist networks and WMD decision-making were also addressed.

2 Accomplishments

Significant accomplishments of our program include the following:

1. Proposed a new mechanism for reaching extreme group decisions based on majority rule outcomes due to symmetry-breaking bifurcation. (Sec 3.1.1)
2. Showed that inclusion of nonlinear influence function can allow lower density networks to better lower discord than higher density ones at high initial disagreement, contrary to linear theory. (Sec. 3.1.2)
3. Developed a novel, nonlinear-based explanation of primacy vs. recency ambiguity observed in attitude change experiments concerning persuasive efficacy and message order. (Sec. 3.1.3)
4. Developed software to manage and conduct group decision making experiments involving online discussion. (Sec. 3.2.1)
5. Conducted five different group discussion experiments involving manipulations of network structure, disagreement level, message order, choice, and task. (Secs. 3.2.2-3.2.6)
6. Experimentally found that chain networks showed superior performance to complete networks in an experiment involving a judgmental task with a correct answer; high disagreement groups were more effective than low disagreement ones. (Sec. 3.2.5).

7. Experimentally observed risky shift effect in a real-world forecasting task that could not be explained by standard informational and normative theories of group polarization toward extreme decisions. Also observed greater risky shift for complete networks than chains and at high disagreement compared to low. (Sec. 3.2.6)
8. Based on experimental results, proposed a new theoretical explanation for group polarization toward extreme decisions based on heuristic issue substitution. Theory led to revision of nonlinear opinion dynamics model which showed qualitative and quantitative agreement with experiment. (Sec. 3.2.6)
9. Developed an analytical framework for terrorist group disruption which can be used to help understand the interaction between counterterrorist actions and small group dynamics within terrorist groups. Generated associated hypotheses. (Sec. 3.3.1)
10. Demonstrated application of group decision making model to terrorist and WMD decision making. Briefed results to DOD analytical personnel. (Secs. 3.3.2 and 3.3.3)
11. Proposed a new research agenda for application of social network analysis to terrorism and insurgency involving political interactions among the groups in a militant movement. Generated hypotheses concerning the impact of terrorist network structure on strategic behaviors. (Sec. 3.3.4)
12. Constructed networks of Syrian militant groups and showed that ideological similarity is an important driver of militant cooperation. (Sec. 3.3.4)
13. Supported transition by training STRATCOM personnel on software implementing group decision making model and other analysis algorithms. STRATCOM is currently conducting an in-house evaluation of the software.

3 Summary of Results

This section highlights significant results from our research. Reference numbers associated with the accomplishments below refer to entries in the Publications list.

3.1 Modeling Opinion Network Dynamics

3.1.1 Majority Rule Due to Symmetry Breaking

Using our nonlinear model of opinion dynamics, we discovered a novel route to extreme group decisions which yields an asymmetric majority rule outcome toward either extreme due to a symmetry breaking bifurcation. Figure 1 shows simulation results for a three-person group in which a centrist is bracketed by two equidistant extremists; all group members are connected with the same coupling strengths and have initial opinions (natural biases) symmetrically distributed around zero. Standard intuition would anticipate either a deadlock or various shades of compromise around the centrist position, consistent with final states that are symmetric about the middle, as shown in Figure 1(a) and (c) for deadlock and compromise respectively. However, at sufficiently high levels of initial disagreement, another outcome can result in which the centrist swings toward one of the extremes (depending on random perturbations), corresponding to a majority rule situation favoring one side of the policy axis. In this case the system reaches an asymmetric final state as observed in Figure 1(b). Even a small change in coupling strength can produce a transition to the majority rule outcome zone from either the deadlock or compromise zones. This result stands in contrast to both psychological theories of group opinion dynamics in

which opinions converge towards the mean and sociological theories in which asymmetric outcomes result from asymmetries in social structure. But the nonlinear model shows that an asymmetric outcome can emerge for symmetric conditions in network structure and initial opinion distribution. This result is important because it implies that: (1) policy outcomes of group debates could be more likely to swing towards the extremes rather than converge toward a compromise solution; and (2) the outcome – on which extreme the majority opinion falls – may be fundamentally unpredictable. If the simulation of a leadership group showed such behavior, analysts could be prepared for an extreme decision rather than deadlock or compromise.

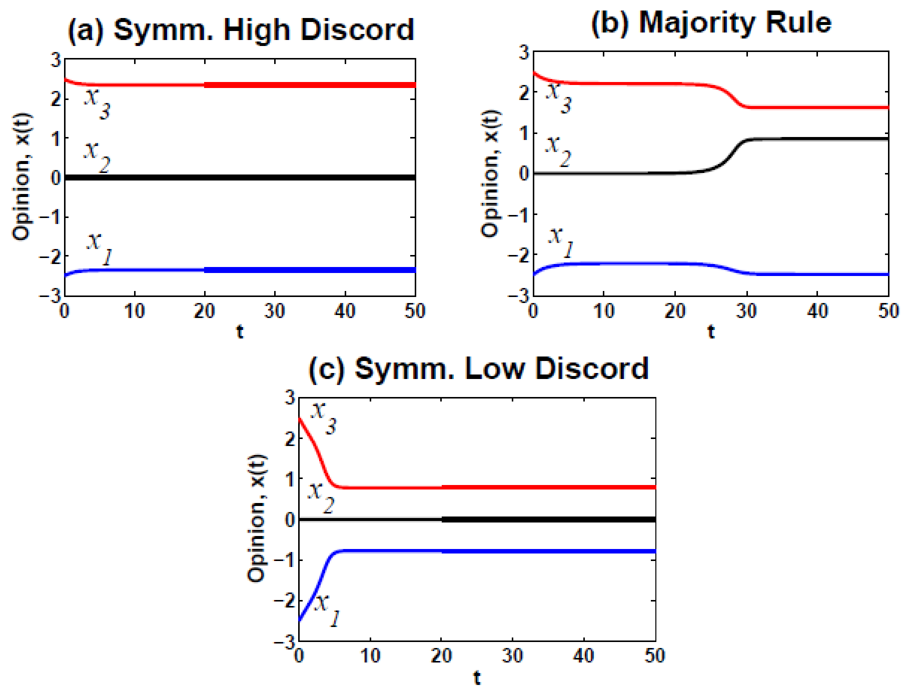

Figure 1. Node opinions vs. time showing equilibrium outcomes in symmetrically-coupled triad chain network with high initial disagreement at different coupling strengths. (a) Symmetric High Discord (deadlock) at low coupling strength; (b) Majority Rule at intermediate coupling strength; (c) Symmetric Low Discord (compromise) at high coupling strength.

A bifurcation analysis of a three person chain or "broker" network – in which two members with extremely opposed positions communicated via a person whose opinion was exactly in the middle – revealed that the majority rule solution results from a pitchfork bifurcation in which the deadlock state becomes unstable as the coupling scale passes a critical value. Analytical expressions derived for the transition boundaries between solution types. The nonlinear model is characterized by multistability in which it is possible to have more than one stable equilibrium solution for a given set of parameters – the actual solution to which the system evolves depends on the initial conditions. The stability diagram shows the regions in the parameter space defined by coupling scale (e.g., communication rate) and natural bias difference (level of ideological disagreement) where the different solution types exist (see Figure 2). The analytical approximations were derived via a stability analysis and show good agreement with numerical simulations of the model.

These results are reported in [2,4,8].

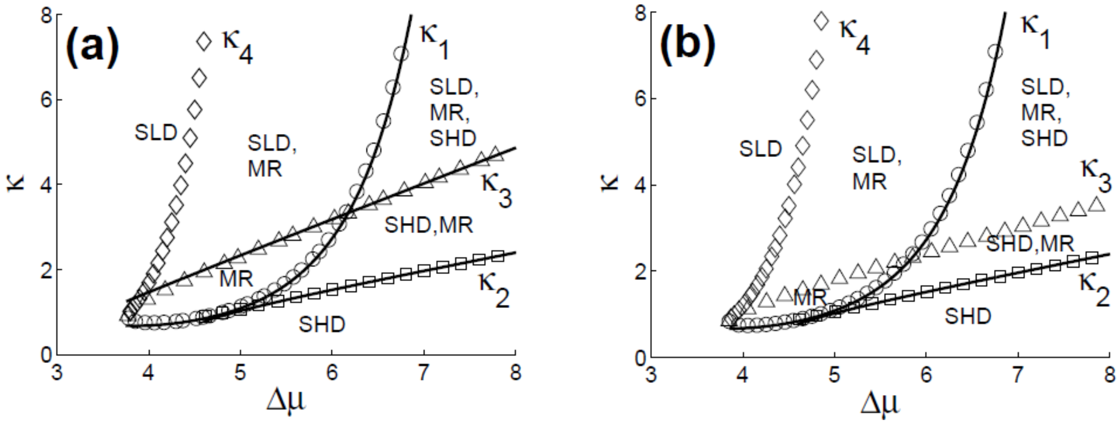

Figure 2. Stability diagram of triad with symmetric coupling showing coupling strength κ vs. natural bias difference Δμ for: (a) chain network and (b) complete network. Open markers are numerically obtained boundaries. Solid lines are chain analytical approximations

3.1.2 Network Density and Discord Reduction

Simulation and analysis of the nonlinear opinion dynamics model showed that lower density networks can reduce discord more effectively at high disagreement levels. This is contrary to the usual assumption in social network theory that denser networks, i.e., those containing more edges, are more cohesive. We extended our result that for triads at high disagreement, chain networks were able to lower the disagreement more efficiently (i.e., using lower communication rates) than complete networks. For example, opinion polarization is observed as the network density is increased in large networks. A schism into two opposed factions was observed in simulations of a 100-node network at high initial disagreement between the extremes. The nodes were uniformly distributed in opinion space and the number of edges they had with their neighbors (on the opinion axis) was varied. The mean coupling strength was kept constant as ties were added in order to compare network topologies for the same total communication rate. Figure 3 shows that increasing the network density past a certain critical value can cause discord to increase rather than decrease as expected in linear theory. The bifurcation as edges are added shows that increased network density may not always be beneficial to group cohesion and that mass polarization can occur without a bimodal initial distribution or exogenous impacts. Opinion fragmentation was also observed in Gaussian-distributed initial opinions.

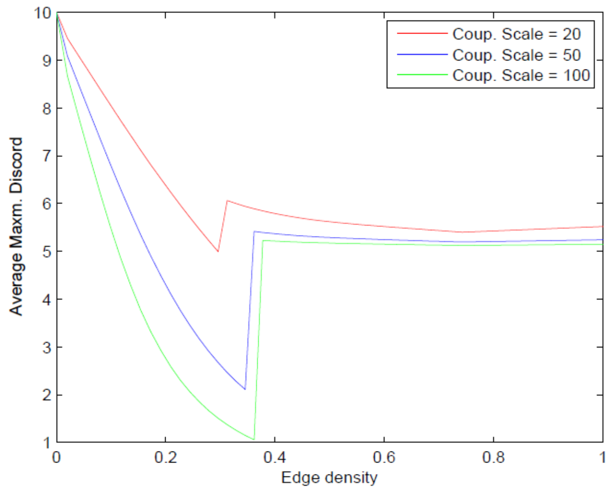

Figure 3. Plot of maximum discord as a function of edge density for high initial disagreement (natural bias difference). Curves are averaged over 100 trials for a linear initial distribution of states.

These results are reported in [2,4].

3.1.3 Asynchronous Communications

The effects of asynchronous communications were simulated for three and five-node networks. The three node network simulations demonstrated a novel explanation of the primacy-recency effect in attitude change. The five node simulations investigated the effects of message order on group consensus and policy outcomes; a route to consensus around an extremist position was identified. Simulation results are reported in [3].

Primacy vs. recency attitude change explanation: We developed a model-based explanation using disagreement level as key variable for primacy-recency effect, i.e., when it is more advantageous to speak first (primacy) or last (recency) when two opposing advocates are trying to persuade someone. Both primacy and recency effects have been observed experimentally in the attitude change literature but no clear explanation has been accepted and no explanation based on disagreement level has been proposed. Model simulation results show that it is better to go first (primacy wins) when the disagreement between the two opposing advocates is high and it is better to go last when the disagreement is low (see Figure 4). Linear models of small group opinion change such as the Friedkin-Johnsen model cannot account for this effect as recency always wins. Note that this primacy vs. recency effect refers to attitude change and not the effect with the same name associated with memory recall.

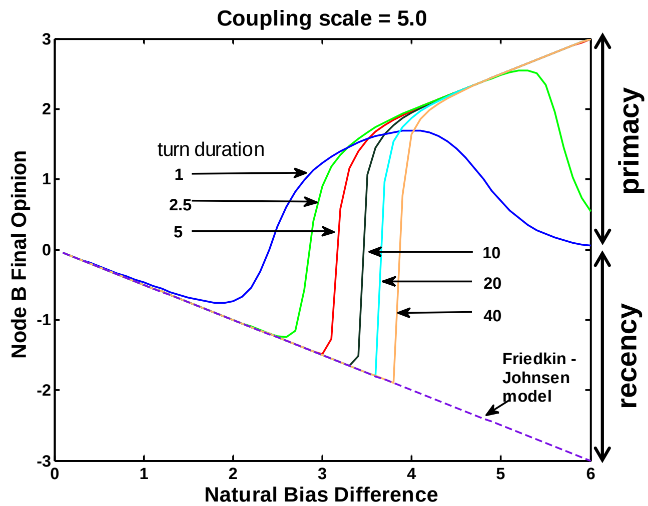

Figure 4. Simulation results showing when speaking first (primacy) or last (recency) is more advantageous for persuading an uncommitted listener. The listener's final opinion is plotted vs. natural bias difference, i.e., the level of disagreement, between the opposing advocates. The advocate who goes first has a positive opinion value and the last one has a negative opinion. For low differences, it is better to go last; for high differences it is better to first. As the speaking turn duration gets longer, the natural bias transition at which the transition between recency and primacy occurs gets larger. The dashed line is the Friedkin-Johnsen model which always favors recency.

<u>Sequence-based consensus around extreme position</u>: For triad networks, simulations in which the speaking sequence for all three members was varied showed that it is possible to obtain a consensus around an extreme position; an outcome not observed for simultaneous communications. The consensus about the extreme was observed for a complete network topology in which one extremist goes first, the centrist second, and the other extremist last; the consensus forms about the final extremist's position (see Figure 5). The centrist's speaking second is key to this route to extremity, which would not occur if the centrist went first or last. This is a nonlinear effect which only occurs at high disagreement levels.

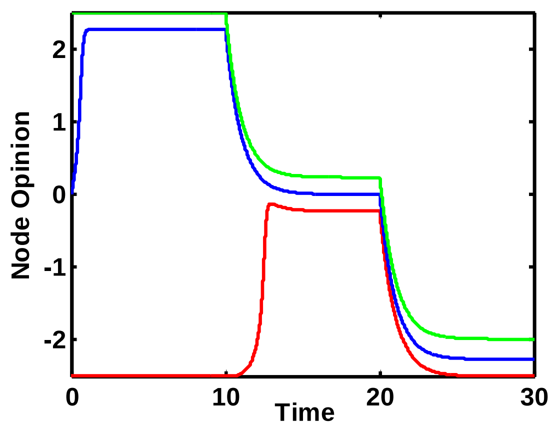

Figure 5. Simulation of asynchronous communications for complete triad network showing route to consensus around an extreme position. The speaking sequence is: green, blue, red. The green extremist, speaking first, cannot form a consensus around its position whereas the red extremist, speaking after the blue centrist, can. Natural bias difference equals 5.

3.2 Experiments

3.2.1 Group Discussion Software

We developed software called Sermo as a platform for small group discussion and decision making experiments (Figure 6). Sermo enables us to vary the random assignment parameters, including network structure, the levels of disagreement, and decision rules and add flexible, instance-specific variables. In addition, it stores transcripts in a machine readable format alongside user's votes and metadata. The software automatically provides users an anonymous login and presents a familiar chat-room style interface. Sermo allows us to vary a large variety of experimental treatments behind the scenes including: the group assignment conditions, discussion topics, network structure, and the group's decision rule. This software was successfully employed to conduct all of our experiments.

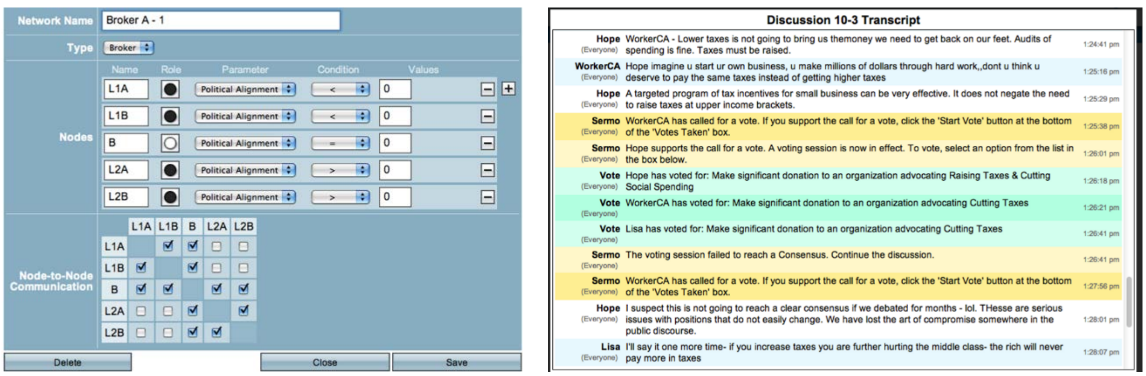

Figure 6. Sermo interface. Administrator view (l) allows manipulation of network size, structure, decision rules, roles, and supplemental conditions. The participant interface (r) is a familiar chat system. Depending on the type of network, participants can specify the direction of their messages.

3.2.2 Experiment 1: Disagreement and Network Structure in Policy Decisions

We conducted experiments with 125 three-person groups using Amazon Mechanical Turk (AMT) workers as subjects to test hypotheses concerning the prevalence of majority rule at high initial disagreement and that chain networks are better at reducing discord at high disagreement. The subject of the discussion involved three different policies regarding the US economy. The group-level analysis did not show any statistically significant effects relating disagreement level to majority rule outcomes or network topology to the efficacy in achieving consensus. However, in the high-disagreement condition, complete network groups were more likely to remain deadlocked than chain groups (17% vs. 6%); chains more readily reached either a majority or consensus decision in accordance with our hypothesis that chains should better reduce discord at high disagreement. A potential complication in the experiment was that one of the extreme policy choices was chosen more frequently than the opposite extreme indicating an unintended choice asymmetry which does not conform to the symmetric opinion distribution assumed in the hypotheses. In addition, the relatively high prevalence of consensus results overall also inhibited our ability to statistically distinguish between the efficacy of the different network structures in reducing discord. The experiment is described in [5].

We also analyzed individual-level effects. Given the importance of the "centrist" or "moderate" in a group debate, we investigated how this person interacts with the rest of the group. Support

was found among both policy moderates and ideological moderates for the hypotheses that: (1) the moderate is likely to be influenced when one of the extreme policy choices prevails; (2) the moderate is the most likely group member generally to report being influenced; and (3) the moderate is most likely to report high process satisfaction. This analysis is detailed in [17].

3.2.3 Experiment 2: Communications Order

We developed a confederate-based design for conduct of communications order effects to test the hypothesized advantage of going first (last) at high (low) disagreement. We conducted experiments on 128 groups where each group consisted of one subject with a moderate initial opinion and one confederate playing the roles of two group members with opposed opinions. We were able to successfully conduct communications order experiments in which a confederate operating from a script played the roles of advocates for the two opposed policy choices presented to the subject. The scripts involved policy attitude statements selected from transcripts of actual three-person discussions (from the previous experiment) which were rated for extremity level by individuals from AMT. The experiment did not show any statistically significant effects relating disagreement level with an advantage in going first or last. This may also be due to the unintended asymmetry in which one of the opposed policy choices appeared inherently more favorable to the subjects as discussed above.

3.2.4 Experiment 3

Experiment 3 was a repeat of Experiment 1 with a different issue – immigration policy – used for discussion. There were too many consensus outcomes to enable us to test our hypotheses.

3.2.5 Experiment 4: Group Performance in Judgmental Task

We developed a revised experimental design which increases subject commitment to discussion and outcome. This was done to address the very strong tendency toward consensus in our earlier experiments and to allow for investigation of group efficacy and groupthink-related phenomena such as the risky shift effect. The experiment presented subjects with a hypothetical foreign policy scenario involving whether or not to launch a preemptive missile strike against cyberterrorists planning an attack against the US. In a pre-survey, subjects were asked (1) to estimate how many Americans surveyed in a separate poll would support the attack and (2) how much they would wager ($0-7) that their answer is correct. Subjects with similar estimates (either <50% or >50%) were placed in three-person groups and engaged in an online discussion at the end of which each group member would make a final individual wager. The experimental conditions manipulated groups according to right (<50%) and wrong (>50%) estimates, network structure (complete vs. chain), and the level of disagreement as to wager amounts – low (wager spread ≤ $5) vs. high (spread = $7). We successfully conducted this experiment using subjects from Amazon Mechanical Turk formed into 138 groups.

The central and most novel finding of the experiment is the superior performance of chain networks over complete networks, particularly at high disagreement level. Groups with a chain network topology consisted of a person with a middle wager at the center of the chain and two people – one with a low wager and the other with a high one – at the opposite ends of the chain. In a complete network, all three group members could communicate with each other. For groups with the right estimate of public opinion, optimal performance was indicated by an increase in

wagers whereas groups with the wrong estimate should decrease their wagers. For chain networks, the difference in the post-discussion means between the right and wrong groups was statistically significant (\$4.47 vs. \$3.01, $p<.002$) strongly indicating correct performance which was larger than the difference for complete networks (Figure 7). Furthermore, chain groups at high disagreement level performed better than any other condition. This result suggests, counterintuitively, that open discussion between all group members can impair performance and that high disagreement can be beneficial particularly when mediated by a broker. Consequently, groupthink phenomena may be mitigated by having a more restricted network topology. This result implies that counterterrorist efforts aimed at disrupting the open communication flow among terrorist group members may induce network changes which make the group more effective at decision making.

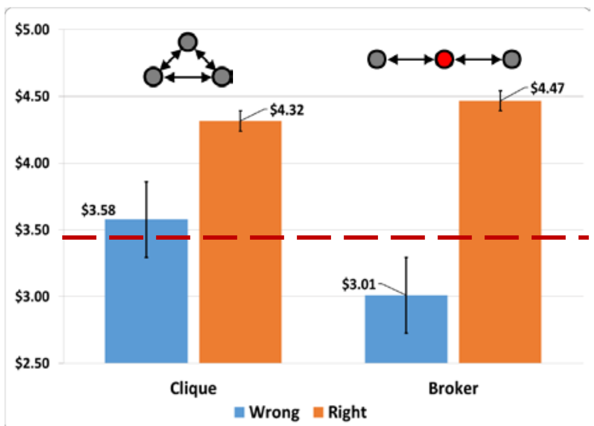

Figure 7. Post-discussion mean wagers for complete ("clique") and chain ("broker") networks. Broker networks show better performance in shifting their wagers in the correct direction from the pre-discussion mean (dashed line).

The experiment and results are described in [9].

3.2.6 Experiment 5: Risky Shift in Forecasting Task

We conducted an experiment to address the interaction of network structure and disagreement levels in producing a risky shift (an example of the broader effect known as group polarization) for discussion groups engaged in a forecasting task. The experiment involved triads discussing how much to wager on the outcome (margin of victory) of National Football League (NFL) games during the 2014-15 season. Our use of football games was motivated by the desire to give participants a task in which they could draw on their natural knowledge base to forecast the outcome of a real-world event. In addition, participants were told that their "winnings" would be donated to a charity, the Wounded-Warrior Project (the donations were actually made), thereby providing real stakes to the task. This combination of real-world forecasting and stakes stands in contrast to the vast majority of group polarization experiments which involved discussion of attitudes or hypothetical situations without consequences tied to decisions. In the experiment, team choice (Favorite/Underdog), initial wager disagreement level (High/Low), and network structure (Complete/Chain) were manipulated.

Focusing on consensus outcomes, statistically significant differences in the amount of wager increase following discussion were found for all three manipulated variables: groups which picked the favorite showed a risky shift whereas Underdog groups did not and, within the Favorite groups, high disagreement showed a greater risky shift than low and complete networks shifted more than chains.

Comparison	n	$\Delta\bar{w}$($)	sd($)	diff.($)	p-val
Favorite	104	1.44	1.91	1.91	10^{-11}***
Null					
Underdog	56	0.19	1.80	0.19	.43
Null					
Favorite	104	1.44	1.91	1.25	.00008***
Underdog	56	0.19	1.80		
Fav./High	60	1.82	2.03	0.89	.014*
Fav./Low	44	0.92	1.60		
Fav./Comp.	37	2.10	1.81	1.02	.008**
Fav./Chain	67	1.07	1.88		

Table 1. Mean wager shifts for consensus outcomes for different variable comparisons. *n* is number of groups; *Δw* is the difference between the post-discussion mean wager and the pre-discussion mean averaged over all groups; *sd* is the standard deviation of the wager shift; *diff* is the difference between the top and bottom variables in the comparison pair; p-val is the statistical significance (*<.05,<.01,***<.001).**

The fact that the Favorite groups showed a risky shift whereas the Underdog groups did not is in contrast to the standard informational and normative explanations of group polarization toward extreme decisions. To account for this, we developed a theoretical argument which represents a novel route to the risky shift and, more generally, group polarization. We argue that a nonlinear, S-shaped relationship between the policy under consideration and the rhetorical issue that the group members actually discuss can lead to group polarization. This occurs via a mechanism of substitution-induced rhetorical asymmetry (SIRA) which causes group members lying more toward the extreme direction to be closer together with respect to the rhetorical issue even when no such asymmetry is present along the policy itself. Consequently, the preferential emergence of majorities at a position more extreme than the mean is facilitated, a position with which the more moderate minority then concurs. Substitution-induced asymmetry is complementary to and can operate in conjunction with the informational and normative routes. We claim that SIRA arises in this experiment because of issue substitution: groups discuss as a rhetorical issue the likelihood of the favorite winning the game rather than the favorite's expected margin of victory which is much more directly related to the wager amount policy. The substitution of the likelihood of victory as a heuristic for the expected margin of victory can be viewed, from the perspective of the heuristics and biases program, as a form of attribute substitution.

We developed two mathematical models which are in both qualitative accord and quantitative agreement with our experimental results. The first is, essentially, a simple phenomenological "rhetorically-proximate majority"' (RPM) model which posits that the final consensus policy for a group is the average policy of the pair of group members which are closest along the rhetorical issue axis. The second model is called the "accept-shift-constrict" (ASC) model and describes

opinion change processes over a network. These processes consist of first the acceptance of a persuasive message which can then lead to a shift in the receiver's opinion and/or a constriction of their uncertainty level which in turn narrows the extent to which messages advocating distant opinions are accepted. Even without SIRA included, ASC represents a new model of opinion network dynamics in which the persistence of majority positions is enhanced via the increase in confidence caused by the reinforcing effect of those with similar opinions, a dynamic missing from existing continuous opinion network models. Incorporating SIRA then produces group polarization similarly to the RPM model but, more fundamentally, reflects the operation of micro-level opinion change processes. Furthermore, shifts toward the extreme arise via the SIRA-ASC combination without reliance upon systematic skews in individual psychological traits such as stubbornness as has been typically assumed within the opinion network modeling literature on extremism. The ASC model was developed under a follow-on ONR-funded grant (Sec. 4).

We are in the process of writing a paper on the experiment and these models [19].

3.3 Application to Terrorist/WMD Contexts

3.3.1 Analytical Framework for Terrorist Group Disruption

A framework which can be used to help understand the interaction between counterterrorist actions and terrorist small group dynamics was developed (Figure 8). The counterterrorist actions are categorized into three types of interventions, each with an associated intended goal: (1) *repression* intended to directly degrade a group's capabilities; (2) *manipulation* intended to cause group dysfunction so that the group can do less with the capabilities it has; (3) *persuasion* intended to produce group moderation. The framework was used to generate a series of hypotheses connecting counterterrorist actions with indirect and potentially unintended, adverse effects as mediated by small group dynamics: e.g. (1) Significantly increasing a terrorist group's communication costs will increase the congruence between leader preferences (possibly extreme) and group decisions; (2) Effective repressive interventions will spur dissolution in those terrorist groups reliant on task-based cohesion but may increase group commitment in those whose cohesion stems from affective social ties among their members.

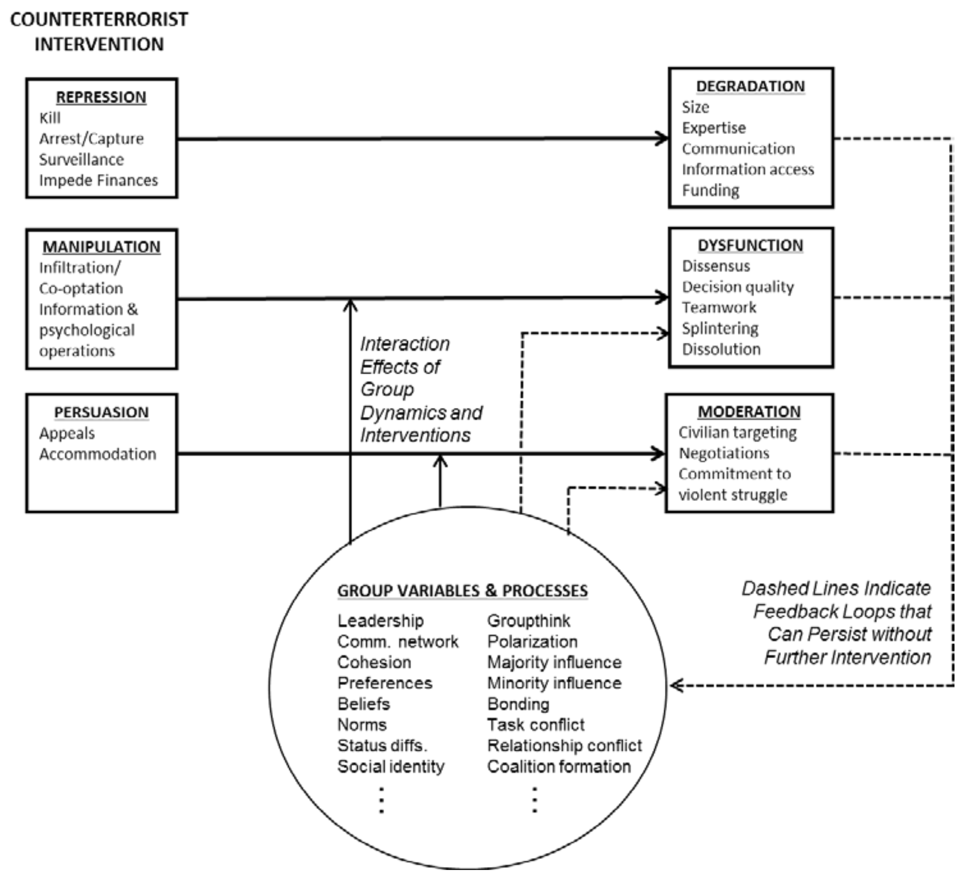

Figure 8. Theoretical framework relating counterterrorist actions, goals, and group variables.

This framework is described in [6].

3.3.2 Afghanistan Decision Making

We continued and documented an application of the nonlinear model and associated methodology to strategic decision making by insurgent and government leaders in Afghanistan [7]. The model implementation methodology used the input of multiple Subject Matter Experts (SMEs) to assess leader ideological and policy positions and network ties. A model of group decision making was used to simulate insurgent and government decision-making dynamics with respect to a number of strategic issues such as negotiations, US presence and influence, the level of state centralization, and support for Al Qaida.

3.3.3 Iranian Nuclear Decision Making

We modeled decision making by Iranian leadership elites on the issue of nuclear technology development in the context of the nuclear negotiations between the US and Iran which were ongoing at the time of the analysis. The analysis was implemented using PORTEND, a Matlab-based software package which provides an interface for the nonlinear group decision making model in addition to network analysis algorithms. The application was done in collaboration with STRATCOM. STRATCOM personnel were briefed on the results and trained in the software. The analysis is described in [16].

3.3.4 Militant Network Analysis

We developed a research agenda aimed at using network analysis to investigate the political interactions of militant groups within a single conflict. This arose out of a review of the literature applying social network analysis to terrorism and insurgency in which we found that the overwhelming emphasis of existing research is on organizational analysis and its implications for militant group operational processes and performance rather than interactions between groups. Our proposed research agenda would apply network analysis to insurgencies consisting of multiple, independent militant groups in order to better understand and anticipate phenomena such as militant infighting, alliance formation, and escalation to more extreme violence. This work is described in [15].

In accord with the above agenda, we collected data on 220 Syrian militant groups based on translated militant claims of attacks. We found that ideological similarity helps explain patterns in the network of (claimed) joint operations between groups (Figure 9). We are preparing a paper documenting this analysis [20]. We also conducted a preliminary analysis of the relationship of network structure to militant infighting.

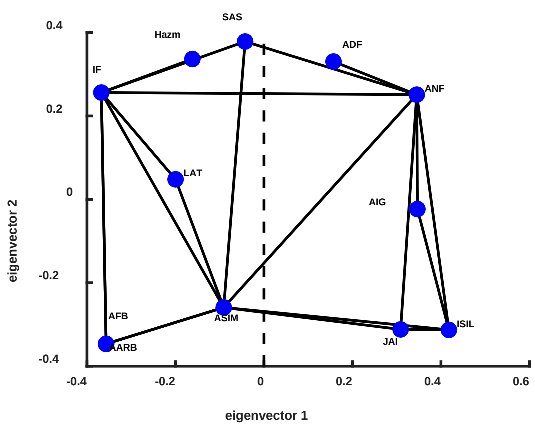

Figure 9. Network of joint operations between 12 Syrian militant groups whose ideologies were scored by a Subject Matter Expert. Plotted are the first two eigenvectors of the modularity matrix. Nationalist groups are found on the left and jihadist groups are on the right.

4 Follow-on Research and Transition

A follow-on grant entitled “Critical Transitions and Adaptation in Group Dynamics” (N00014-15-1-2549) has been awarded by ONR to continue aspects of this research.

STRATCOM personnel were trained on the PORTEND software package which implements the group decision making model in addition to other structural and network analysis methods. STRATCOM is in the process of applying PORTEND to case studies. The results of this effort were also presented to personnel in the intelligence community.

5 Personnel

The following UW personnel received support under this project:

- Dr. Michael Gabbay (APL-UW): Principal Investigator
- Dr. Arindam Das (APL-UW): Research Engineer
- Troy Tanner (APL-UW): Software Engineer
- Zane Kelly (APL-UW): Postdoc
- Prof. John Gastil (Communication Dept.): Co-PI
- Justin Reedy (Communication Dept.): Graduate Research Assistant and Postdoc
- Whitney Anspach (Communication Dept.): Graduate Research Assistant
- Steven Zech (Political Science): Graduate Research Assistant
- Emily Gade (Political Science): Graduate Research Assistant

The following Pennsylvania State University personnel were supported under a subcontract:

- Prof. John Gastil (Communication Dept.): Co-PI
- Robert Richards (Communication Dept.): Graduate Research Assistant

The following University of Oklahoma personnel were supported under a subcontract:

- Prof. Justin Reedy (Communication Dept.)
- Cheryl Maiorca (Communication Dept.): Graduate Research Assistant

6 Publications and Presentations

6.1 Publications and Reports

Selected documents appear as appendices to this report as noted.

1. Reedy, J. & Gastil, J., “Terrorism and small groups: How group communication research can inform the study of terrorist cells and leadership teams,” paper presented at the annual convention of the National Communication Association, New Orleans, LA, USA, November 2011.
2. Das, A., “Large Group Nonlinear Opinion Simulations,” unpublished report, June 2012. **Appendix 1**.
3. Das, A., “Simulation of Asynchronous Communications in Nonlinear Opinion Dynamics,” unpublished report, July 2012. **Appendix 2.**
4. Gabbay, M. & Das. A., “Majority Rule as Spontaneous Symmetry Breaking in Small Opinion Networks,” unpublished report, December 2012. **Appendix 3**.
5. Reedy, J., Gabbay, M., and Gastil, J., “Experimental Study of Persuasion and Decision Making in Small Networks,” paper presented at the Annual Meeting of the American Political Science Association, Chicago, IL, August 2013. **Appendix 4.**
6. Reedy, J., Gastil, J., & Gabbay, M., “Terrorism and small groups: An analytical framework for group disruption,” *Small Group Research* 44(6), 599-626, 2013. **Appendix 5.**

7. Gabbay, M., "Modeling Decision-Making Outcomes in Political Elite Networks," in K. Glass et al (Eds.), *Complex Sciences* (LNICST 126, 95-110), Springer International Publishing, 2013. **Appendix 6.**
8. Gabbay, M. & Das, A., "Majority rule in nonlinear opinion dynamics," in In, V, Palacios, A. & Longhini, P. (Eds.), *International Conference on Theory and Application in Nonlinear Dynamics (ICAND 2012)*, 167-179, Springer International Publishing, 2014. **Appendix 7.**
9. Kelly, Z., Gabbay, M., Reedy, J., and Gastil, J., "An Experimental Study of Persuasion, Confidence, and Choice Shift in Small Networks," paper presented at the Annual Meeting of the American Political Science Association, Washington, DC, Aug. 31, 2014. **Appendix 8.**
10. Gabbay, M., "Data Processing for Applications of Dynamics-Based Models to Forecasting," in Egeth, J.E., Klein, G.L., and Schmorrow, D. (Eds.), *Sociocultural Behavior Sensemaking: State of the Art in Understanding the Operational Environment.* McLean, VA: The MITRE Corporation, 2014.
11. Kelly, Z., Zech, S., Gabbay, M., Thirkill-Mackelprang, A., "Modeling Insurgent Networks using Exponential Random Graphs and Targeting Policies," paper presented at Midwestern Political Science Assoc. conference, Chicago, IL, April 2015.
12. Kelly, Z., Gabbay, M., Reedy, J., and Gastil, J., "Experimental Study of Persuasion, Decision Making, and Risky Shift in Small Networks," paper presented at Midwestern Political Science Assoc. conference, Chicago, IL, April 2015.
13. Gastil, J., Richards, R.C., Reedy, J., Gabbay, M., Kelly, Z., "Deliberative and Cultural Orientations in Small Decision-Making Groups," National Communication Assoc. Annual Conference, Las Vegas, Nov. 2015.
14. Kelly, Z., Reedy, J., Gabbay, M., Gastil, J., "Persuasion Confidence, and Choice Shifts: An Experimental Study of Decision Making and Communication Structure in Small Groups," Natl. Comm. Assoc. Annual Conference, Las Vegas, Nov. 2015.
15. Zech, S. & Gabbay, M., "Social Network Analysis in the Study of Terrorism and Insurgency: From Organization to Politics," *International Studies Review*, doi: 10.1093/isr/viv011, 2016. **Appendix 9.**
16. Gabbay, M., "Leadership Network Structure and Policy Dynamics," in *Handbook of Research Methods in Complexity Science: Theory & Application*, E. Mitleton-Kelly, A. Paraskevas, C. Day (eds.), Edward Elgar Publishing (to appear). **Appendix 10**.
17. Richards, R.C. & Gastil, J., "Moderates Results," unpublished report, October 2016.
18. Richards, R.C. & Gastil, J., "Abstracts for Individual Differences," unpublished report, October 2016.
19. Gabbay, M., Kelly, Z., Reedy, J. & Gastil, J., "Substitution-induced risky shift in small group opinion dynamics," in preparation.
20. Gade, E.K, Gabbay, M., Hafez, M., & Kelly, Z., "Bringing Ideology Back In: Militant Networks and Alliance Formation in Syria's Civil War," in preparation.

6.2 Presentations

1. Gabbay, M. and Das, A., "A nonlinear model of decision making in small social networks," presented at Political Networks 2011 Conference, Ann Arbor, MI, June 2011.

2. Reedy, J., Gastil, J., Shuffler, M., Wildman, J., Grossman, R., & Salas, E., "Groups turned violent: Applying research on groups and teams to the study of terrorism," panel organized for the annual convention of the Interdisciplinary Network for Group Research (INGRoup), Minneapolis, MN, July 2011.
3. Reedy, J., "Online group experiments in the study of terrorism: Using a web-based chat tool and semi-automated scripts to conduct mock group discussions," poster presented at the annual convention of the Interdisciplinary Network for Group Research (INGRoup), Minneapolis, MN, July 2011.
4. Reedy, J. & Gastil, J., "Terrorism and small groups: How group communication research can inform the study of terrorist cells and leadership teams," annual convention of the National Communication Association, New Orleans, LA, USA, November 2011.
5. Gabbay, M., "Network and nonlinear effects in small group decision making," UC Davis, Lectures in Network Sciences, December 2011.
6. Gabbay, M., "Modeling the structure and dynamics of insurgent and political networks," Pacific Northwest National Laboratory, May 24, 2012.
7. Reedy, J., Gastil, J., & Gabbay, M., "Group research and terrorism: The interaction of network structure and disagreement in online political discussion groups," annual conference of the Interdisciplinary Network for Group Research in Chicago, IL, July 2012.
8. Gabbay, M. & Das, A., "The nonlinear dynamics of opinion networks," International Conference on Theory and Applications in Nonlinear Dynamics, Seattle, WA, August 2012.
9. Gabbay, M., "Nonlinear dynamics of opinion networks," U. Washington Applied Physics Laboratory seminar, October 25, 2012.
10. Gabbay, M., "Modeling Decision-Making Outcomes in Political Elite Networks," 2nd International Conference on Complex Sciences: Theory and Applications, Santa Fe, NM, Dec. 5-7, 2012.
11. Gabbay, M., "Nonlinear Dynamics on Opinion Networks," American Physical Society March Meeting, Baltimore, BD, March 19, 2013.
12. Gabbay, M. "Modeling Strategic Decision Making in the Afghan Insurgency," 6th Annual Political Networks Conference, Bloomington, IN, June 28, 2013.
13. Reedy, J., Gabbay, M., and Gastil, J., "An Experimental Study of Persuasion and Decision Making in Small Networks," Annual Meeting of the American Political Science Association, Chicago, IL, August 31, 2013.
14. Gabbay, M., "A Simulation of Cooperation and Competition in Insurgent Networks," American Physical Society March Meeting, Denver, CO, March 7, 2014.
15. Reedy, J., Gabbay, M., Kelly, Z., and Gastil, J., "An Experimental Study of Persuasion and Decision Making in Small Networks," Midwestern Political Science Association conference, Chicago, IL, April 3, 2014.
16. Reedy, J., Gabbay, M., Kelly, Z., and Gastil, J., "An Experimental Study of Persuasion and Decision Making in Small Networks," 7th Annual Political Networks Conference, Montreal, May 31, 2014.
17. Kelly, Z., Gabbay, M., Reedy, J., and Gastil, J., "An Experimental Study of Persuasion, Confidence, and Choice Shift in Small Networks," Annual Meeting of the American Political Science Association, Washington, DC, Aug. 31, 2014.

18. Kelly, Z., "Decision Making in Terrorist Cells," seminar at Political Science Dept., Boise State University, Boise, ID, Dec. 5, 2014.
19. Gabbay, M. "Theoretical and Empirical Research on Decision Making and Cooperation in Terrorist Networks," ONR Future Force S&T Expo, Washington, DC, Feb. 5, 2015.
20. Kelly, Z., Zech, S., Gabbay, M., Thirkill-Mackelprang, A., "Modeling Insurgent Networks using Exponential Random Graphs and Targeting Policies," Midwestern Political Science Assoc. conference, Chicago, IL, April 2015.
21. Kelly, Z., Gabbay, M., Reedy, J., and Gastil, J., "Experimental Study of Persuasion, Decision Making, and Risky Shift in Small Networks," Midwestern Political Science Assoc. conference, Chicago, IL, April 2015.
22. Kelly, Z., Gabbay, M., Reedy, J., and Gastil, J., "Choice Shift in Small Networks," Political Networks Conference, Portland, OR, June 19, 2015.
23. Gade, E.K., Kelly, Z., and Gabbay, M., "Militant Networks and Violence in Syria," Political Networks Conference, Portland, OR, June 19, 2015.
24. Gastil, J., Richards, R.C., Reedy, J., Gabbay, M., Kelly, Z., "Deliberative and Cultural Orientations in Small Decision-Making Groups," National Communication Assoc. Annual Conference, Las Vegas, Nov. 2015.
25. Kelly, Z., Reedy, J., Gabbay, M., Gastil, J., "Persuasion Confidence, and Choice Shifts: An Experimental Study of Decision Making and Communication Structure in Small Groups," Natl. Comm. Assoc. Annual Conference, Las Vegas, Nov. 2015.

Applied Physics Laboratory
University of Washington

APPENDICES TO FINAL TECHNICAL REPORT

DTRA/ONR Grant HDTRA1-10-1-0075
Theoretical and Experimental Investigation of Opinion Dynamics in Small Social Networks

<u>Principal Investigator</u>
Dr. Michael Gabbay
Applied Physics Laboratory
University of Washington
1013 NE 40th Street
Seattle, WA 98105

<u>Period of Performance</u>: 14 MAY 2010 – 31 DEC 2015

Table of Contents

Appendix 1

Large Group Opinion Simulations

Large Group Nonlinear Opinion Simulations

Arindam Das
Applied Physics Laboratory, University of Washington

June 12, 2012

1 Overview of Influence Model

The nonlinear model we consider is due to Gabbay [3]:

$$\dot{x}_i = -\gamma_i(x_i - \mu_i) + \sum_{j=1}^{N} \kappa_{ij} [x_j - x_i] \exp\left[-\frac{(x_j - x_i)^2}{2\lambda_i^2}\right], \tag{1}$$

where μ_i is the *natural bias* of node i, λ_i is the *latitude of acceptance* of node i, γ_i is the *commitment* of node i, and κ_{ij} is the *coupling strength* (weight) of the directed link $j \to i$. The first expression on the r.h.s of (1) can be viewed as a *self-bias force* while the second expression can be interpreted as a *coupling force* which is the influence that node j exerts on node i because of a discrepancy between their positions. The smaller the latitude of acceptance of a node, the smaller is the impact of any state discrepancy with its neighbors, $(x_j - x_i)$, on itself. In this model, the commitments, γ_i, and the coupling strengths, κ_{ij}, are non-negative. Although we are free to set the self-coupling strengths to an arbitrary value without changing the dynamics, we demand that $\kappa_{ii} = 0$ in order to allow for extensions such as shading.

The coupling strength, κ_{ij}, can be considered to be the product of two factors, $\kappa_{ij} = v_{ij}\rho_{ij}$, where v_{ij} is the *communication rate* at which j sends persuasive messages to i, and ρ_{ij} is the *regard* of i for j that accounts for how susceptible i is to influence from j due to factors such as j's perceived credibility on the issue of concern. The ratio $\sum_j \kappa_{ij}/\gamma_i$ gives a measure of how responsive node i is to the influence of its neighbors as compared to its commitment to its initial bias. Now, assuming that $\rho_{ij} = \rho$ and $v_{ij} = v_0\mathbf{A}_{ij}$, we have

$$\kappa_{ij} = \rho(v_0\mathbf{A}_{ij}),$$

where $\mathbf{A}_{ij}$ is the $(i,j)^{th}$ element of the adjacency matrix $\mathbf{A}$ and is equal to 1 if nodes i and j share a link and 0 otherwise. This expression, however, is not in a form that places comparable network topologies on an equal footing, which is necessary in order to compare their efficacies in reducing discord. Reasoning that communications are costly, we will require that the average communication rate, $\bar{v} = \sum_{i,j} v_{ij}/N$, of different networks be equal. Noting that node i's degree, d_i, is $d_i = \sum_j \mathbf{A}_{ij}$, and that the mean degree is $\bar{d} = \sum_i d_i/N = \sum_{i,j} \mathbf{A}_{ij}/N$, we can write:

$$\bar{v} = \frac{\sum_{i,j} v_{ij}}{N} = v_0 \left(\frac{\sum_{i,j} \mathbf{A}_{ij}}{N}\right) = v_0\bar{d}.$$

The $(i,j)^{th}$ element of the coupling strength matrix can therefore be rewritten in terms of average communication rate as follows:

$$\kappa_{ij} = \rho(v_0 \mathbf{A}_{ij}) = \rho \left(\bar{v}/\bar{d}\right) \mathbf{A}_{ij}.$$

Subsequently, we refer to the parameter $\rho\bar{v}$ as the *coupling scale* and denote it by α. The coupling strength matrix (or simply, the weight matrix) is therefore given by:

$$\kappa = \frac{\alpha \mathbf{A}}{\bar{d}} \tag{2}$$

A discretized version of (1) can be obtained as follows by using the forward difference approximation $\dot{x} \approx \frac{x(k+1)-x(k)}{\Delta}$, where Δ is the time-step parameter:

$$x_i(k+1) - x_i(k) = \left[-\Delta\gamma_i\left(x_i(k) - \mu_i\right)\right] + \Delta \sum_{j=1}^{N} \kappa_{ij}\left[x_j(k) - x_i(k)\right]\ \exp\left[-\frac{\left(x_j(k) - x_i(k)\right)^2}{2\lambda_i^2}\right], k \geq 0,$$

which implies that:

$$x_i(k+1) = \left[(1-\Delta\gamma_i)x_i(k) + \Delta\gamma_i\mu_i\right] + \Delta \sum_{j=1}^{N} \kappa_{ij}\left[x_j(k) - x_i(k)\right]\ \exp\left[-\frac{\left(x_j(k) - x_i(k)\right)^2}{2\lambda_i^2}\right], k \geq 0. \tag{3}$$

If all nodes have infinite latitudes of acceptance ($\lambda_i \to \infty$), the model reduces to the linear form:

$$x_i(k+1) = \left[(1-\Delta\gamma_i)x_i(k) + \Delta\gamma_i\mu_i\right] + \Delta \sum_{j=1}^{N} \kappa_{ij}\left[x_j(k) - x_i(k)\right], k \geq 0. \tag{4}$$

In matrix-vector notation, Eqn. (4) can be written as:

$$\mathbf{x}(k+1) = \left[\mathbf{I} + \Delta(\kappa - \mathbf{D}_\gamma - \mathbf{D}_\kappa)\right]\mathbf{x}(k) + \Delta\mathbf{D}_\gamma\mu,\ \ k \geq 0, \tag{5}$$

where:

- $\mathbf{I}$ is the identity matrix,
- $\kappa = [\kappa_{ij} : (i,j) = 1, 2, \ldots N]$ is the coupling strength matrix,
- $\mathbf{D}_\gamma := diag(\gamma_1, \gamma_2, \ldots \gamma_N)$ is a diagonal matrix of node commitments,
- $\mathbf{D}_\kappa := diag\left(\sum_{j=1}^{N}\kappa_{1,j},\ \sum_{j=1}^{N}\kappa_{2,j}, \ldots\ \sum_{j=1}^{N}\kappa_{N,j}\right)$ is a diagonal matrix of the row sums of the coupling matrix, and
- $\mu = [\mu_1, \mu_2, \ldots \mu_N]^T$ is the natural bias vector.

2 Simulation results

In this section, we present simulation results for networks of size $N = 100$. We consider two different distributions of the initial node states - (a) uniformly spaced in the range $[-\frac{\Delta_\mu}{2}, +\frac{\Delta_\mu}{2}]$, and (b) Gaussian distributed with zero mean and variance σ^2. But first, a couple of definitions are in order.

Definition 1 *Given a set of state values $\mathbf{x} = x_1, x_2, \cdots x_N$, the* ***maximum discord****, e, is defined as:*

$$e = max(\mathbf{x}) - min(\mathbf{x}).$$

Definition 2 *Given a set of state values $\mathbf{x} = x_1, x_2, \cdots x_N$, let $\mathbf{x}^+$ and $\mathbf{x}^-$ denote the sets of nonnegative and negative state values. That is, $\mathbf{x}^+$ is the set of all state values which are greater than or equal to zero and $\mathbf{x}^-$ is the set of all state values which are strictly less than zero. Then, the* ***polarization index****, p, is defined as follows:*

$$p = \frac{\mid mean(\mathbf{x}^+) - mean(\mathbf{x}^-) \mid}{\sqrt{var(\mathbf{x}^+) + var(\mathbf{x}^+)}},$$

where 'mean' and 'var' denote the statistical mean and standard deviation respectively.

We end this section with a discussion of the clustering algorithm we have used to determine the number of clusters from a set of final state values. Let $\pi_1, \pi_2, \cdots \pi_N$ denote an ordered set of state values, sorted in an ascending order, and let $a_1, a_2, \cdots a_N$ denote the corresponding node labels. That is, a_1 is the label of the node with the smallest state value, π_1, and a_N is the label of the node with the highest state value, π_N. Initially, we set the number of clusters to 1 and allocate node a_1 to that cluster (say $\mathcal{C}_1$). Next, we examine π_2. Assuming that the latitudes of acceptance of all nodes are equal to λ, we assign node a_2 to $\mathcal{C}_1$ if the condition $\pi_2 - \pi_1 \leq \lambda$ is satisfied[1]. Otherwise, we increment the number of clusters by one and assign node a_2 to a new cluster $\mathcal{C}_2$. This procedure is repeated until all the nodes have been assigned to a cluster.

2.1 Network generation procedure and weight matrix computation

Our network generation procedure is a modified version of a k-regular graph, also known as an (n, k) regular lattice, where k is the regularity parameter and n is the number of nodes. A k-regular graph is a regular graph where the degree of each node is equal to k (we assume that k is even). For example, Fig. 1a shows a 4-regular graph on 10 nodes with state values evenly spaced between $+5$ and -5.

Generally speaking, our networks are generated based on the initial state values and can be considered to be modified k-regular graphs. The modification is necessary since it is not tenable that agents at the positive and negative ends of the opinion spectrum share a link. Referring to Fig. 1a, it is highly unlikely that the agents with state values $+5$ and -5 will be communicating with each other. The easiest way to visualize our modification is by representing the nodes (in an ordered manner) on a ring and snapping the link between the nodes at positive and negative extremes. Now, if we imagine an imaginary barrier between these two extreme nodes, links can be

[1]For arbitrary latitudes of acceptance, this condition can be replaced by $\pi_j - \pi_i \leq min(\lambda_j, \lambda_i)$ where λ_i and λ_j are the latitudes of acceptance of nodes i and j respectively.

added (in the usual way as for k-regular graphs) with the proviso that a link cannot be added if it entails crossing the virtual barrier. Fig. 1b shows a modified 4-regular graph on 10 nodes with state values evenly spaced between $+5$ and -5. Note that, strictly speaking, the network is not a regular graph and therefore usage of the term 'modified k-regular' is a misnomer. However, we use this term given the general resemblance of the networks generated to k-regular graphs.

For a more mathematical description of the network generation procedure, we invoke the nomenclature system indicated in the previous sub-section during our discussion of the clustering algorithm. That is, let $\pi_1, \pi_2, \cdots \pi_N$ denote an ordered set of initial state values, sorted in an ascending order, and let $a_1, a_2, \cdots a_N$ denote the corresponding node labels. A family of graphs with different edge densities can be generated by varying the parameter k (pertaining to k-regularity). For a given node a_q, let a_r belong to the set of nodes such that $a_r > a_q$ and let a_p belong to the set of nodes such that $a_p < a_q$. Also, let d_{pq} denote the number of hops (*hop distance*) between nodes p and q based on the sorted initial state vector. For example, referring to Fig. 1b, the hop distance between the node with state value $+5$ and those with state values -5, -1.67 and $+1.67$ are 9, 6 and 3 respectively (traveling along the ring). Then, for a given k (assumed even for $k < N-1$, or equal to $N-1$ in which case we have a complete graph), a link is added between nodes a_q and a_r if:

$$d_{qr} \leq k/2 \quad \textbf{and} \quad a_r \leq a_N. \tag{6}$$

Similarly, a link is added between nodes a_p and a_q if:

$$d_{pq} \leq k/2 \quad \textbf{and} \quad a_p \geq a_1. \tag{7}$$

Note that the conditions $a_r \leq a_N$ and $a_p \geq a_1$ jointly represent the stipulation that the virtual barrier between the two extreme nodes not be crossed during the link addition process.

After the graph has been generated, the coupling strength (weight) matrix is computed according to eqn. (2). For $k < N-1$ and even, straightforward enumeration shows that the total number of edges that will be added during the network generation process is given by (cf. Fig. 2):

$$\begin{aligned}
&(N-k)k + 2\left[(k-1) + (k-2) + \cdots + (k/2)\right] \\
=&(N-k)k + 2\left[\left(\frac{k}{2}+\frac{k}{2}-1\right) + \left(\frac{k}{2}+\frac{k}{2}-2\right) + \cdots + \left(\frac{k}{2}+1\right) + \left(\frac{k}{2}\right)\right] \\
=&(N-k)k + 2\left[\left(\frac{k}{2}\right)^2 + \left(1+2+\cdots+\left(\frac{k}{2}-1\right)\right)\right] \\
=&(N-k)k + \left[\frac{k^2}{2} + \left(\frac{k}{2}\right)\left(\frac{k}{2}-1\right)\right] \\
=&(N-k)k + \left[\frac{3k^2}{4} - \frac{k}{2}\right] \\
=&k\left[N - \frac{1}{4}(k+2)\right]
\end{aligned} \tag{8}$$

Note that the reduction from line 2 to line 3 in the above derivation is based on the fact that there are $k/2$ terms in the expression within the square brackets. The average node degree, $\bar{d}$, is therefore

given by:

$$\bar{d} = \left(\frac{k}{N}\right)\left[N - \frac{1}{4}(k+2)\right] \tag{9}$$

Fig. 3 shows the plots of $\bar{d}$ vs. N for four different values of the regularity parameter k.

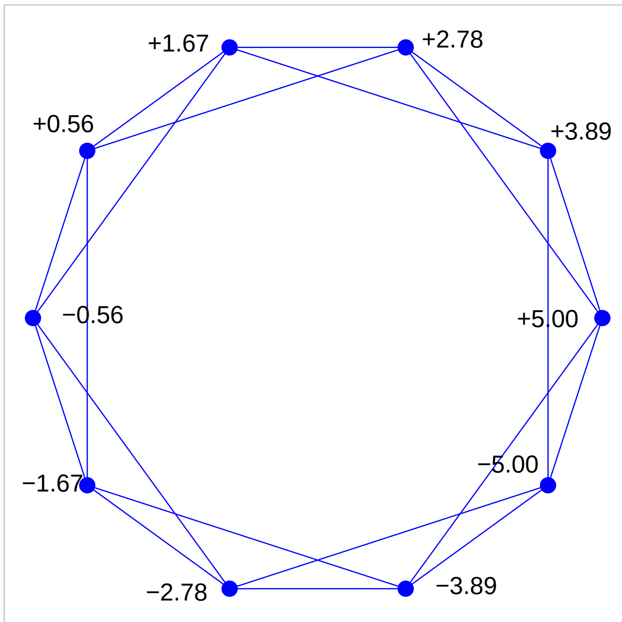

Figure 1a: A k-regular graph over 10 nodes, $k = 4$.

2.2 Simulation parameter values

Unless explicitly noted otherwise, all simulation results presented in the subsequent subsections are based on the following parameters:

1. Discretization time step parameter $\Delta = 0.01$.
2. Latitudes of acceptance of all nodes $\lambda_i = 1$, for all $i \in \mathcal{N}$, where $\mathcal{N}$ is the set of all nodes.
3. Commitments of all nodes $\gamma_i = 1$, for all $i \in \mathcal{N}$.
4. Natural biases of all nodes $\mu_i = x_i(0)$, for all $i \in \mathcal{N}$, where $x_i(0)$ is the initial state of node i.

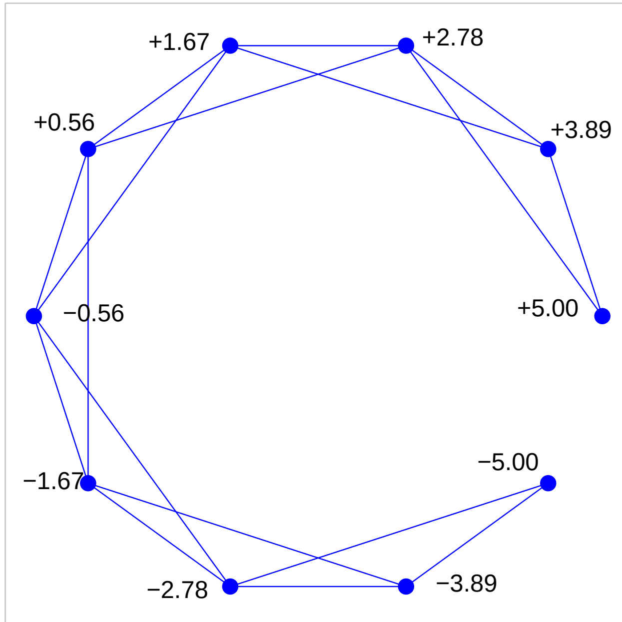

Figure 1b: A modified k-regular graph over 10 nodes, $k = 4$.

2.3 $N = 100$, $\Delta_\mu = 10$, Linearly spaced initial states

Figs. 4a-4c show the maximum discord, polarization index and number of clusters (all computed from the final state values) as a function of edge density, averaged over 100 trials (with 5000 iterations for each trial), for $N = 100$, $\Delta_\mu = 10$ and linearly spaced initial states. From the plots of discord and polarization index vs. edge density, we can observe a transition' effect which occurs between the edge densities 0.3 and 0.4. This transition coincides with the emergence of an additional cluster, as is evident from the plot of no. of clusters vs. edge density. Another point to note is that the transition point occurs at a slighter higher edge density for larger coupling scales, presumably because of stronger affinities between nodes which act to postpone the emergence of polarization.

The emergence of additional clustering (or polarization of opinion) at the transition point is further illustrated in Figs. 5a-5d and Figs. 6a-6d, which correspond to edge densities of 0.295758 and 0.312525 respectively from a typical sample run with coupling scale $\alpha = 20$. For each of these edge densities, we have shown the temporal state evolutions, a superimposed plot and individual histograms of initial and final state values. From Fig. 5a, we can observe that the solution for edge density 0.295758 (at the transition point) can be classified as high discord but symmetric, while for edge density 0.312525 (immediately after the transition), the solution clearly indicates a polarization of opinion (see Fig. 6a).

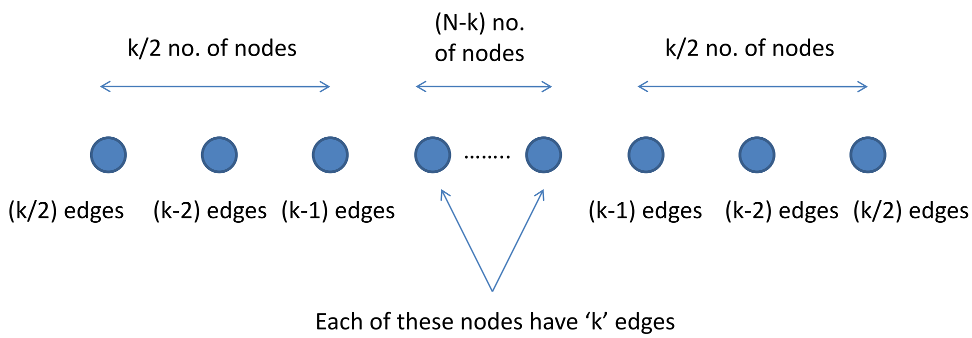

Figure 2: Illustration for determining the total number of edges in a modified k-regular graph.

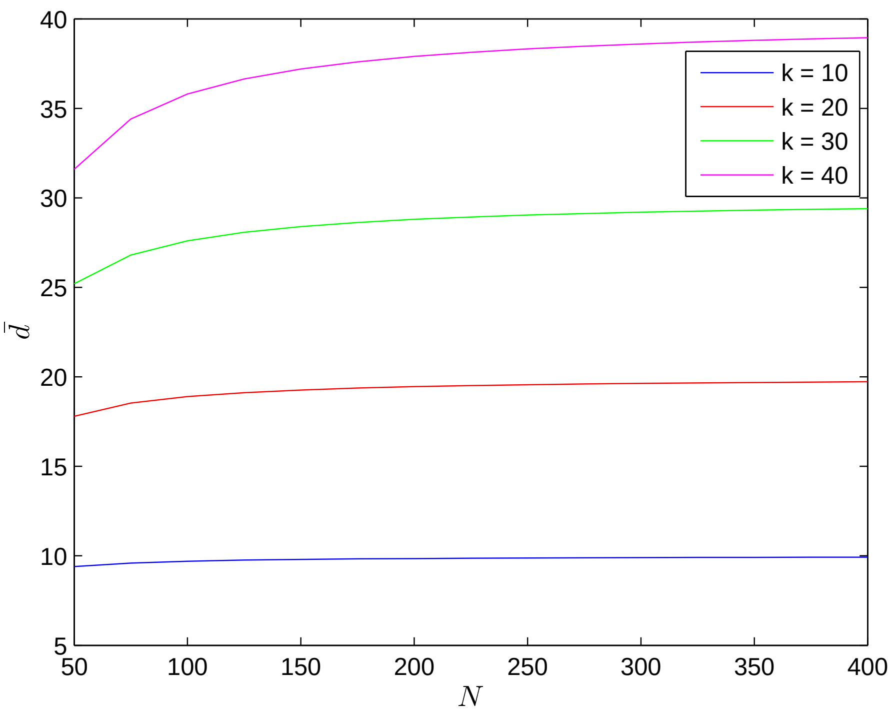

Figure 3: Plot of mean node degree vs. N, for four different values of k (regularity parameter).

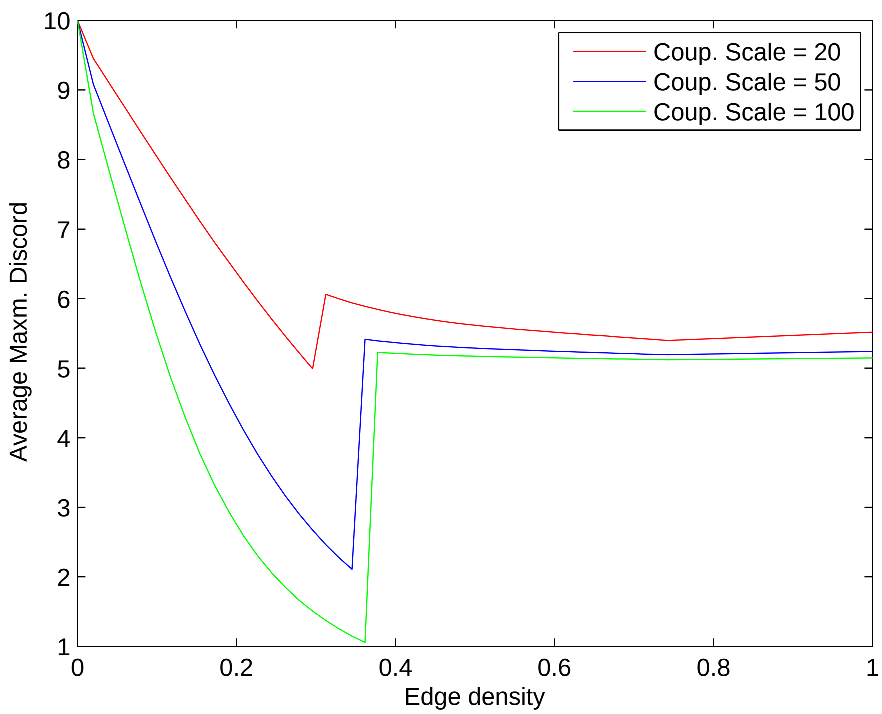

Figure 4a: Plot of maximum discord as a function of edge density, averaged over 100 trials, for linearly distributed initial states.

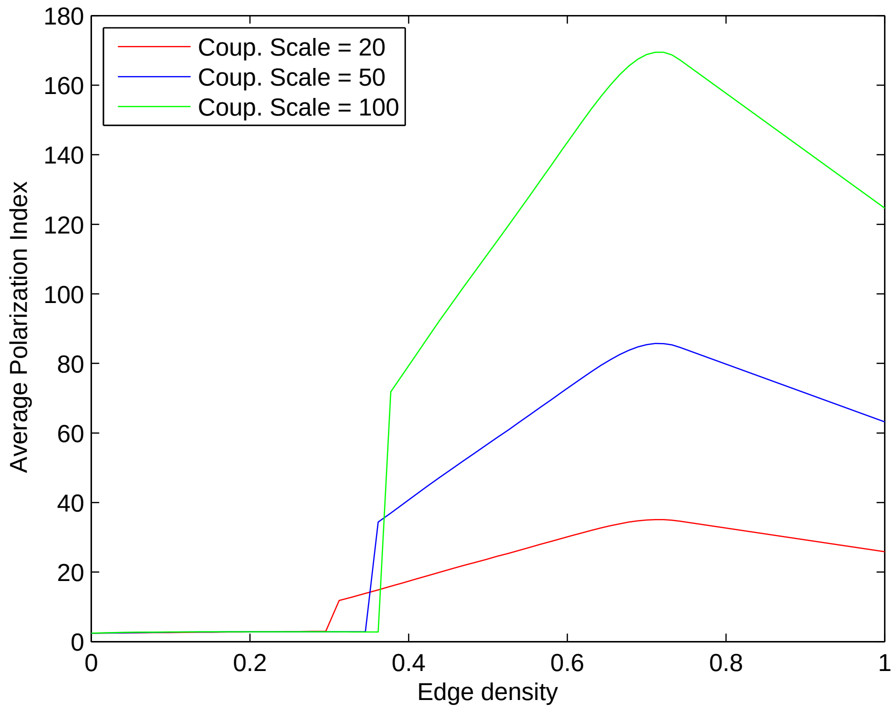

Figure 4b: Plot of polarization index as a function of edge density, averaged over 100 trials, for linearly distributed initial states.

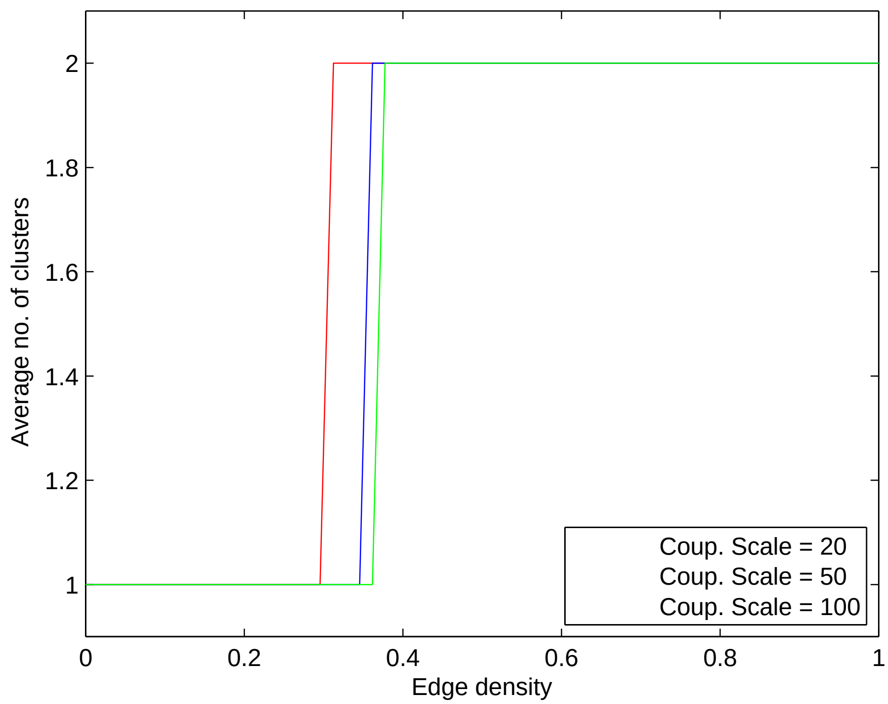

Figure 4c: Plot of number of clusters as a function of edge density, averaged over 100 trials, for linearly distributed initial states.

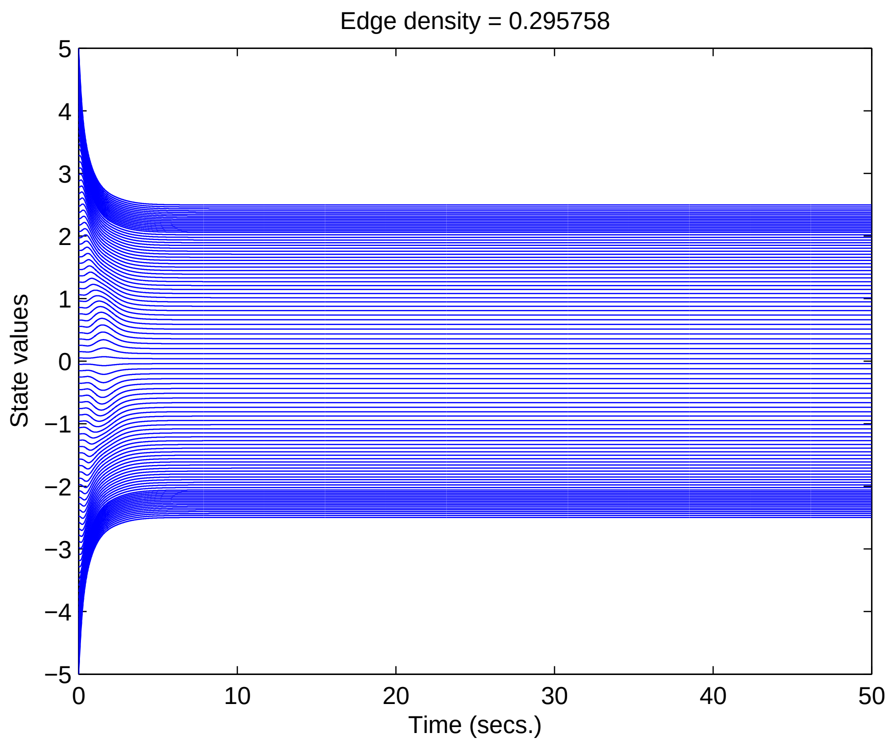

Figure 5a: Plot of temporal state evolution from a sample run for edge density = 0.295758.

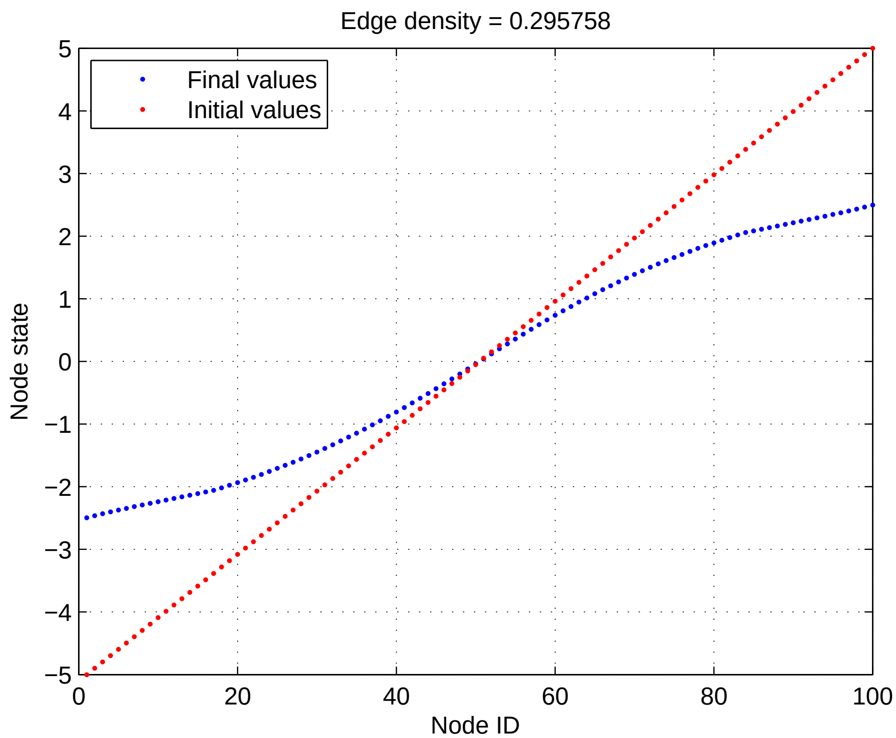

Figure 5b: Plot of initial and final state values from a sample run for edge density = 0.295758.

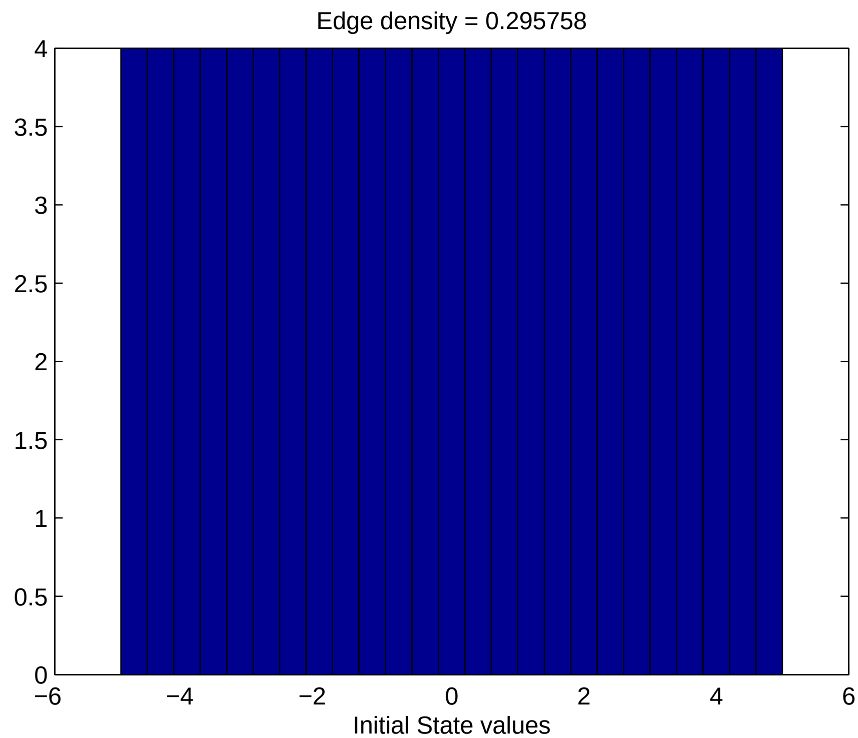

Figure 5c: Histogram of initial state values from a sample run for edge density = 0.295758.

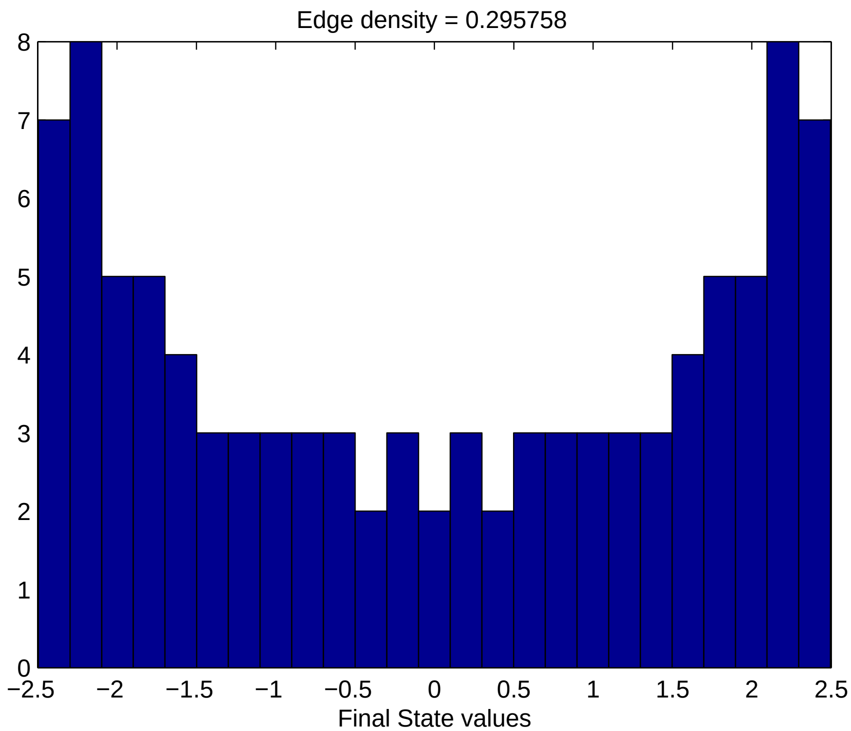

Figure 5d: Histogram of final state values from a sample run for edge density = 0.295758.

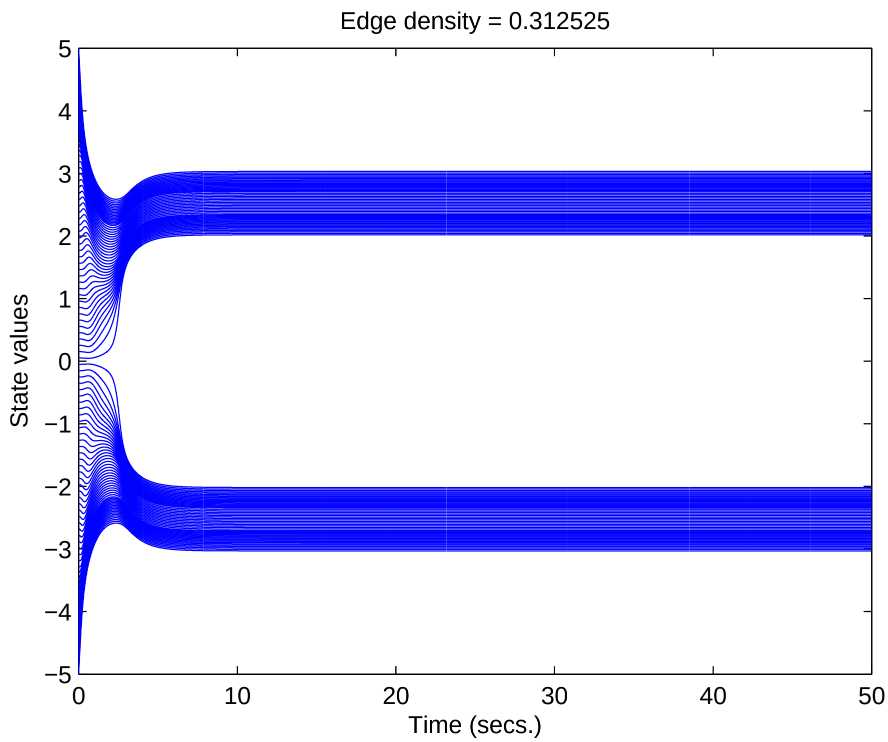

Figure 6a: Plot of temporal state evolution from a sample run for edge density = 0.312525.

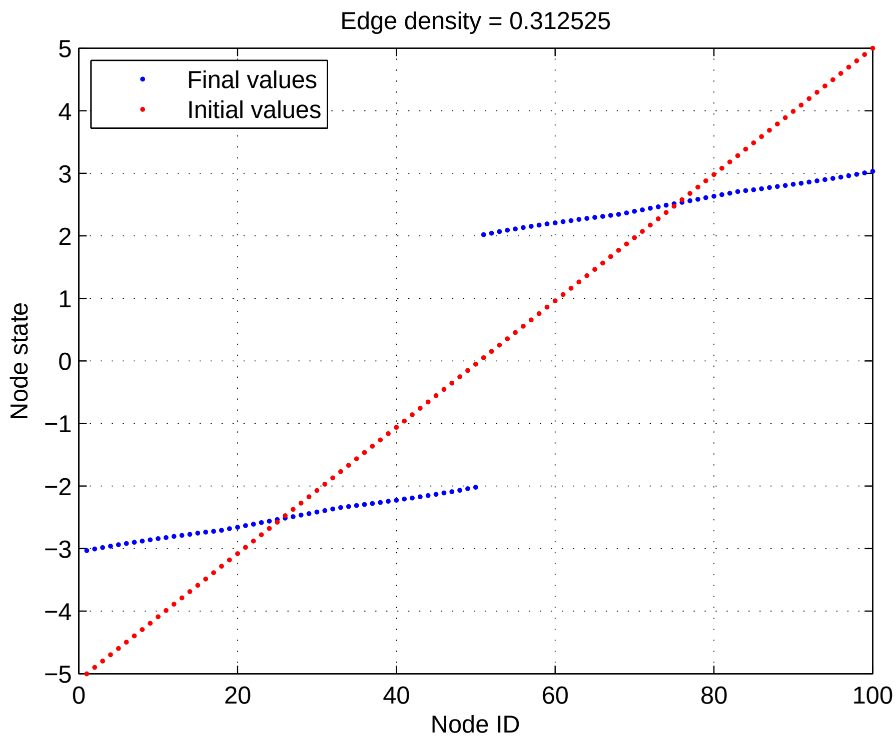

Figure 6b: Plot of initial and final state values from a sample run for edge density = 0.312525.

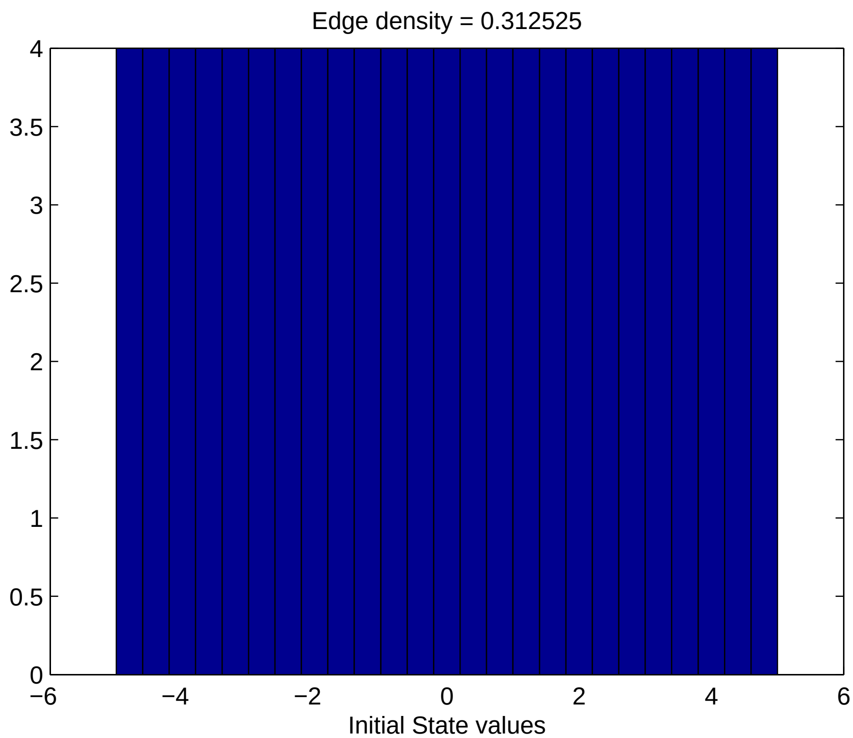

Figure 6c: Histogram of initial state values from a sample run for edge density = 0.312525.

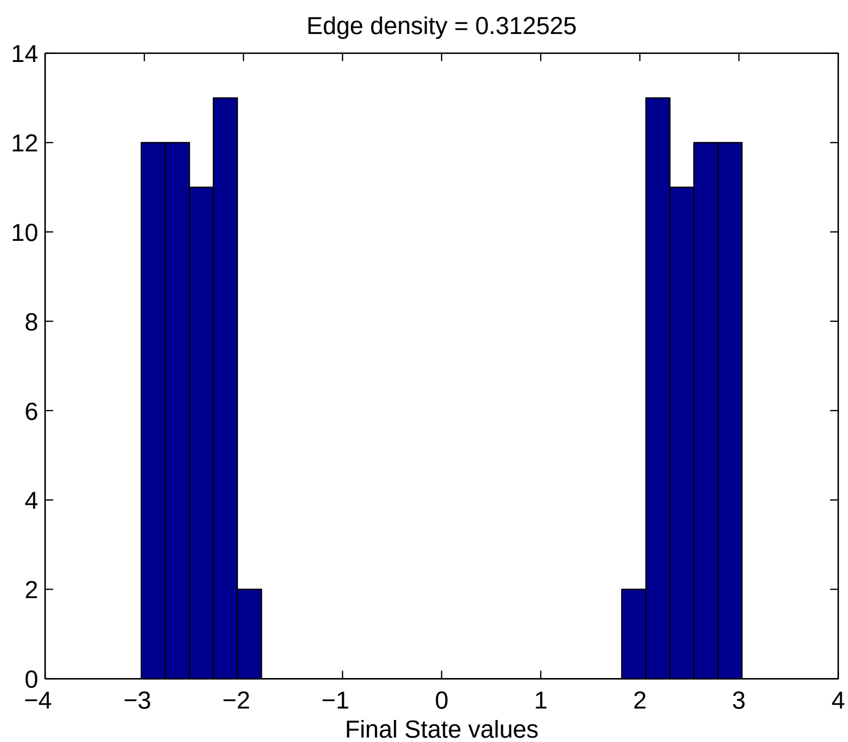

Figure 6d: Histogram of final state values from a sample run for edge density = 0.312525.

2.4 $N = 100$, $\sigma = 2.5$, Gaussian distributed initial states

Figs. 7a-7c show the maximum discord, polarization index and number of clusters (computed from the final state values) as a function of edge density, averaged over 100 trials (5000 iterations for each trial), for $N = 100$ and Gaussian distributed (zero mean and standard deviation = 2.5) initial states. From the plots of discord and polarization index vs. edge density, we can observe a similar transition' effect which occurs between the edge densities 0.1 and 0.2. However, compared to the linear case, the trough is generally shallower, but gets more pronounced with increasing values of coupling scale. This transition generally coincides with the emergence of an additional cluster (both positive and negative groups tend to develop beyond this transition point).

This emergence of additional clustering (or bifurcation of opinion) at the transition point is further illustrated in Figs. 8a-8d and Figs. 9a-9d, which correspond to edge densities of 0.172727 and 0.190909 respectively from a typical sample run with coupling scale $\alpha = 100$. For each of these edge densities, we have shown the temporal state evolutions, a superimposed plot and individual histograms of initial and final state values. From Fig. 8a, we can observe that the solution for edge density 0.172727 (at the transition point) can be classified as heavily neutral with a splinter negative group, while for edge density 0.190909 (immediately after the transition), the solution clearly indicates a bifurcation of opinion into positive and negative groups, along with a heavily neutral majority (see Fig. 9a).

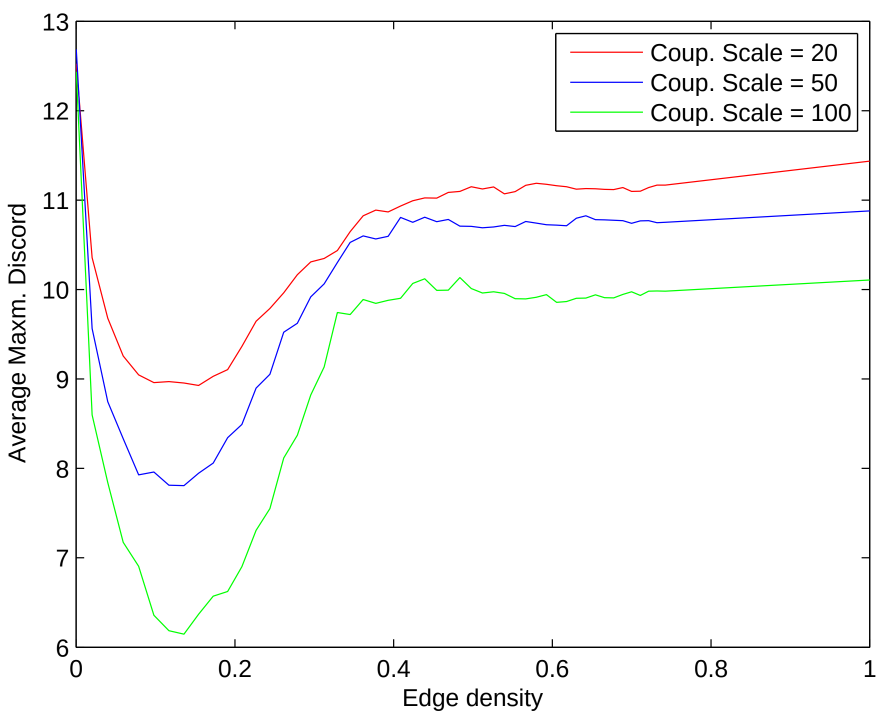

Figure 7a: Plot of maximum discord as a function of edge density, averaged over 100 trials, for Gaussian distributed initial states.

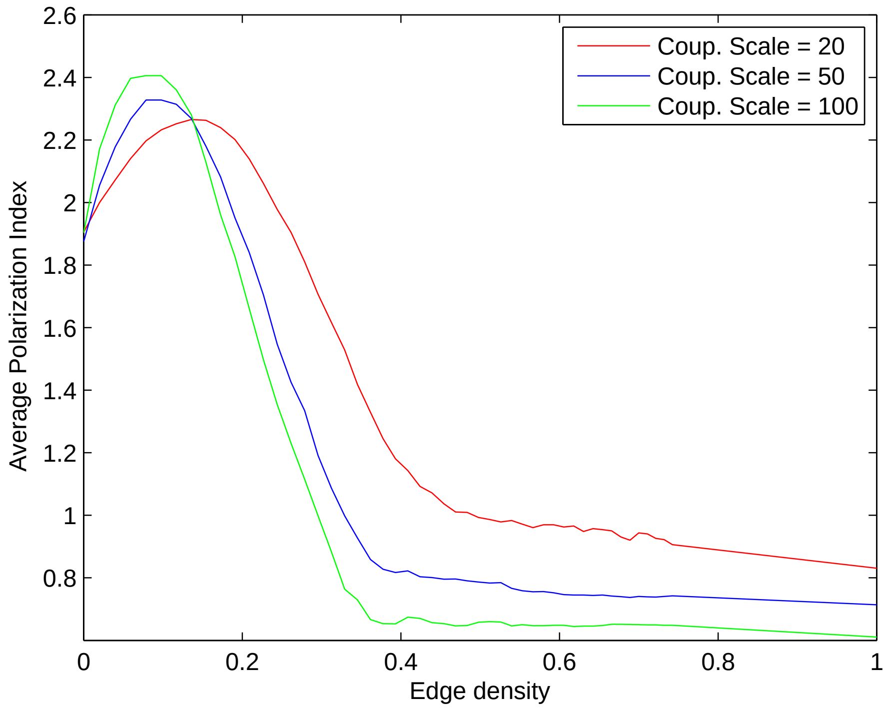

Figure 7b: Plot of polarization index as a function of edge density, averaged over 100 trials, for Gaussian distributed initial states.

References

[1] Noah E. Friedkin, "Norm formation in social influence networks", *Social Networks*, 23 (2001), pp. 167-189.

[2] Noah E. Friedkin, "A structural theory of social influence", *Cambridge University Press*, Cambridge.

[3] Michael Gabbay, "The effects of nonlinear interactions and network structure in small group opinion dynamics", *Physica A*, 378 (2007), pp. 118-126.

[4] Michael Gabbay, "*A dynamical systems model of small group decision making*", in Diplomacy Games: Formal Models and International Negotiations, R. Avenhaus and I.W. Zartman (eds.), Springer, New York, 2007.

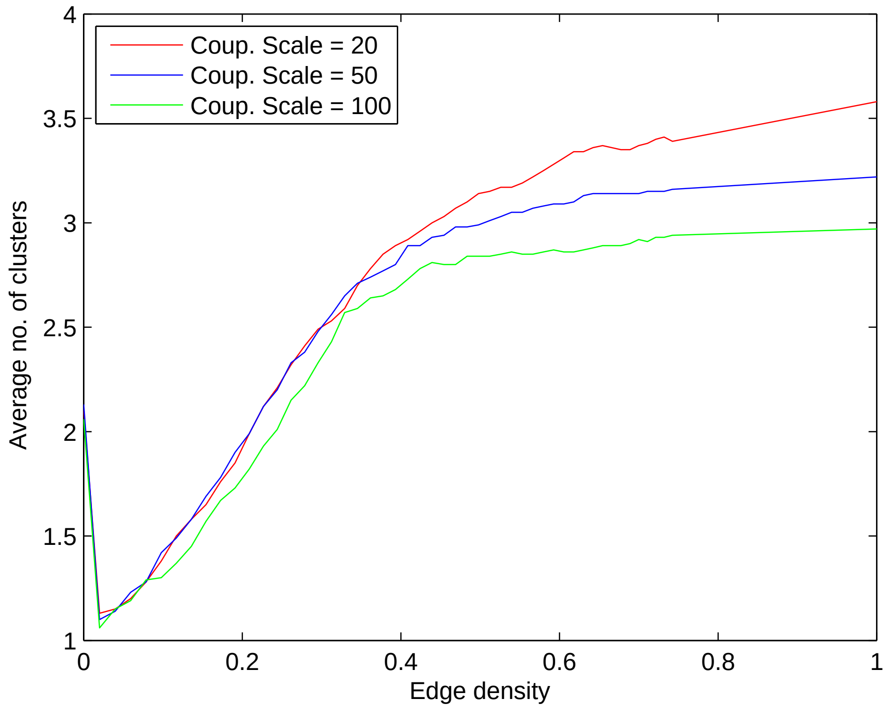

Figure 7c: Plot of number of clusters as a function of edge density, averaged over 100 trials, for Gaussian distributed initial states.

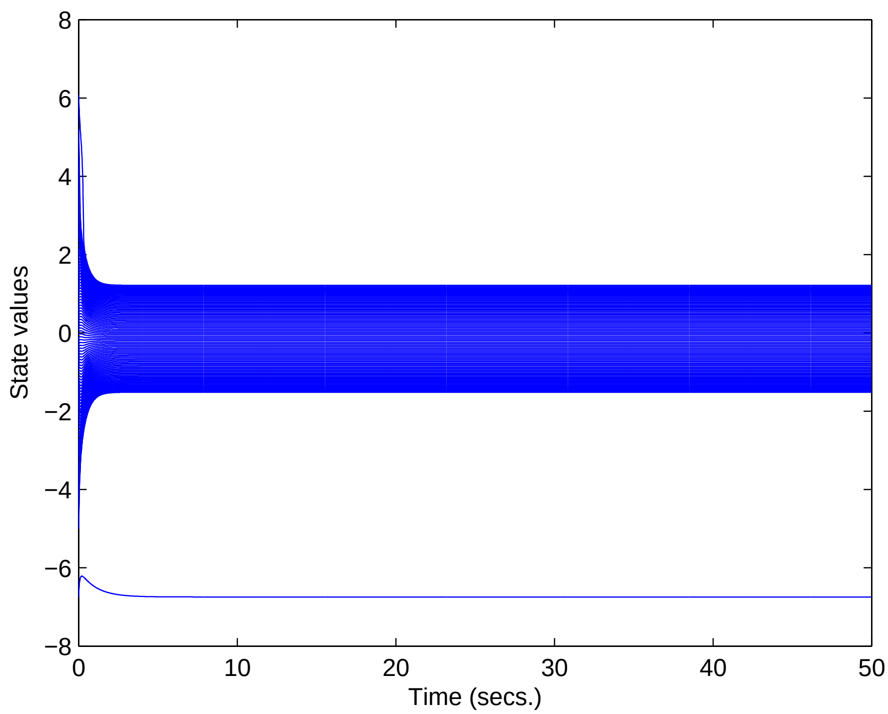

Figure 8a: Plot of temporal state evolution from a sample run for edge density = 0.172727.

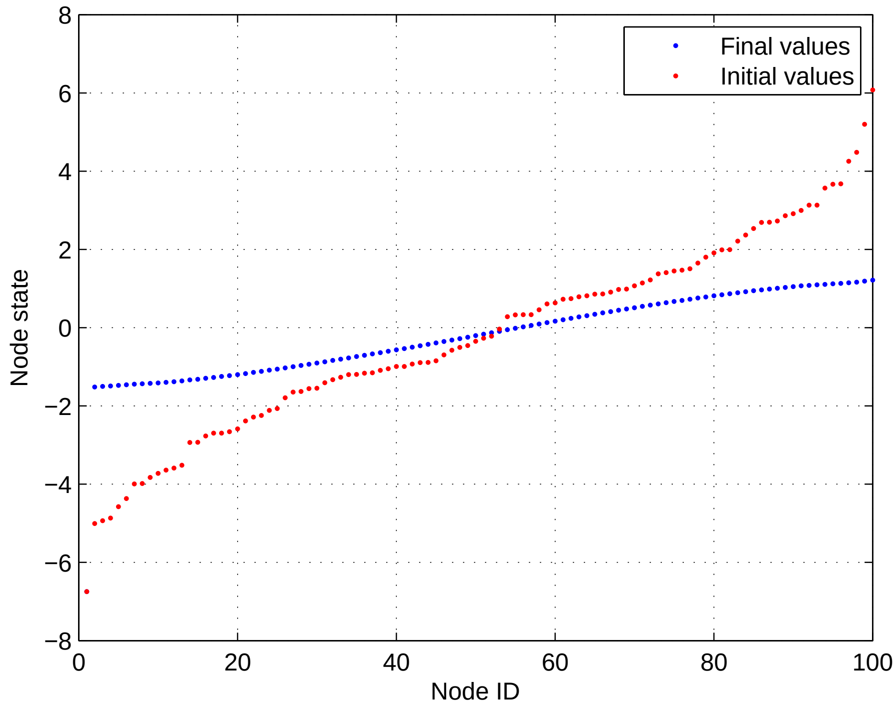

Figure 8b: Plot of initial and final state values from a sample run for edge density = 0.172727.

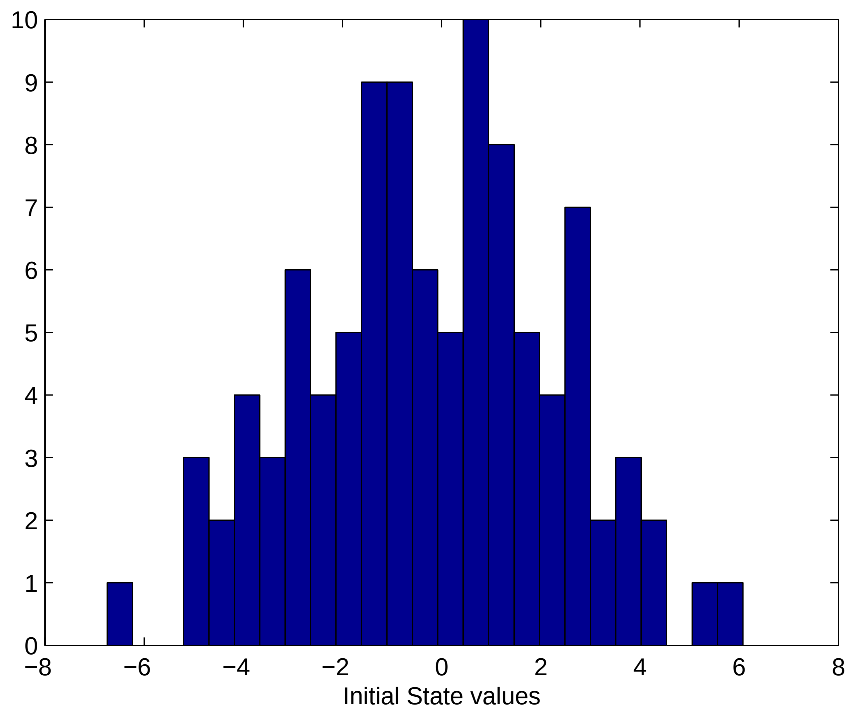

Figure 8c: Histogram of initial state values from a sample run for edge density = 0.172727.

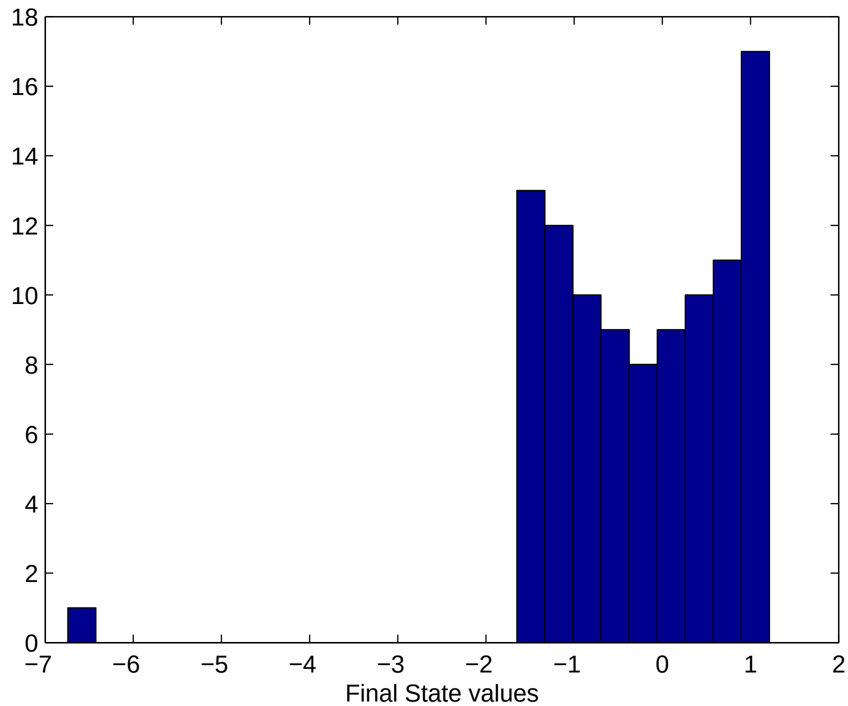

Figure 8d: Histogram of final state values from a sample run for edge density = 0.172727.

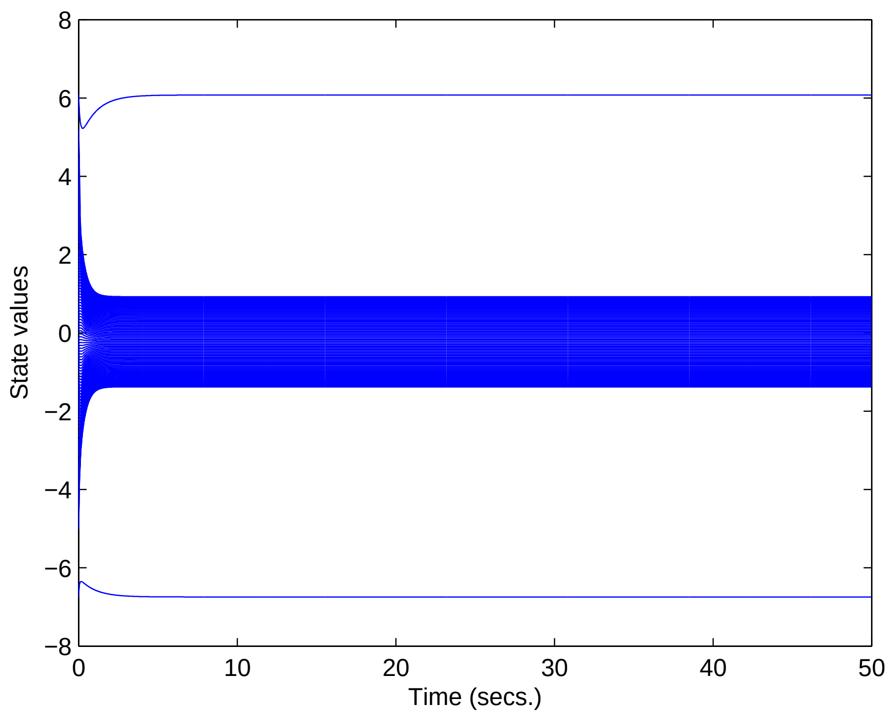

Figure 9a: Plot of temporal state evolution from a sample run for edge density = 0.190909.

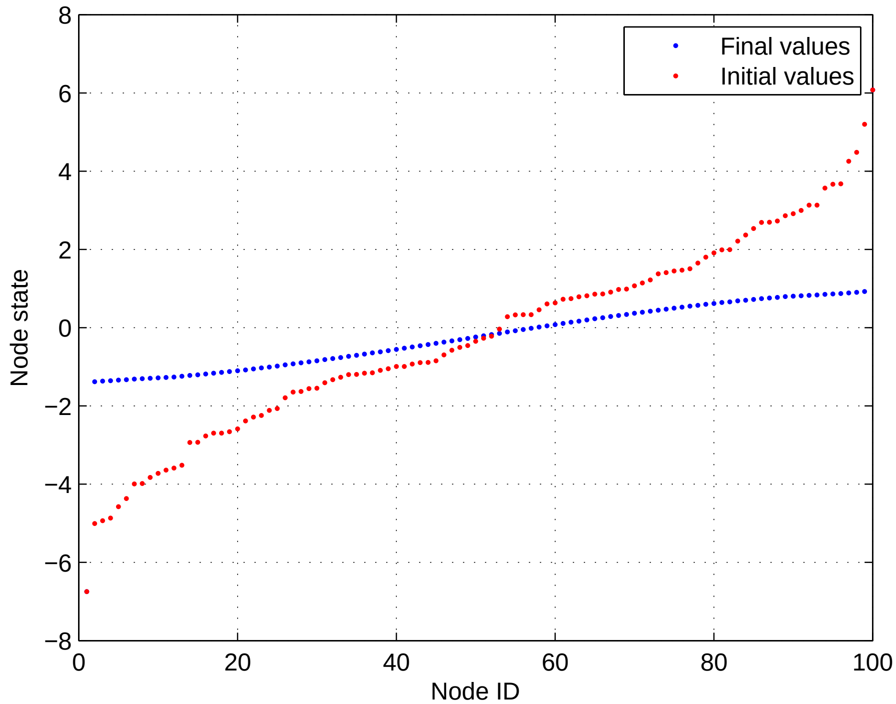

Figure 9b: Plot of initial and final state values from a sample run for edge density = 0.190909.

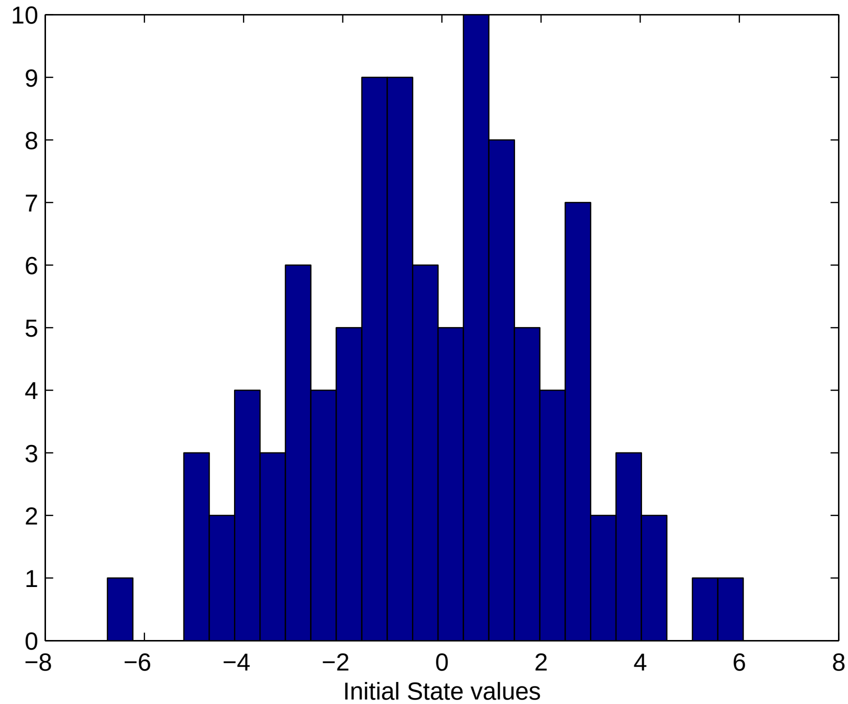

Figure 9c: Histogram of initial state values from a sample run for edge density = 0.190909.

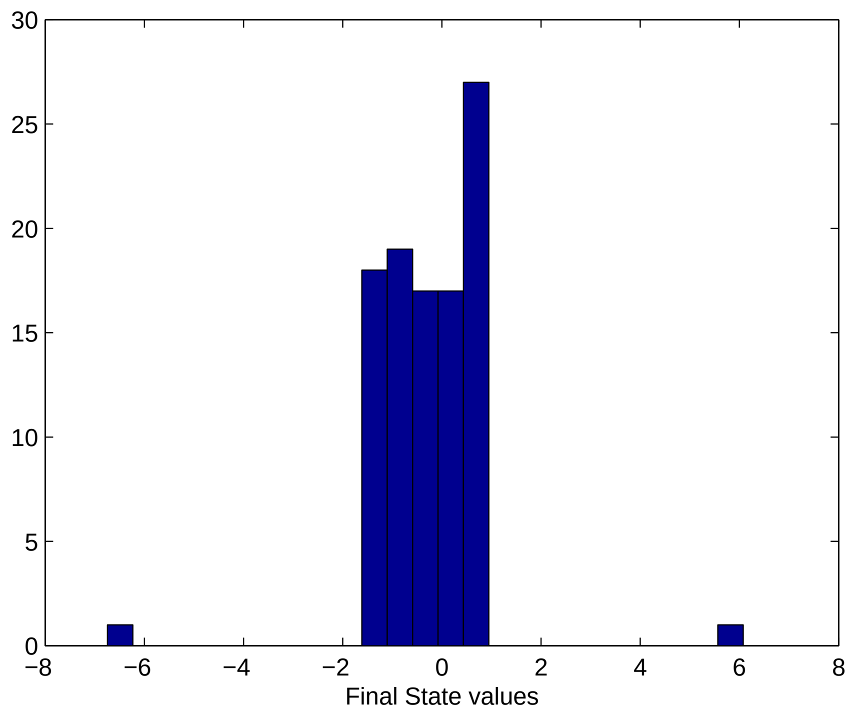

Figure 9d: Histogram of final state values from a sample run for edge density = 0.190909.

Appendix 2

Simulation of Asynchronous Communications in Nonlinear Opinion Dynamics

Simulation of Asynchronous Communications in Nonlinear Opinion Dynamics

Arindam Das
Applied Physics Laboratory, University of Washington

July 12, 2012

1 Overview of Influence Model

The nonlinear model we consider is due to Gabbay [3]:

$$\dot{x}_i = -\gamma_i(x_i - \mu_i) + \sum_{j=1}^{N} \kappa_{ij} [x_j - x_i] \exp\left[-\frac{(x_j - x_i)^2}{2\lambda_i^2}\right], \tag{1}$$

where μ_i is the *natural bias* of node i, λ_i is the *latitude of acceptance* of node i, γ_i is the *commitment* of node i, and κ_{ij} is the *coupling strength* (weight) of the directed link $j \to i$. The first expression on the r.h.s of (1) can be viewed as a *self-bias force* while the second expression can be interpreted as a *coupling force* which is the influence that node j exerts on node i because of a discrepancy between their positions. The smaller the latitude of acceptance of a node, the smaller is the impact of any state discrepancy with its neighbors, $(x_j - x_i)$, on itself. In this model, the commitments, γ_i, and the coupling strengths, κ_{ij}, are non-negative. Although we are free to set the self-coupling strengths to an arbitrary value without changing the dynamics, we demand that $\kappa_{ii} = 0$ in order to allow for extensions such as shading.

The coupling strength, κ_{ij}, can be considered to be the product of two factors, $\kappa_{ij} = v_{ij}\rho_{ij}$, where v_{ij} is the *communication rate* at which j sends persuasive messages to i, and ρ_{ij} is the *regard* of i for j that accounts for how susceptible i is to influence from j due to factors such as j's perceived credibility on the issue of concern. The ratio $\sum_j \kappa_{ij}/\gamma_i$ gives a measure of how responsive node i is to the influence of its neighbors as compared to its commitment to its initial bias. Now, assuming that $\rho_{ij} = \rho$ and $v_{ij} = v_0\mathbf{A}_{ij}$, we have

$$\kappa_{ij} = \rho(v_0\mathbf{A}_{ij}),$$

where $\mathbf{A}_{ij}$ is the $(i,j)^{th}$ element of the adjacency matrix $\mathbf{A}$ and is equal to 1 if nodes i and j share a link and 0 otherwise. This expression, however, is not in a form that places comparable network topologies on an equal footing, which is necessary in order to compare their efficacies in reducing discord. Reasoning that communications are costly, we will require that the average communication rate, $\bar{v} = \sum_{i,j} v_{ij}/N$, of different networks be equal. Noting that node i's degree, d_i, is $d_i = \sum_j \mathbf{A}_{ij}$, and that the mean degree is $\bar{d} = \sum_i d_i/N = \sum_{i,j} \mathbf{A}_{ij}/N$, we can write:

$$\bar{v} = \frac{\sum_{i,j} v_{ij}}{N} = v_0 \left(\frac{\sum_{i,j} \mathbf{A}_{ij}}{N}\right) = v_0\bar{d}.$$

The $(i,j)^{th}$ element of the coupling strength matrix can therefore be rewritten in terms of average communication rate as follows:

$$\kappa_{ij} = \rho(v_0 \mathbf{A}_{ij}) = \rho \left(\bar{v}/\bar{d}\right) \mathbf{A}_{ij}. \tag{2}$$

Subsequently, we refer to the parameter $\rho\bar{v}$ as the *coupling scale* and denote it by α. The coupling strength matrix (or simply, the weight matrix) is therefore given by:

$$\kappa = \frac{\alpha \mathbf{A}}{\bar{d}} \tag{3}$$

For 3-node broker and clique graphs as shown in Fig. 1, the average node degrees are 4/3 and 2 respectively.

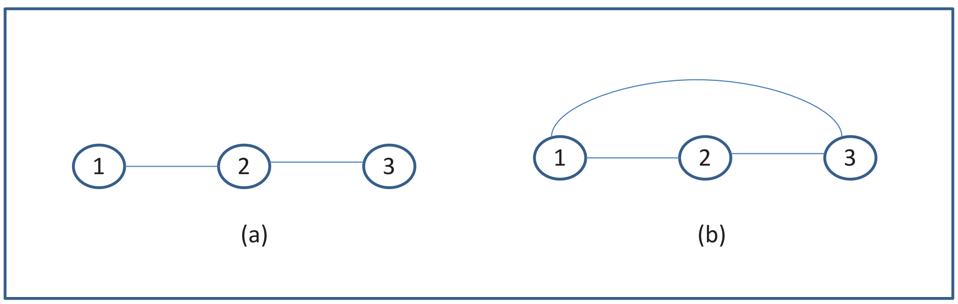

Figure 1: 3-node (a) broker and (b) clique graphs. We refer to the central node in the broker configuration as the *broker node.*

A discretized version of (1) can be obtained as follows by using the forward difference approximation $\dot{x} \approx \frac{x(k+1)-x(k)}{\Delta}$, where Δ is the time-step parameter:

$$x_i(k+1) - x_i(k) = \left[-\Delta\gamma_i \left(x_i(k) - \mu_i\right)\right] + \Delta \sum_{j=1}^{N} \kappa_{ij} \left[x_j(k) - x_i(k)\right] \ \exp\left[-\frac{\left(x_j(k) - x_i(k)\right)^2}{2\lambda_i^2}\right], k \geq 0,$$

which implies that:

$$x_i(k+1) = [(1-\Delta\gamma_i)x_i(k) + \Delta\gamma_i\mu_i] + \Delta\sum_{j=1}^{N}\kappa_{ij}\left[x_j(k) - x_i(k)\right]\ \exp\left[-\frac{(x_j(k)-x_i(k))^2}{2\lambda_i^2}\right], k \geq 0. \tag{4}$$

If all nodes have infinite latitudes of acceptance ($\lambda_i \to \infty$), the model reduces to the linear form:

$$x_i(k+1) = [(1-\Delta\gamma_i)x_i(k) + \Delta\gamma_i\mu_i] + \Delta\sum_{j=1}^{N}\kappa_{ij}\left[x_j(k) - x_i(k)\right], k \geq 0. \tag{5}$$

In matrix-vector notation, Eqn. (5) can be written as:

$$\mathbf{x}(k+1) = [\mathbf{I} + \Delta(\kappa - \mathbf{D}_\gamma - \mathbf{D}_\kappa)]\,\mathbf{x}(k) + \Delta\mathbf{D}_\gamma\mu,\ \ k \geq 0, \tag{6}$$

where:

- $\mathbf{I}$ is the identity matrix,
- $\kappa = [\kappa_{ij} : (i,j) = 1, 2, \ldots N]$ is the coupling strength matrix,
- $\mathbf{D}_\gamma := diag(\gamma_1, \gamma_2, \ldots \gamma_N)$ is a diagonal matrix of node commitments,
- $\mathbf{D}_\kappa := diag\left(\sum_{j=1}^{N}\kappa_{1,j},\ \sum_{j=1}^{N}\kappa_{2,j}, \ldots\ \sum_{j=1}^{N}\kappa_{N,j}\right)$ is a diagonal matrix of the row sums of the coupling matrix, and
- $\mu = [\mu_1, \mu_2, \ldots \mu_N]^T$ is the natural bias vector.

2 Asynchronous communications

Implicit in the eqns. 1 and 4 is an assumption that all nodes update their states synchronously. However, examples abound where synchronous updates are not possible; e.g., if information exchanges are carried out over a shared wireless channel, communications may need to be staggered to minimize interference between simultaneous transmissions. A similar scenario may arise when modeling jury deliberations or round robin discussions. Various models of asynchrony are possible - the one we adopt in this paper is a *slotted time sequential model* where time is divided into contiguous equal (may also be unequal) sized slots of arbitrary duration and only one user is permitted to speak during a slot. Generally speaking, N slots (where N is the number of nodes/users) will be required to accommodate all users if all are permitted to speak. The number of slots required to accommodate all speakers is called a *frame*. Communications/discussions may be limited to one frame or multiple frames, as shown in Fig. 2. For multiple frame communications, it is not necessary that the communication sequence be identical for all frames. For any given slot k, only those users which share an edge with the speaker update their states in accordance with eqn. 1. However, the behavior of the speaker and all users that do not share an edge with it is different. Specifically, the update model for these nodes includes only the first term on the r.h.s of eqn. 1. Let $\mathbf{x}^k$ denote the state vector at the end of slot k, T^k the time duration of slot k and let node j be

the speaker for slot $k+1$. The update equation for slot k can therefore be summarized as follows ($\mathbf{A}$ is the adjacency matrix):

$$\dot{x}_i^{k+1}(t) = \begin{cases} -\gamma_i(x_i^k - \mu_i) + \kappa_{ij}\left[x_j^k - x_i^k\right] \exp\left[-\frac{\left(x_j^k - x_i^k\right)^2}{2\lambda_i^2}\right], \ t \in [0, T^{k+1}], & \text{if } \mathbf{A}_{ij} = 1 \text{ and } i \neq j \\ -\gamma_i(x_i^k - \mu_i), \ t \in [0, T^{k+1}], & \text{otherwise.} \end{cases} \tag{7}$$

Since $\kappa_{ij} = 0$ if $\mathbf{A}_{ij} = 0$ (cf. eqn. (2)), the above equation can simply be written as:

$$\dot{x}_i^{k+1}(t) = -\gamma_i(x_i^k - \mu_i) + \kappa_{ij}\left[x_j^k - x_i^k\right] \exp\left[-\frac{\left(x_j^k - x_i^k\right)^2}{2\lambda_i^2}\right]; \ t \in [0, T^{k+1}]. \tag{8}$$

Note that, for $i = j$, only the first term is applicable since $x_j^k - x_i^k = 0$. A difference equation approximation of the above update model can be obtained straightforwardly as in eqn. 4.

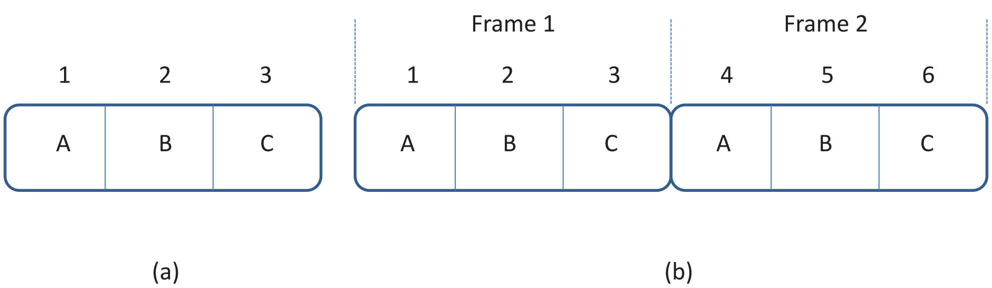

Figure 2: Example of a (a) single frame slotted time asynchronous communication sequence and (b) two-frame slotted time asynchronous communication sequence, for an arbitrary 3-node network (the labels A, B and C are the node ID's). For multiple frames, it is not necessary for all frames to have the same communication sequence. For example, the communication sequence for the second frame could be [B,C,A]. The numeric labels represent the slot numbers. Note that we have adopted a continuous slot numbering system for multiple frame communications.

3 Simulation results

3.1 Simulation parameter values

Unless explicitly noted otherwise, all simulation results presented in the subsequent subsections are based on the following parameters:

1. Discretization time step parameter $\Delta = 0.001$.
2. Coupling scale $\alpha = 20$.
3. Latitudes of acceptance of all nodes $\lambda_i = 1$, for all $i \in \mathcal{N}$, where $\mathcal{N}$ is the set of all nodes.
4. Commitments of all nodes $\gamma_i = 1$, for all $i \in \mathcal{N}$.
5. Natural biases of all nodes $\mu_i = x_i(0)$, for all $i \in \mathcal{N}$, where $x_i(0)$ is the initial state of node i.

3.2 Simulation results - Single frame, 3-node broker vs. clique

For the notes below, A is the node with positive bias, B is the node with zero bias and C is the node with negative bias.

- Fig. 3a for a clique shows a consensus in the negative opinion space, whereas Fig. 3b for a broker shows a 2-1 split with a negative majority. These figures correspond to $\Delta_\mu = 2.5$ and communication sequence [A, B, C].
- Fig. 4a for a clique and Fig. 4b for a broker both show a consensus around zero. These figures correspond to $\Delta_\mu = 2.5$ and communication sequence [A, C, B]. However, the final discord is smaller for the broker.
- Fig. 5a for a clique shows a consensus in the negative opinion space, whereas Fig. 5b for a broker shows a 2-1 split with a negative majority. These figures correspond to $\Delta_\mu = 5.0$ and communication sequence [A, B, C].
- Fig. 6a for a clique and Fig. 6b for a broker both show a consensus around zero. These figures correspond to $\Delta_\mu = 5.0$ and communication sequence [A, C, B]. However, the final discord is smaller for the broker.
- Fig. 7a shows the final states as a function of coupling scale for a clique, $\Delta_\mu = 2.5$ and communication sequence [A, B, C]. For larger values of coupling scale, we see evidence of consensus in the negative opinion space. Also, node A exhibits a transition effect around $\alpha = 5$. In contrast, Fig. 7b for a broker network shows a 2-1 split with a negative majority. Similar observations hold for Figs. 7c and 7d which are for $\Delta_\mu = 5.0$, except, the transition point for node A for the clique network is much more pronounced and occurs around $\alpha = 12.5$.
- Fig. 8a shows the final state of the broker node as a function of Δ_μ for $\alpha = 20$. The commitments of nodes A, B and C are 1, 0 and 1 respectively. The communication sequence is [A, C], where A is the node with positive bias and C is the node with negative bias and the speaking durations are as labeled. For example, the label $[1, 0, 1]$ implies that the speaking duration of nodes A and C is 1 sec. for each and node B is silent (listener only). It can be

seen that for smaller values of Δ_μ, it is better to speak last (the final state of the broker node is in the negative opinion space), whereas for larger values of Δ_μ, it is better to speak first. Temporal plots in support of this observation are shown in Figs. 8b and 8b for $\delta_\mu = 2.5$ and 5 respectively.

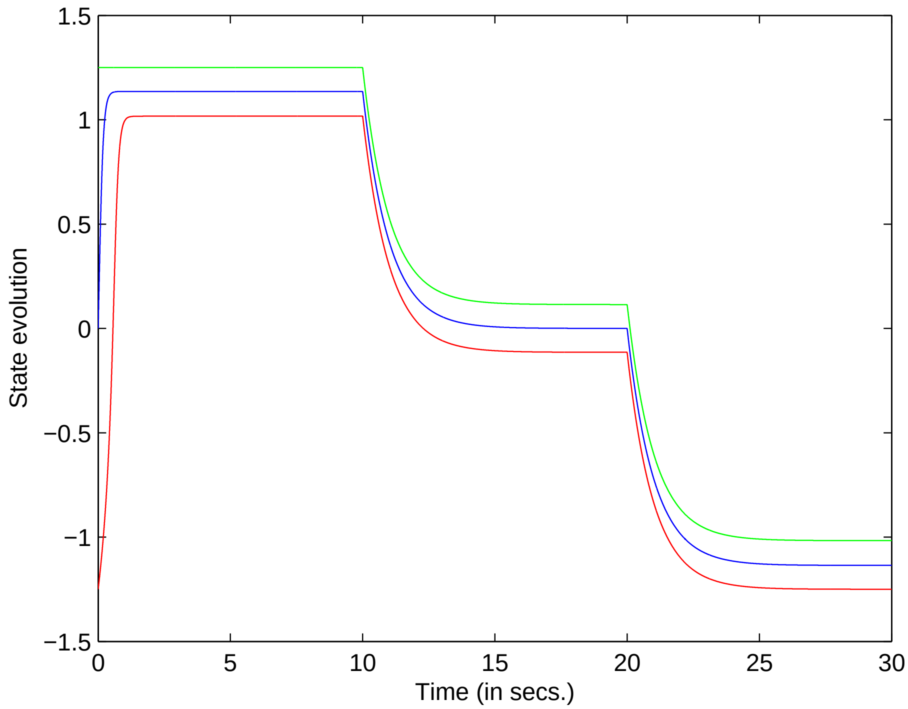

Figure 3a: Plot of temporal state evolution for a 3-node clique, $\Delta_\mu = 2.5$, and all speaking durations $= 10$ secs. The communication sequence is $[A,\ B,\ C]$, where A is the node with positive bias, B is the node with zero bias and C is the node with negative bias.

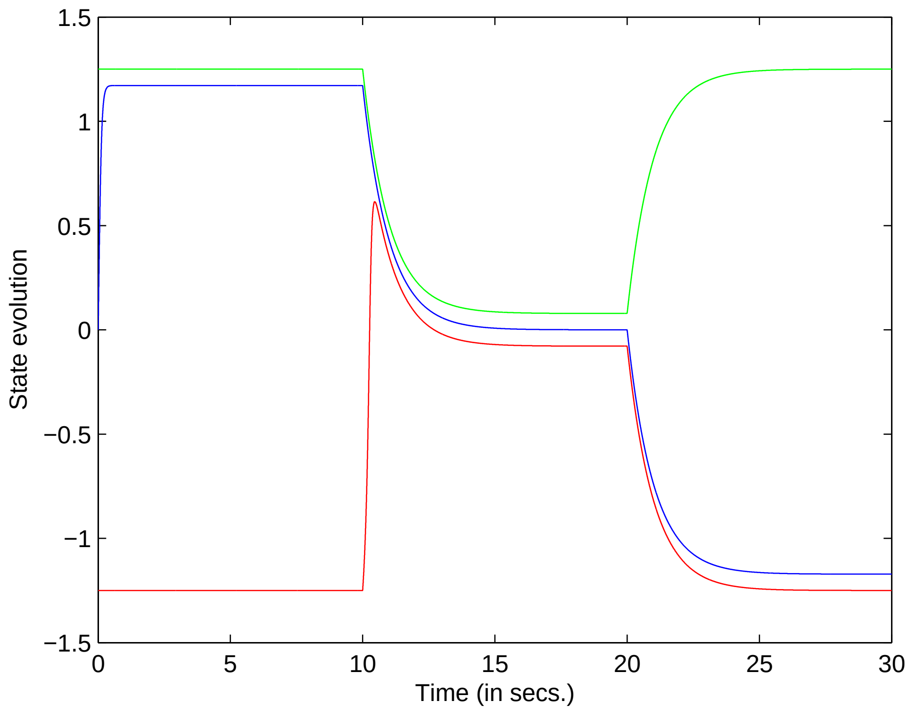

Figure 3b: Plot of temporal state evolution for a 3-node broker, $\Delta_\mu = 2.5$, and all speaking durations = 10 secs. The communication sequence is $[A,\ B,\ C]$, where A is the node with positive bias, B is the broker node with zero bias and C is the node with negative bias.

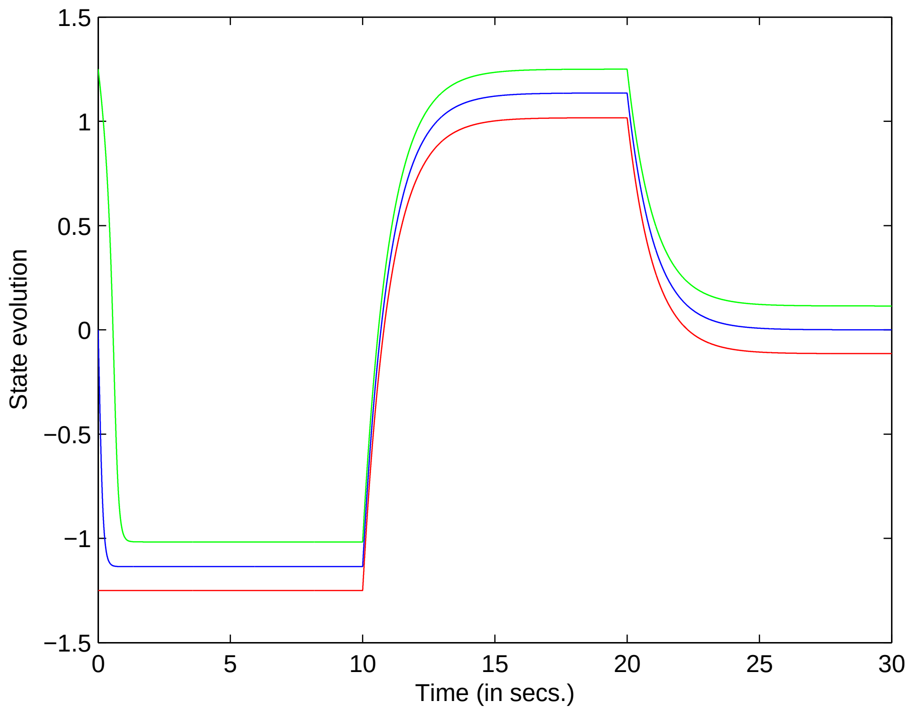

Figure 4a: Plot of temporal state evolution for a 3-node clique, $\Delta_\mu = 2.5$, and all speaking durations $= 10$ secs. The communication sequence is $[A,\ C,\ B]$, where A is the node with positive bias, B is the node with zero bias and C is the node with negative bias.

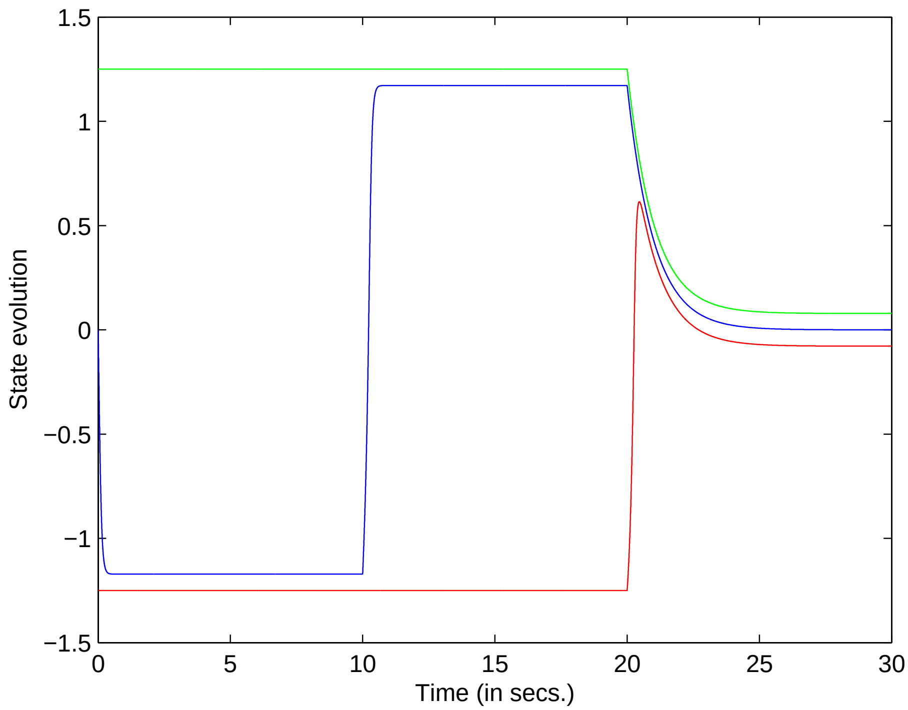

Figure 4b: Plot of temporal state evolution for a 3-node broker, $\Delta_\mu = 2.5$, and all speaking durations = 10 secs. The communication sequence is $[A, C, B]$, where A is the node with positive bias, B is the broker node with zero bias and C is the node with negative bias.

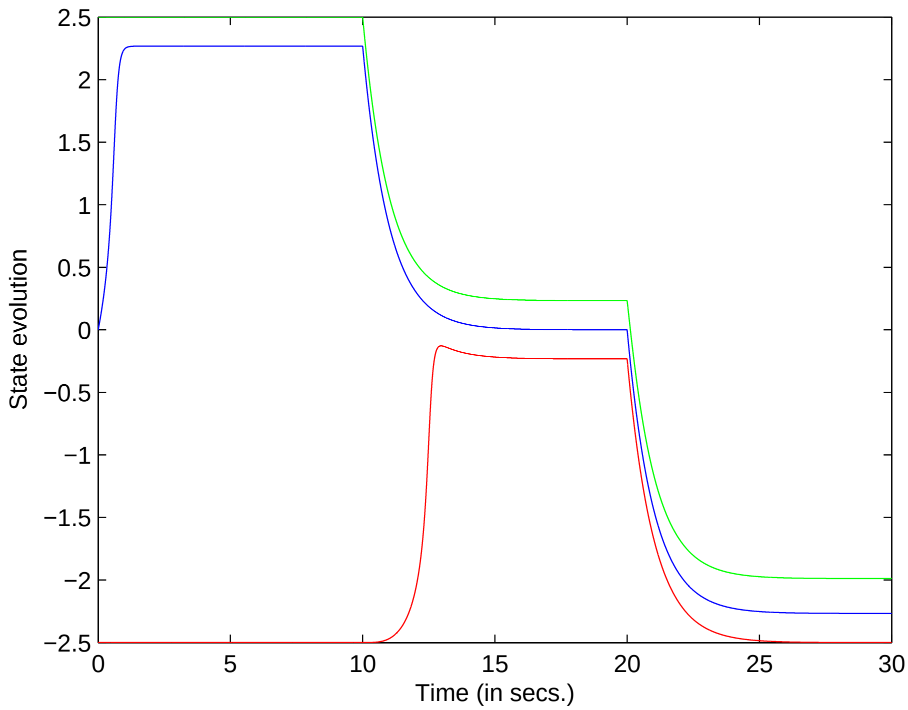

Figure 5a: Plot of temporal state evolution for a 3-node clique, $\Delta_\mu = 5.0$, and all speaking durations $= 10$ secs. The communication sequence is $[A,\ B,\ C]$, where A is the node with positive bias, B is the node with zero bias and C is the node with negative bias.

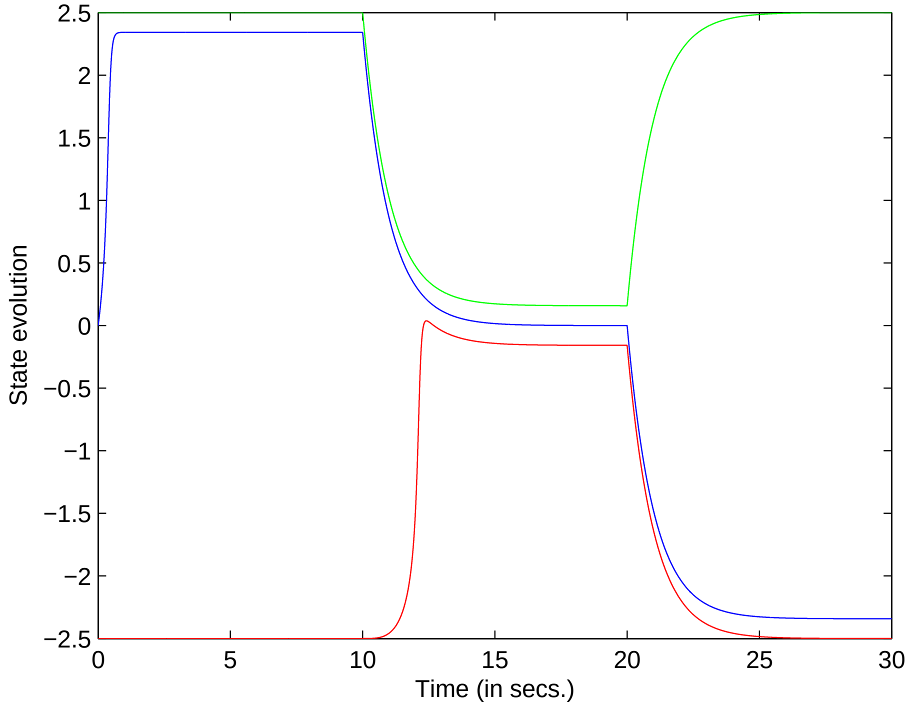

Figure 5b: Plot of temporal state evolution for a 3-node broker, $\Delta_\mu = 5.0$, and all speaking durations = 10 secs. The communication sequence is $[A,\ B,\ C]$, where A is the node with positive bias, B is the broker node with zero bias and C is the node with negative bias.

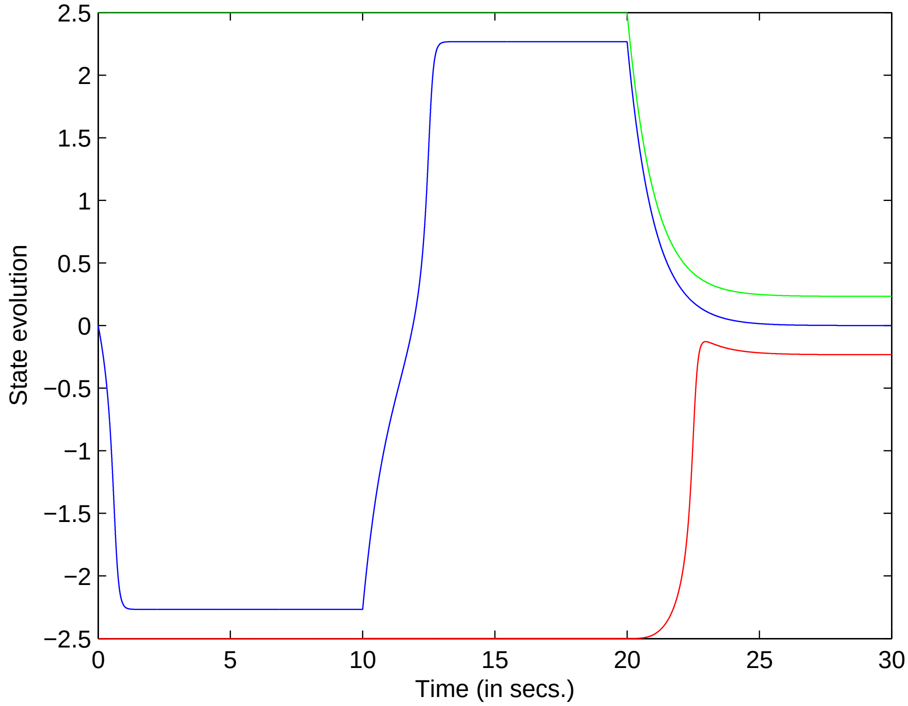

Figure 6a: Plot of temporal state evolution for a 3-node clique, $\Delta_\mu = 5.0$, and all speaking durations $= 10$ secs. The communication sequence is $[A,\, C,\, B]$, where A is the node with positive bias, B is the node with zero bias and C is the node with negative bias.

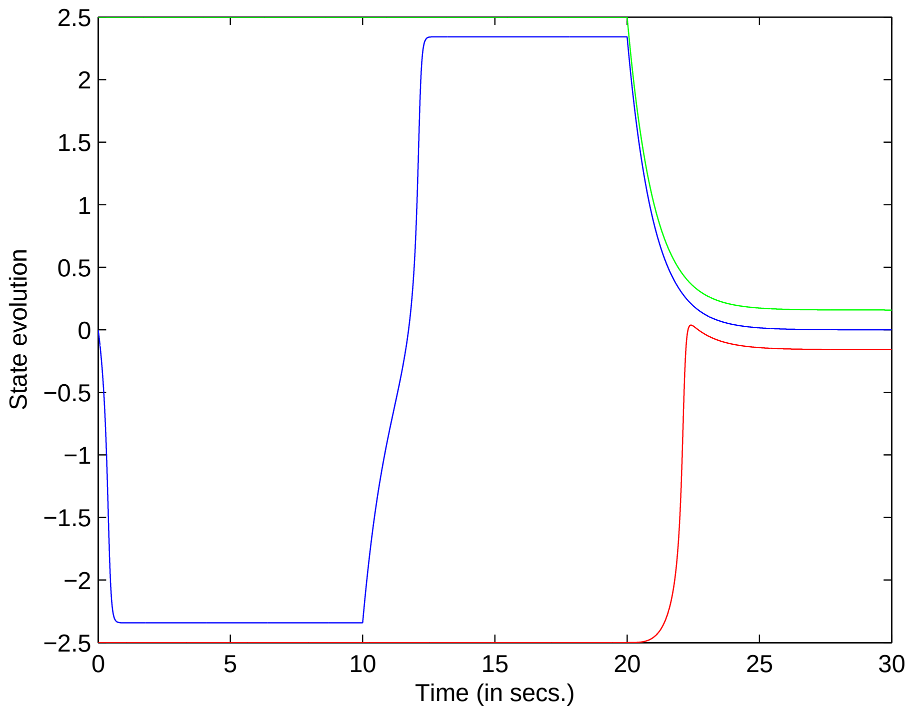

Figure 6b: Plot of temporal state evolution for a 3-node broker, $\Delta_\mu = 5.0$, and all speaking durations = 10 secs. The communication sequence is [A, C, B], where A is the node with positive bias, B is the broker node with zero bias and C is the node with negative bias.

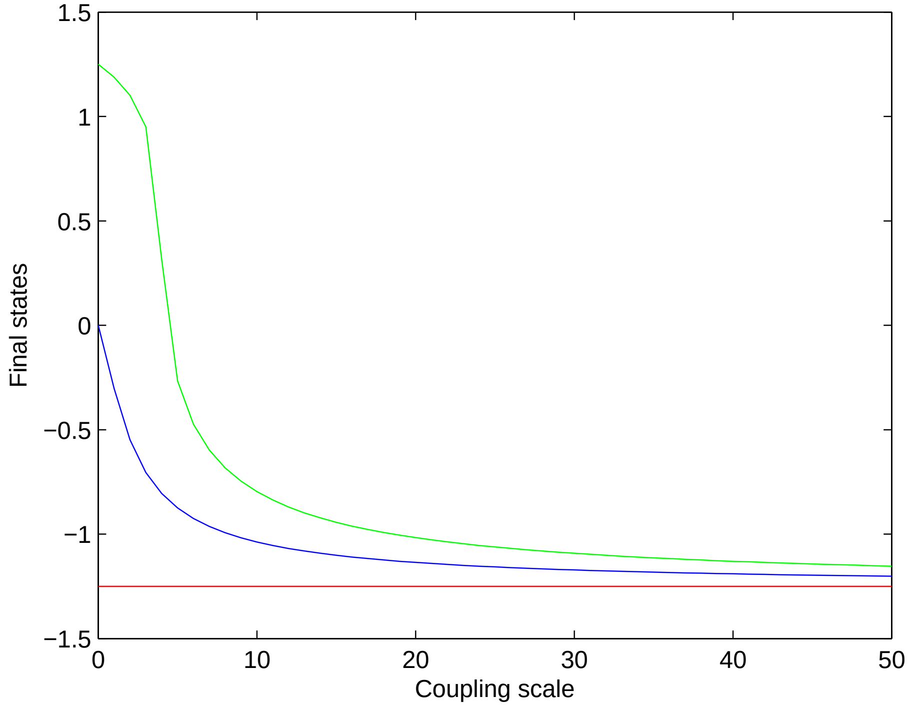

Figure 7a: Plot of final states as a function of coupling scale for a 3-node clique, $\Delta_\mu = 2.5$. All speaking durations = 10 secs. and the communication sequence is $[A,\ B,\ C]$, where A is the node with positive bias, B is the node with zero bias and C is the node with negative bias.

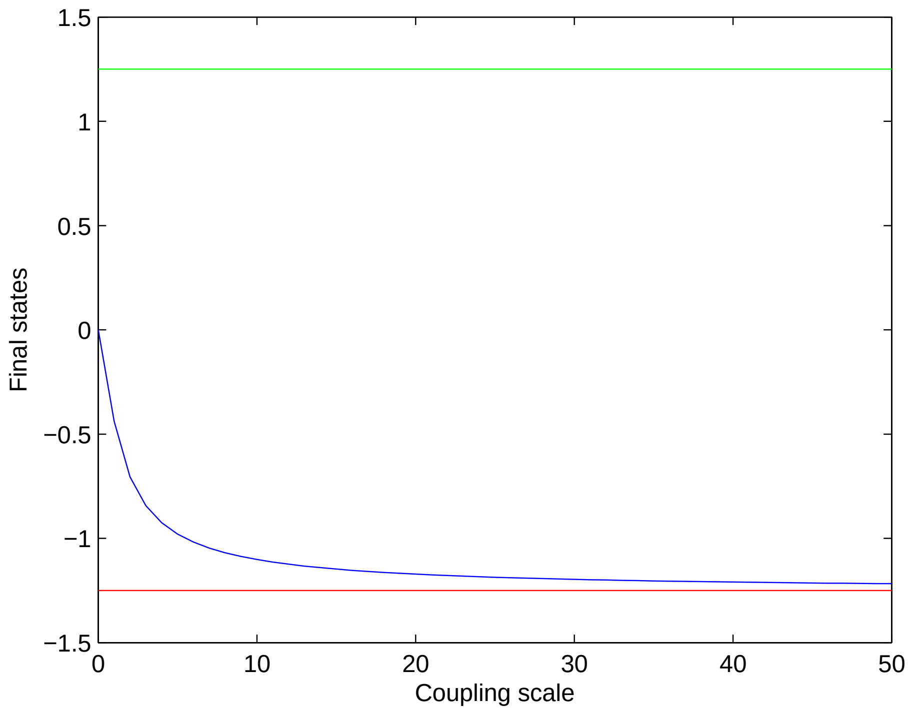

Figure 7b: Plot of final states as a function of coupling scale for a 3-node broker, $\Delta_\mu = 2.5$. All speaking durations = 10 secs. and the communication sequence is $[A, B, C]$, where A is the node with positive bias, B is the broker node with zero bias and C is the node with negative bias.

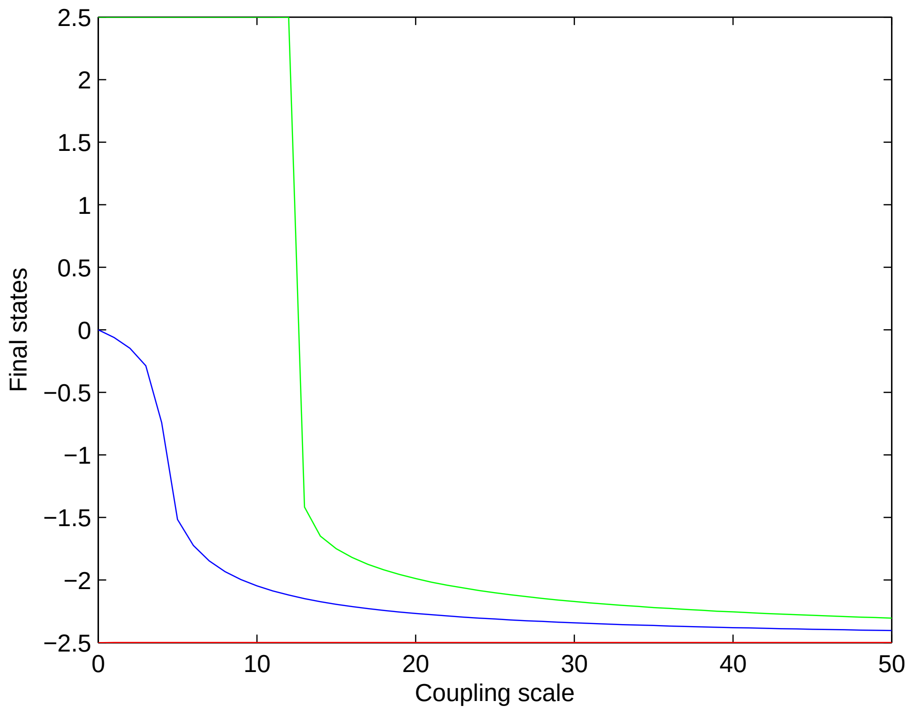

Figure 7c: Plot of final states as a function of coupling scale for a 3-node clique, $\Delta_\mu = 5.0$. All speaking durations = 10 secs. and the communication sequence is [A, B, C], where A is the node with positive bias, B is the node with zero bias and C is the node with negative bias.

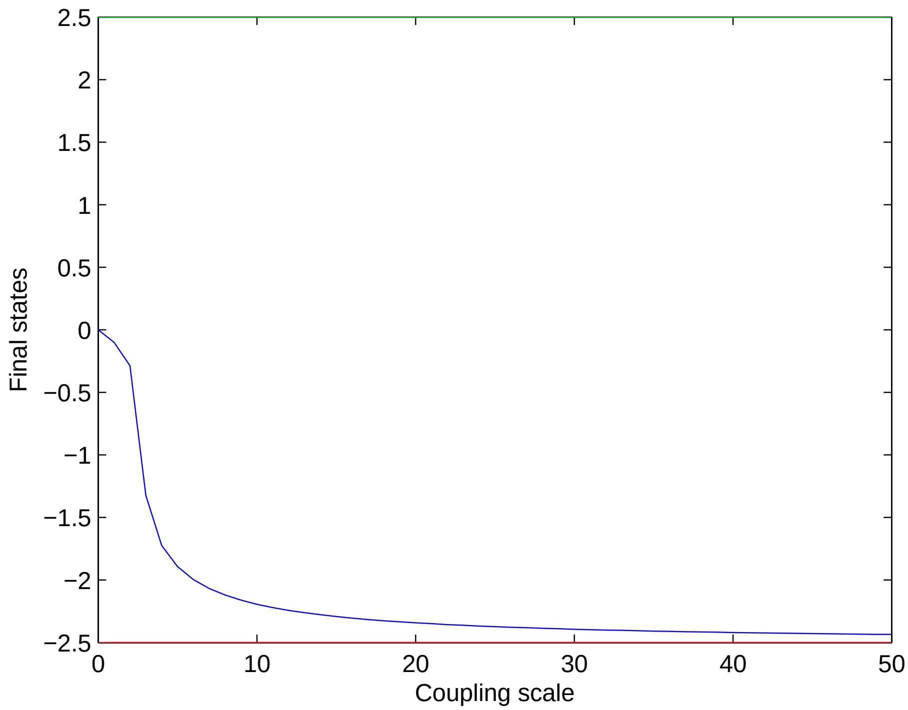

Figure 7d: Plot of final states as a function of coupling scale for a 3-node broker, $\Delta_\mu = 5.0$. All speaking durations = 10 secs. and the communication sequence is [A, B, C], where A is the node with positive bias, B is the broker node with zero bias and C is the node with negative bias.

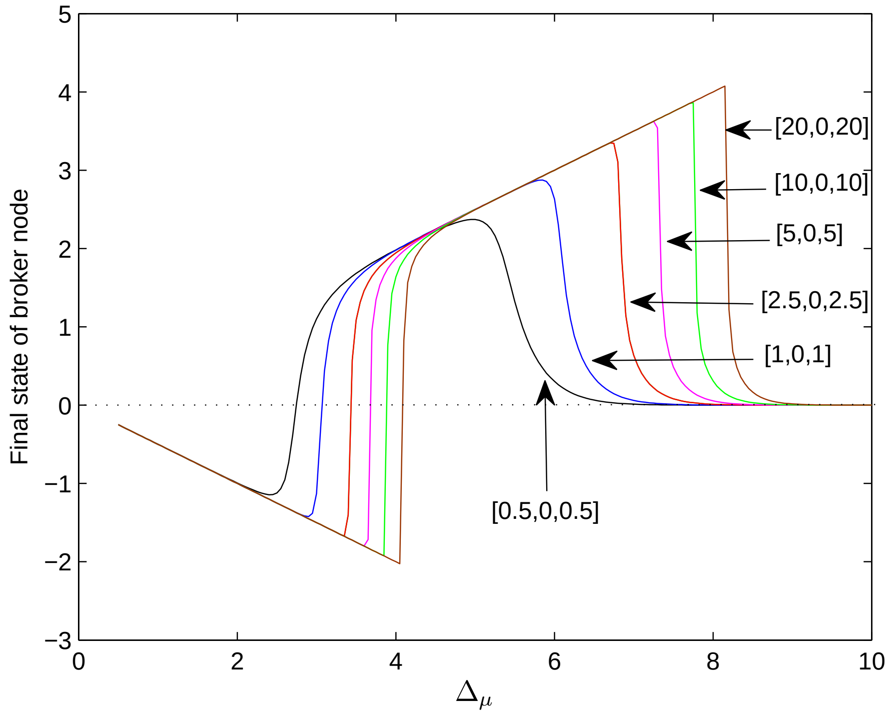

Figure 8a: Plot of the final state of the broker node as a function of Δ_μ for $\alpha = 20$. The commitments of nodes A, B and C are 1, 0 and 1 respectively. The communication sequence is [A, C], where A is the node with positive bias and C is the node with negative bias and the speaking durations are as labeled. For example, the label $[1, 0, 1]$ implies that the speaking duration of nodes A and C is 1 sec. for each and node B is silent (listener only).

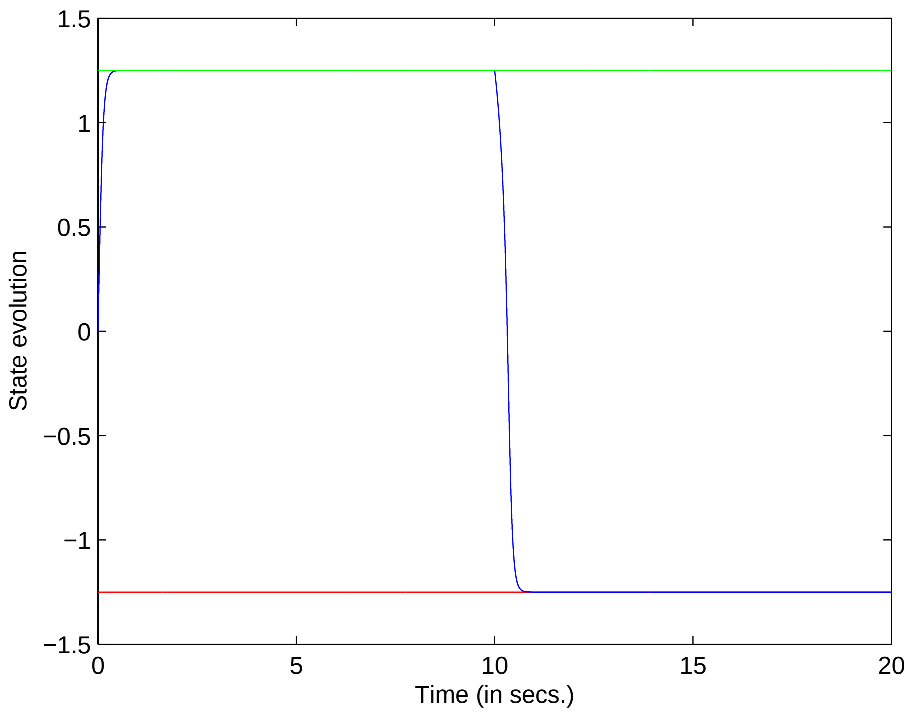

Figure 8b: Plot of temporal state evolution for a 3-node broker for $\Delta_\mu = 2.5$ and $\alpha = 20$. The commitments of nodes A, B and C are 1, 0 and 1 respectively. The communication sequence is $[A, C]$, where A is the node with positive bias and C is the node with negative bias, and each node speaks for 10 secs. Node B is silent.

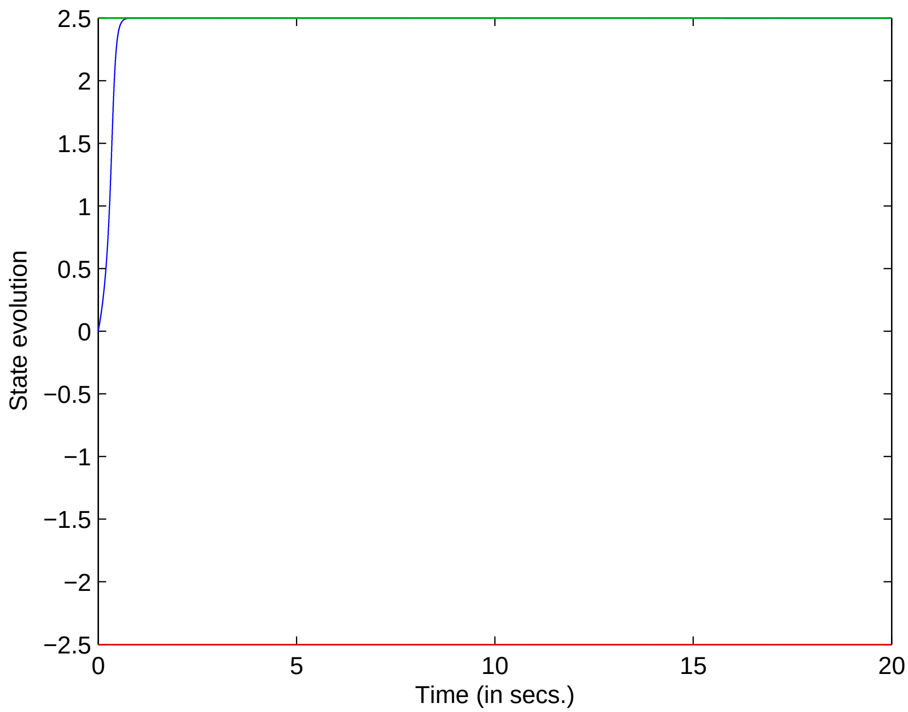

Figure 8c: Plot of temporal state evolution for a 3-node broker for $\Delta_\mu = 5.0$ and $\alpha = 20$. The commitments of nodes A, B and C are 1, 0 and 1 respectively. The communication sequence is [A, C], where A is the node with positive bias and C is the node with negative bias, and each node speaks for 10 secs. Node B is silent.

3.3 Simulation results - Multiple frames, 3-node broker vs. clique

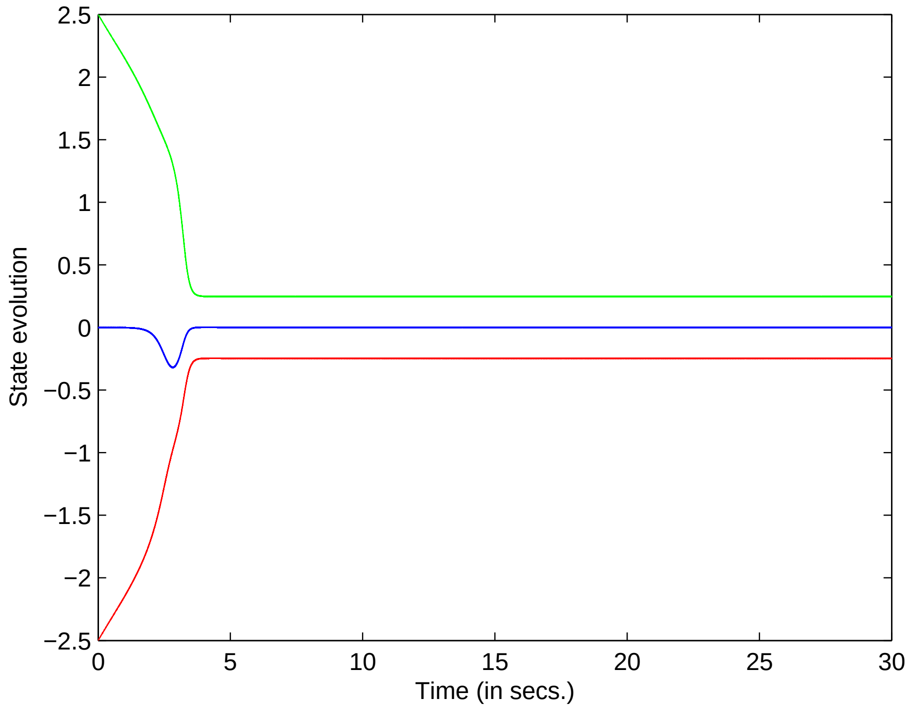

Figure 9a: Plot of temporal state evolution for a 3-node clique, $\Delta_\mu = 5.0$. No. of frames = 10000 and all speaking durations = 0.001 sec. The communication sequence for all frames is $[C, B, A]$, where A is the node with positive bias, B is the node with zero bias and C is the node with negative bias.

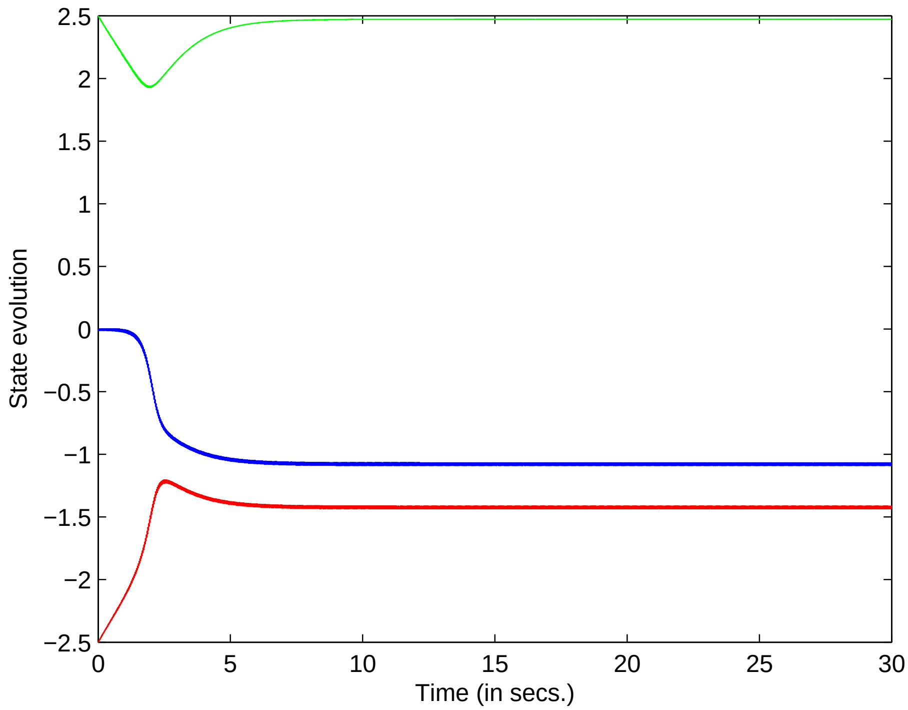

Figure 9b: Plot of temporal state evolution for a 3-node clique, $\Delta_\mu = 5.0$. No. of frames = 1000 and all speaking durations = 0.01 sec. The communication sequence for all frames is $[C,\ B,\ A]$, where A is the node with positive bias, B is the node with zero bias and C is the node with negative bias.

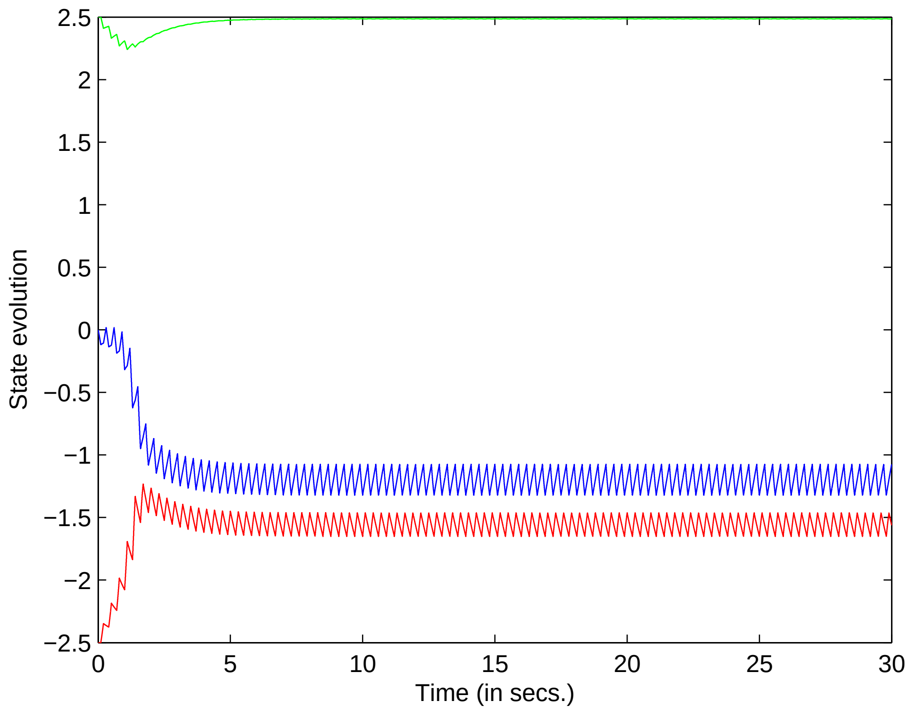

Figure 9c: Plot of temporal state evolution for a 3-node clique, $\Delta_\mu = 5.0$. No. of frames $= 100$ and all speaking durations $= 0.1$ sec. The communication sequence for all frames is $[C, B, A]$, where A is the node with positive bias, B is the node with zero bias and C is the node with negative bias.

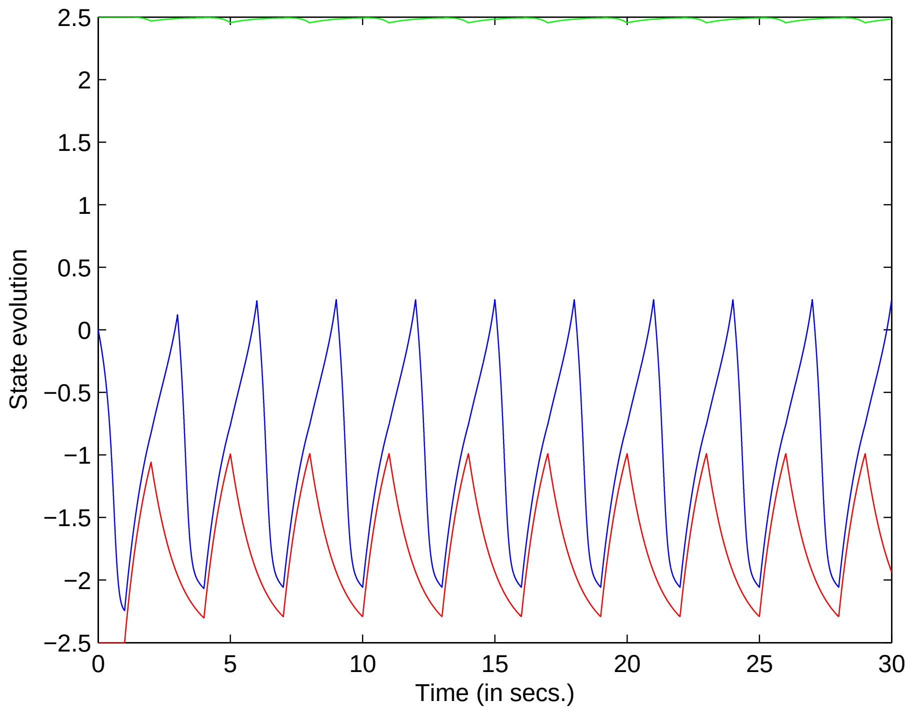

Figure 9d: Plot of temporal state evolution for a 3-node clique, $\Delta_\mu = 5.0$. No. of frames = 10 and all speaking durations = 1 sec. The communication sequence for all frames is [C, B, A], where A is the node with positive bias, B is the node with zero bias and C is the node with negative bias.

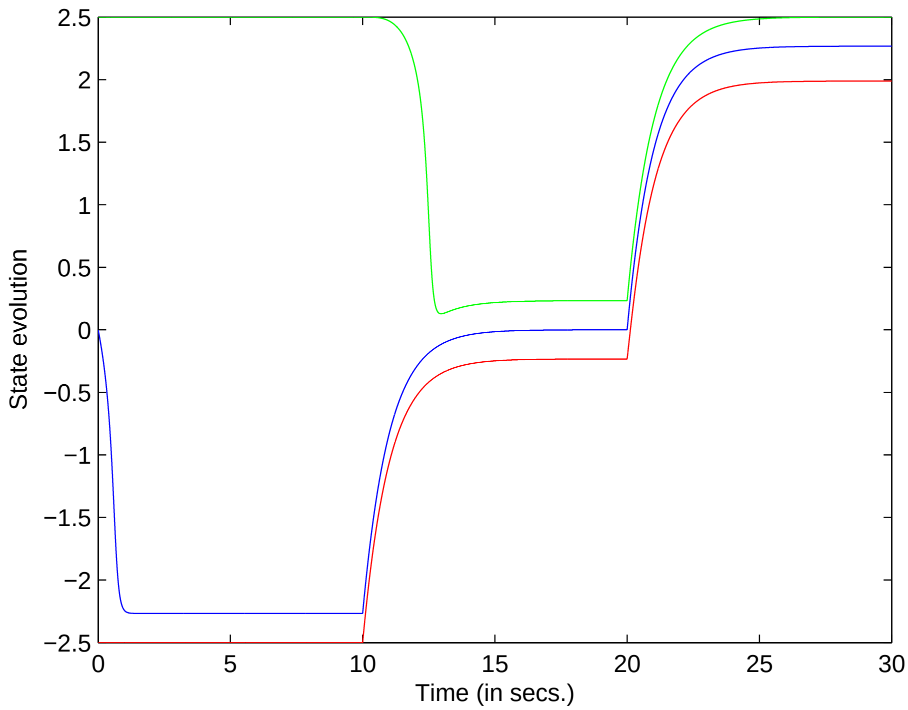

Figure 9e: Plot of temporal state evolution for a 3-node clique, $\Delta_\mu = 5.0$. No. of frames = 1 and all speaking durations = 10 secs. The communication sequence for all frames is [C, B, A], where A is the node with positive bias, B is the node with zero bias and C is the node with negative bias.

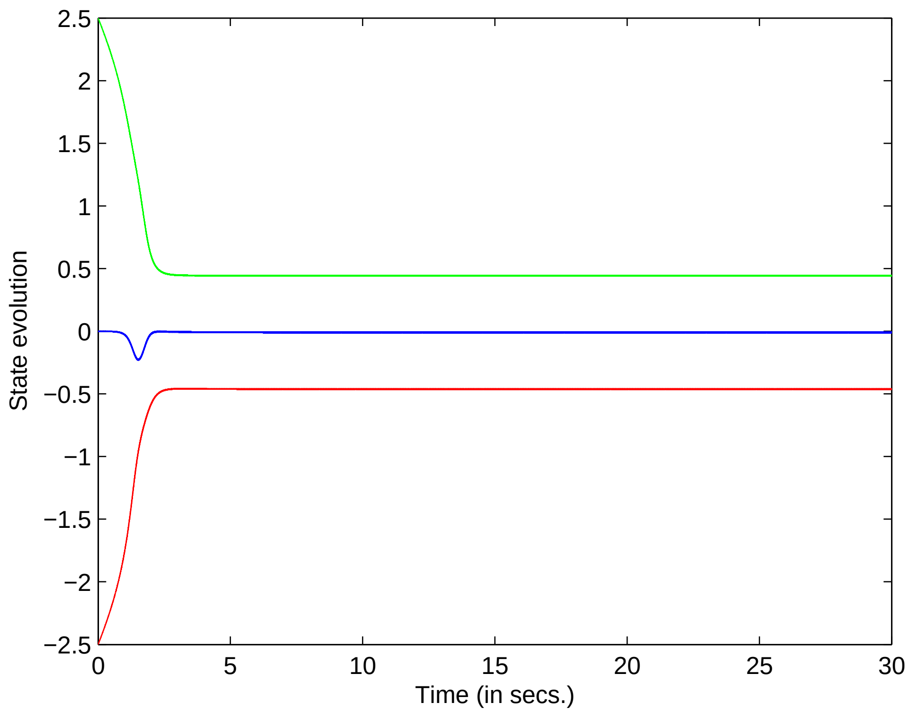

Figure 10a: Plot of temporal state evolution for a 3-node broker, $\Delta_\mu = 5.0$. No. of frames = 10000 and all speaking durations = 0.001 sec. The communication sequence for all frames is $[C, B, A]$, where A is the node with positive bias, B is the broker node with zero bias and C is the node with negative bias.

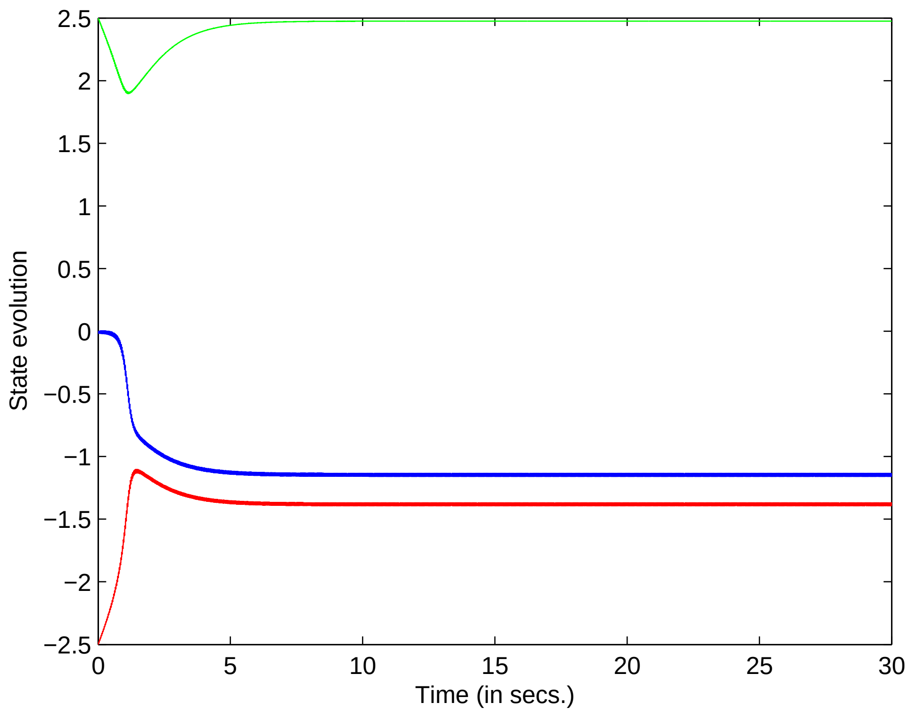

Figure 10b: Plot of temporal state evolution for a 3-node broker, $\Delta_\mu = 5.0$. No. of frames = 1000 and all speaking durations = 0.01 sec. The communication sequence for all frames is [C, B, A], where A is the node with positive bias, B is the broker node with zero bias and C is the node with negative bias.

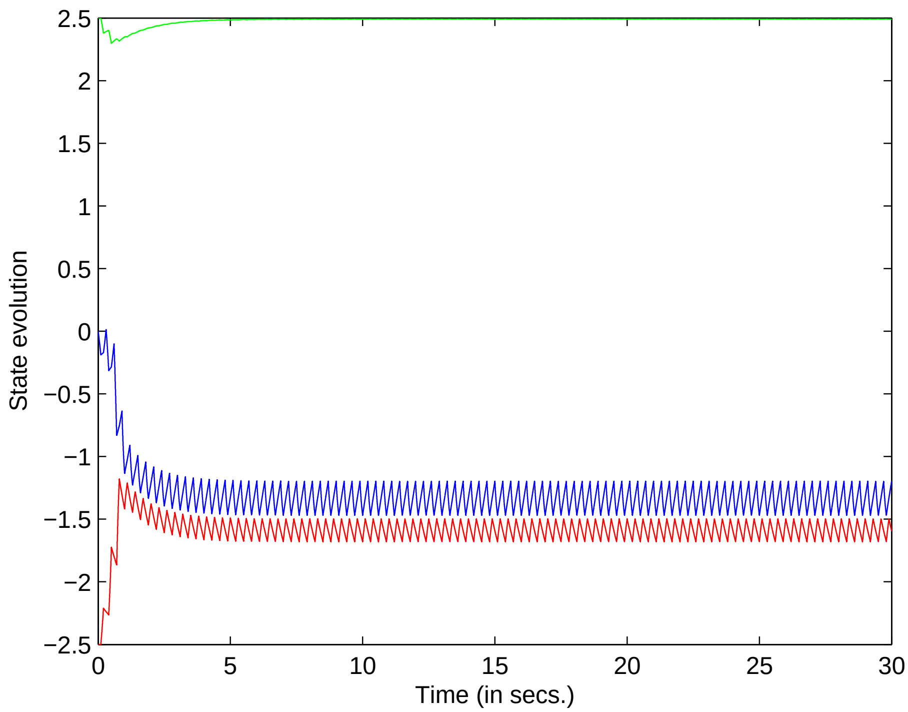

Figure 10c: Plot of temporal state evolution for a 3-node broker, $\Delta_\mu = 5.0$. No. of frames = 100 and all speaking durations = 0.1 sec. The communication sequence for all frames is $[C,\ B,\ A]$, where A is the node with positive bias, B is the broker node with zero bias and C is the node with negative bias.

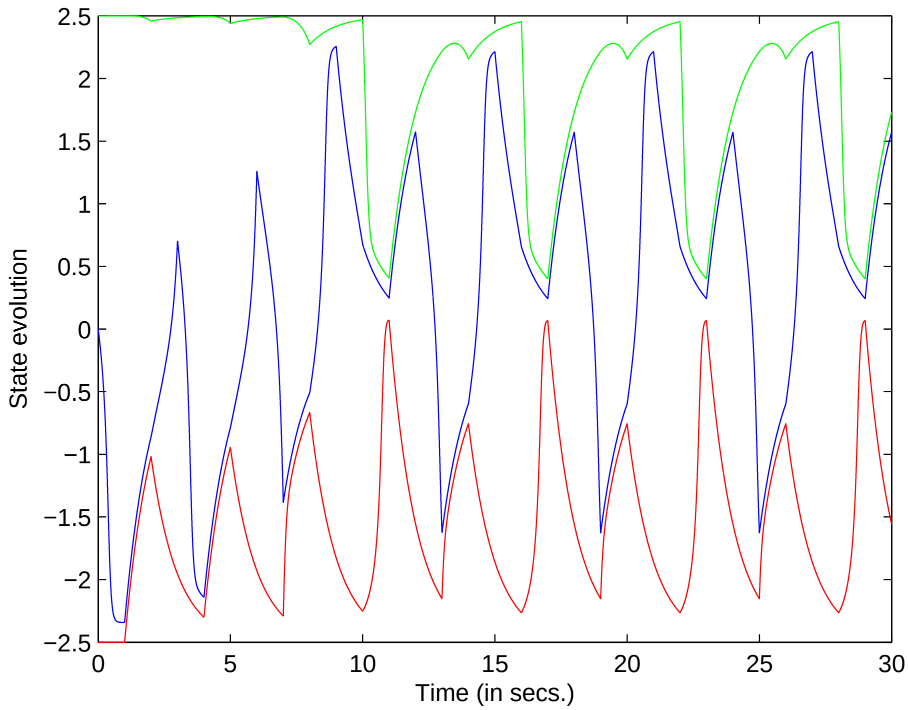

Figure 10d: Plot of temporal state evolution for a 3-node broker, $\Delta_\mu = 5.0$. No. of frames = 10 and all speaking durations = 1 sec. The communication sequence for all frames is $[C,\ B,\ A]$, where A is the node with positive bias, B is the broker node with zero bias and C is the node with negative bias.

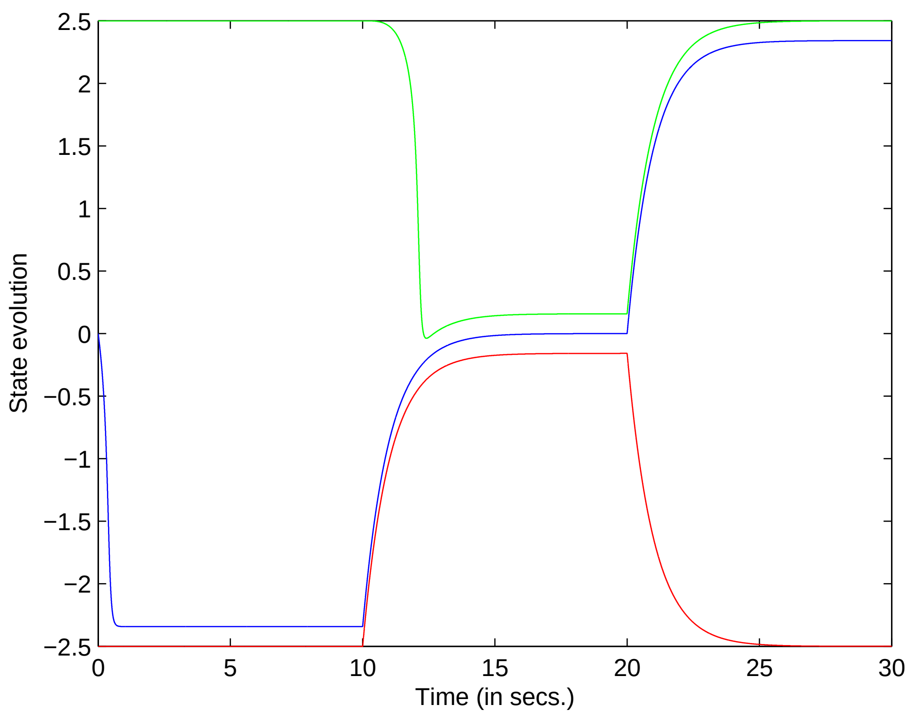

Figure 10e: Plot of temporal state evolution for a 3-node broker, $\Delta_{\mu} = 5.0$. No. of frames = 1 and all speaking durations = 10 secs. The communication sequence for all frames is [C, B, A], where A is the node with positive bias, B is the broker node with zero bias and C is the node with negative bias.

3.4 Simulation results - Single frame, 5-node canonical graphs

In the notes below, A is the node with maximum positive bias, B is the node with moderate positive bias, C is the node with zero bias, D is the node with moderate negative bias and E is the node with maximum negative bias.

- Fig. 11a which plots the temporal state evolution for a complete graph, $\Delta_\mu = 2.5$ and communication sequence [A, B, C, D, E], shows a consensus in the negative opinion space. Fig. 11b for a star graph shows a 3-2 negative majority evolving. A rather curious phenomenon we observe for the star graph is that the hub node (node C) ends up with a more negative state than node D. Fig. 11c for a chain graph shows a 2-1-1-1 grouping. Fig. 11d for a broker-A graph shows a 3-2 negative majority evolving, but the 3 nodes in the negative group are much closer to each other in opinion space compared to the star.

- Fig. 12a which plots the temporal state evolution for a complete graph, $\Delta_\mu = 5.0$ and communication sequence [A, B, C, D, E], shows a 4-1 negative majority evolve (compare with Fig. 11a). Fig. 12b for a star graph shows a 3-2 negative majority evolving. In this case too, we observe that the hub node (node C) ends up with a more negative state than node D. Fig. 12c for a chain graph shows a 2-1-1-1 grouping. Fig. 12d for a broker-A graph shows a 3-2 negative majority evolving, but the 3 nodes in the negative group are much closer to each other in opinion space compared to the star.

- Fig. 13a which plots the final states as a function of coupling scale for a complete graph, $\Delta_\mu = 2.5$ and communication sequence [A, B, C, D, E], shows a consensus in the negative opinion space for relatively larger values of coupling scale. Fig. 13b for a star graph shows a 3-2 negative majority grouping for larger values of coupling scale. We also observe that the hub node (node C) generally ends up with a more negative state than node D. Fig. 13c for a chain graph shows a 2-1-1-1 grouping. Fig. 13d for a broker-A graph shows a 3-2 negative majority evolving, but the 3 nodes in the negative group are much closer to each other in opinion space compared to the star.

- Fig. 14a which plots the final states as a function of coupling scale for a complete graph, $\Delta_\mu = 2.5$ and communication sequence [A, B, C, D, E], also shows a consensus in the negative opinion space for relatively larger values of coupling scale. Sharp transition effects can also be observed for the nodes with positive bias. Fig. 14b for a star graph shows a 3-2 negative majority grouping for larger values of coupling scale. We also observe that the hub node (node C) generally ends up with a more negative state than node D. Fig. 14c for a chain graph shows a 2-1-1-1 grouping. Fig. 14d for a broker-A graph shows a 3-2 negative majority evolving, but the 3 nodes in the negative group are much closer to each other in opinion space compared to the star.

References

[1] Noah E. Friedkin, "Norm formation in social influence networks", *Social Networks*, 23 (2001), pp. 167-189.

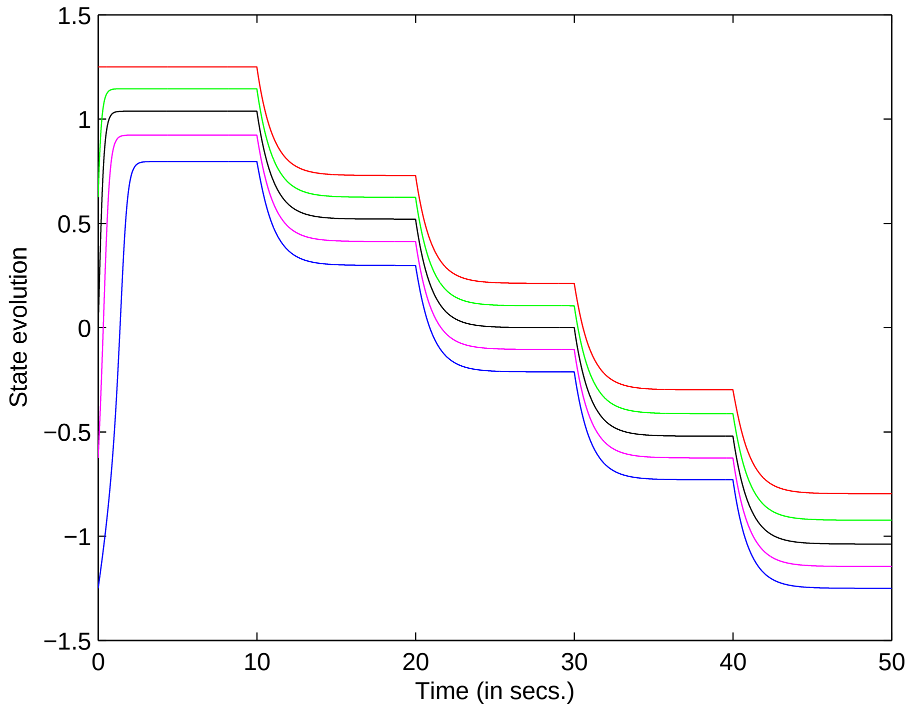

Figure 11a: Plot of temporal state evolution for a 5-node complete graph, $\Delta_\mu = 2.5$, and all speaking durations = 10 secs. The communication sequence is $[A,\ B,\ C,\ D,\ E]$, where A is the node with maximum positive bias, B is the node with moderate positive bias, C is the node with zero bias, D is the node with moderate negative bias and E is the node with maximum negative bias.

[2] Noah E. Friedkin, "A structural theory of social influence", *Cambridge University Press*, Cambridge.

[3] Michael Gabbay, "The effects of nonlinear interactions and network structure in small group opinion dynamics", *Physica A*, 378 (2007), pp. 118-126.

[4] Michael Gabbay, "*A dynamical systems model of small group decision making*", in Diplomacy Games: Formal Models and International Negotiations, R. Avenhaus and I.W. Zartman (eds.), Springer, New York, 2007.

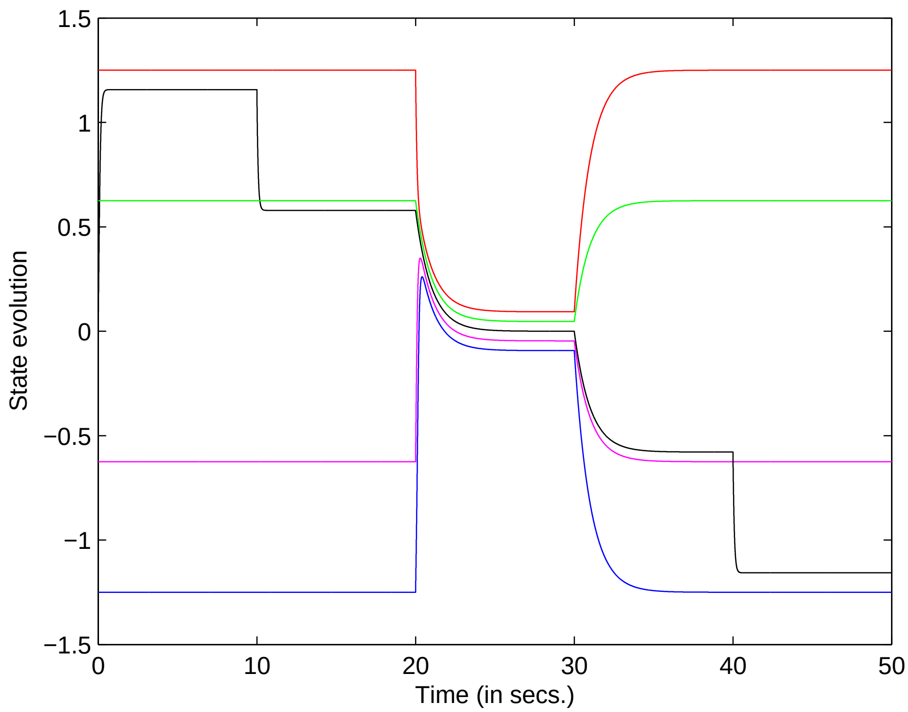

Figure 11b: Plot of temporal state evolution for a 5-node star graph, $\Delta_\mu = 2.5$, and all speaking durations = 10 secs. The communication sequence is $[A,\ B,\ C,\ D,\ E]$, where A is the node with maximum positive bias, B is the node with moderate positive bias, C is the hub node with zero bias, D is the node with moderate negative bias and E is the node with maximum negative bias.

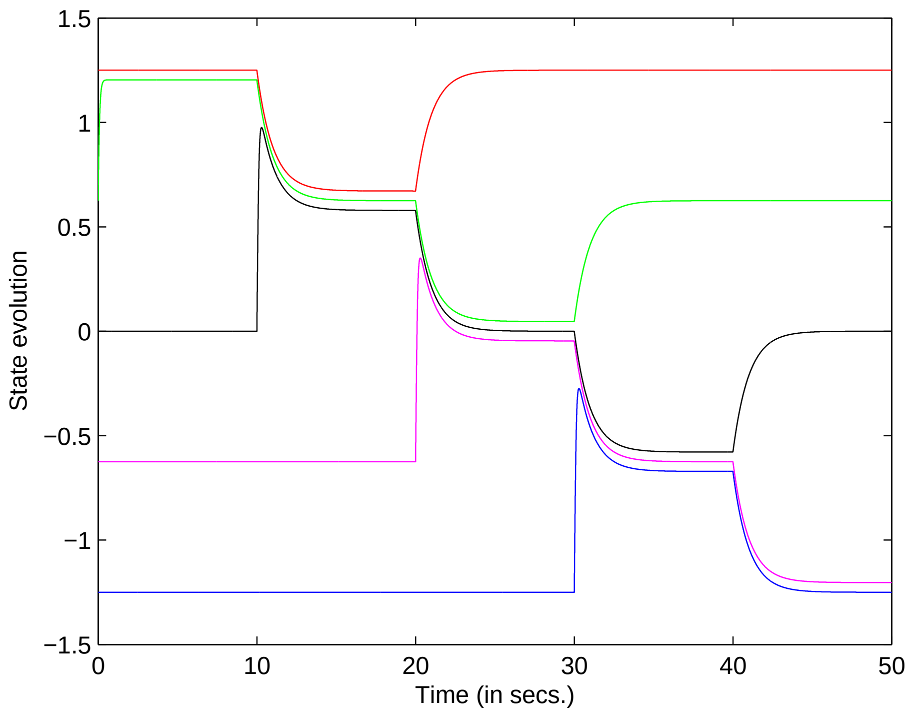

Figure 11c: Plot of temporal state evolution for a 5-node chain graph, $\Delta_\mu = 2.5$, and all speaking durations $= 10$ secs. The communication sequence is $[A,\ B,\ C,\ D,\ E]$, where A is the node with maximum positive bias, B is the node with moderate positive bias, C is the central node with zero bias, D is the node with moderate negative bias and E is the node with maximum negative bias.

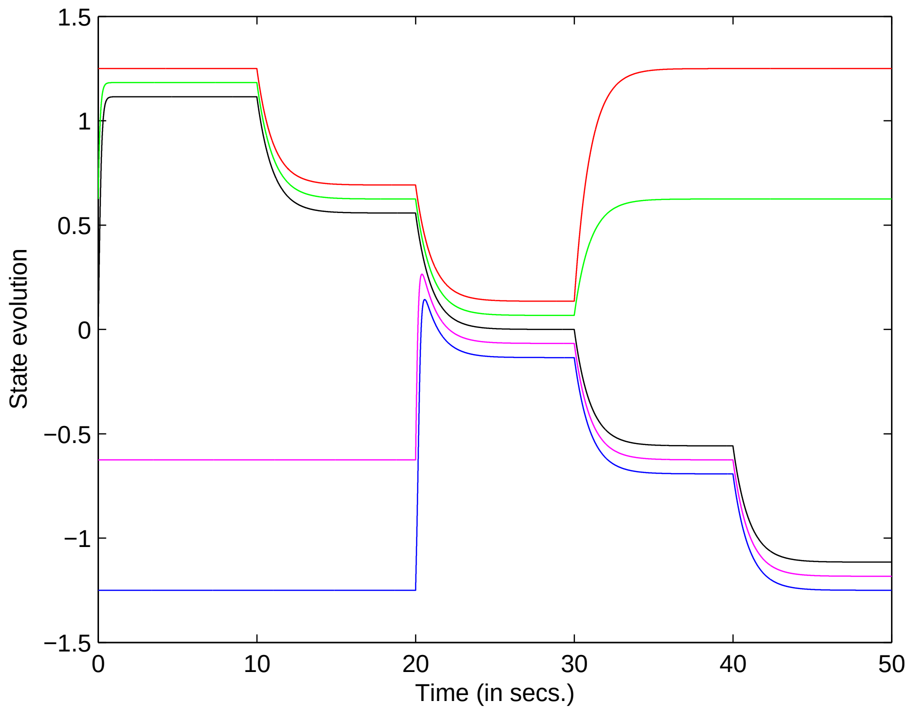

Figure 11d: Plot of temporal state evolution for a 5-node broker-A graph, $\Delta_\mu = 2.5$, and all speaking durations $= 10$ secs. The communication sequence is $[A,\ B,\ C,\ D,\ E]$, where A is the node with maximum positive bias, B is the node with moderate positive bias, C is the broker node with zero bias, D is the node with moderate negative bias and E is the node with maximum negative bias.

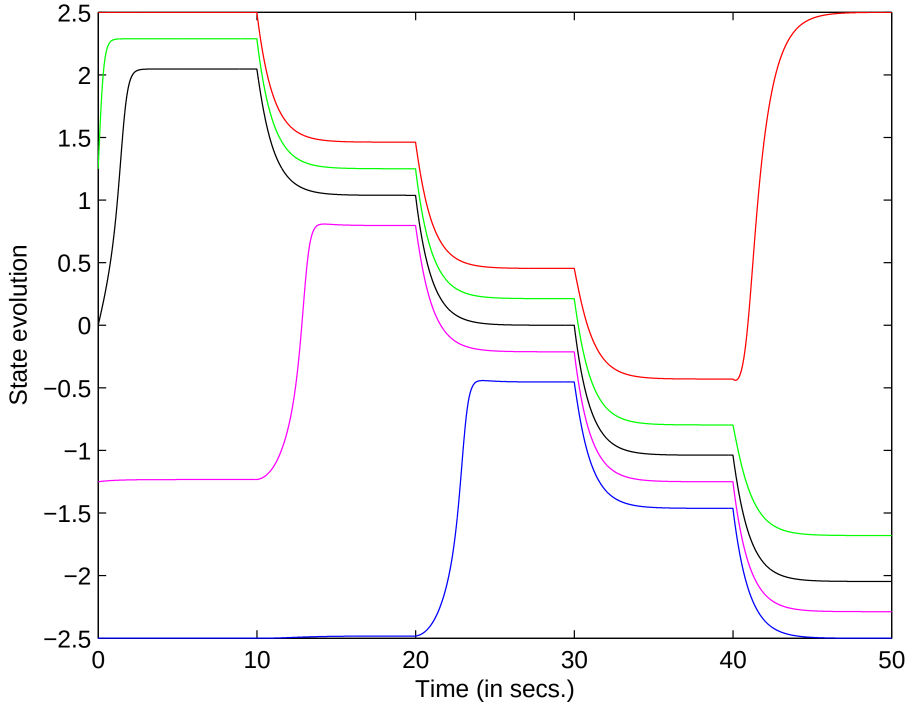

Figure 12a: Plot of temporal state evolution for a 5-node complete graph, $\Delta_\mu = 5.0$, and all speaking durations $= 10$ secs. The communication sequence is $[A,\ B,\ C,\ D,\ E]$, where A is the node with maximum positive bias, B is the node with moderate positive bias, C is the node with zero bias, D is the node with moderate negative bias and E is the node with maximum negative bias.

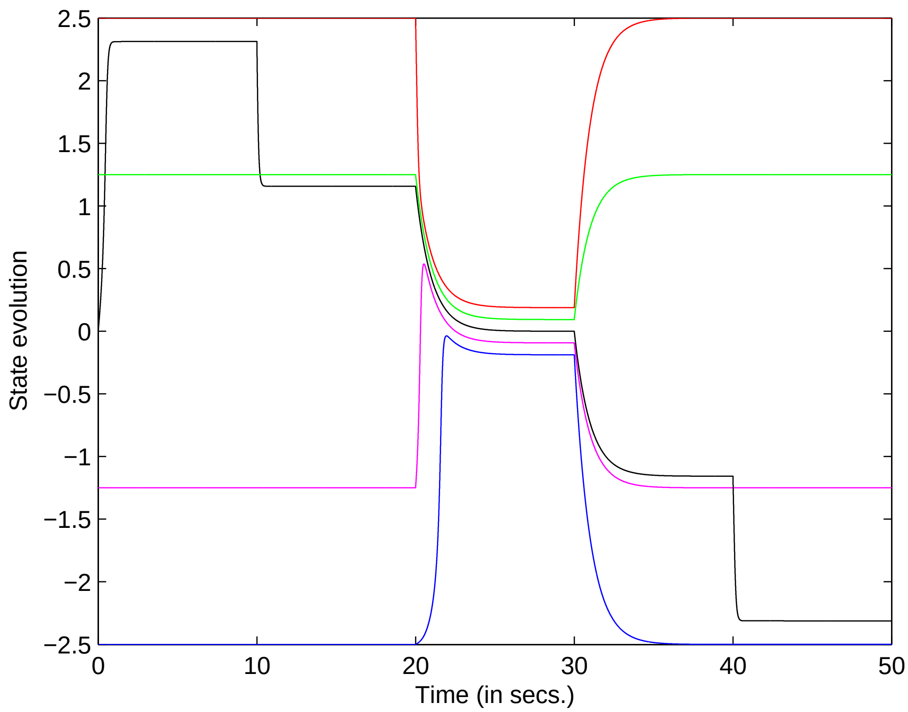

Figure 12b: Plot of temporal state evolution for a 5-node star graph, $\Delta_\mu = 5.0$, and all speaking durations = 10 secs. The communication sequence is $[A,\ B,\ C,\ D,\ E]$, where A is the node with maximum positive bias, B is the node with moderate positive bias, C is the hub node with zero bias, D is the node with moderate negative bias and E is the node with maximum negative bias.

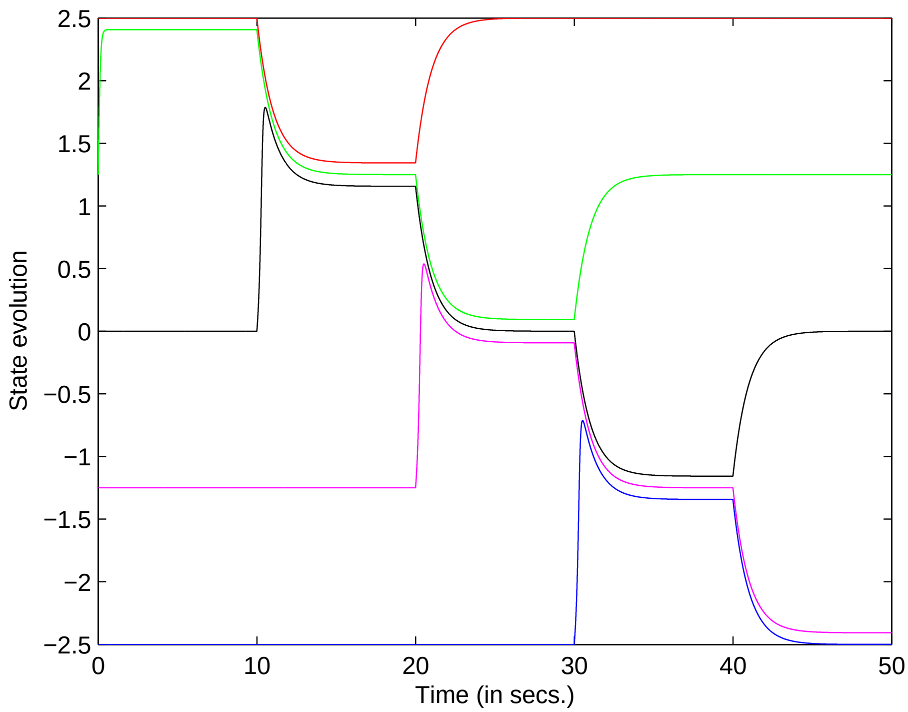

Figure 12c: Plot of temporal state evolution for a 5-node chain graph, $\Delta_\mu = 5.0$, and all speaking durations $= 10$ secs. The communication sequence is $[A,\ B,\ C,\ D,\ E]$, where A is the node with maximum positive bias, B is the node with moderate positive bias, C is the central node with zero bias, D is the node with moderate negative bias and E is the node with maximum negative bias.

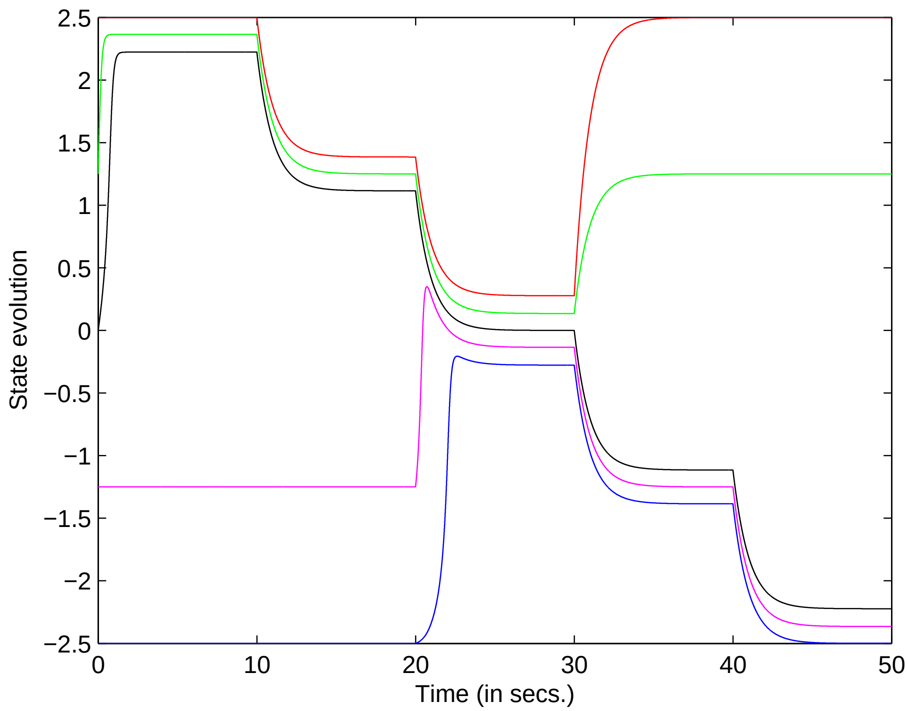

Figure 12d: Plot of temporal state evolution for a 5-node broker-A graph, $\Delta_\mu = 5.0$, and all speaking durations = 10 secs. The communication sequence is $[A, B, C, D, E]$, where A is the node with maximum positive bias, B is the node with moderate positive bias, C is the broker node with zero bias, D is the node with moderate negative bias and E is the node with maximum negative bias.

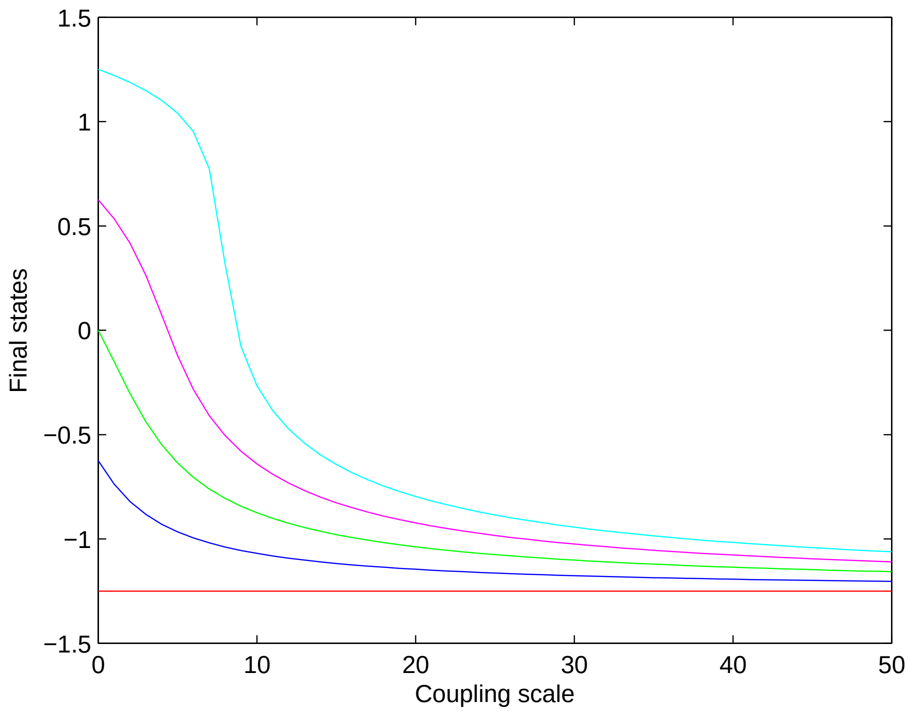

Figure 13a: Plot of final states as a function of coupling scale for a 5-node complete graph, $\Delta_\mu = 2.5$, and all speaking durations $= 10$ secs. The communication sequence is $[A,\ B,\ C,\ D,\ E]$, where A is the node with maximum positive bias, B is the node with moderate positive bias, C is the node with zero bias, D is the node with moderate negative bias and E is the node with maximum negative bias.

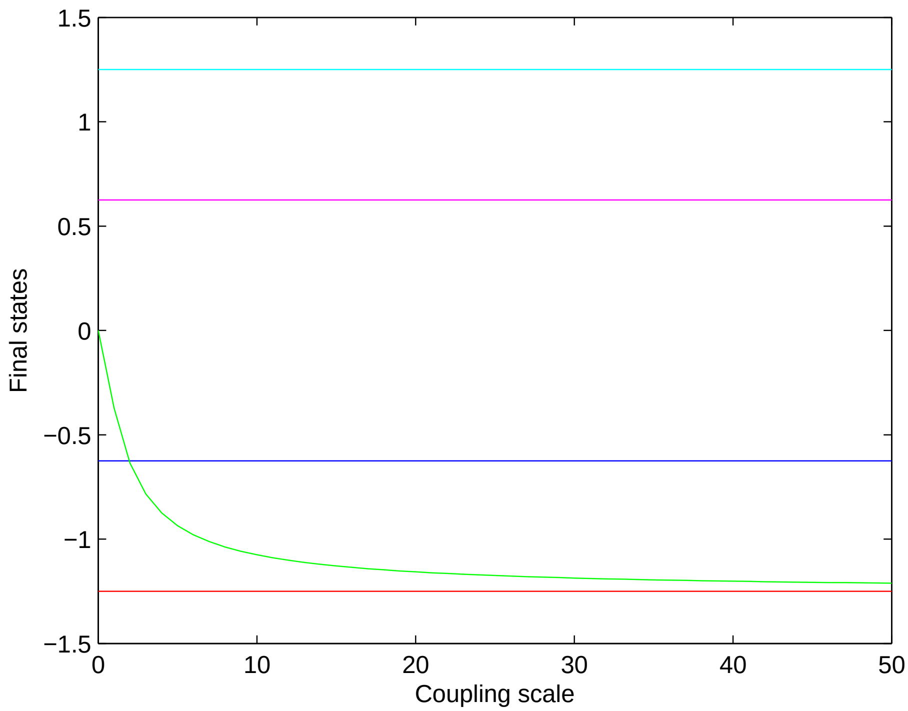

Figure 13b: Plot of final states as a function of coupling scale for a 5-node star graph, $\Delta_\mu = 2.5$, and all speaking durations = 10 secs. The communication sequence is $[A,\ B,\ C,\ D,\ E]$, where A is the node with maximum positive bias, B is the node with moderate positive bias, C is the hub node with zero bias, D is the node with moderate negative bias and E is the node with maximum negative bias.

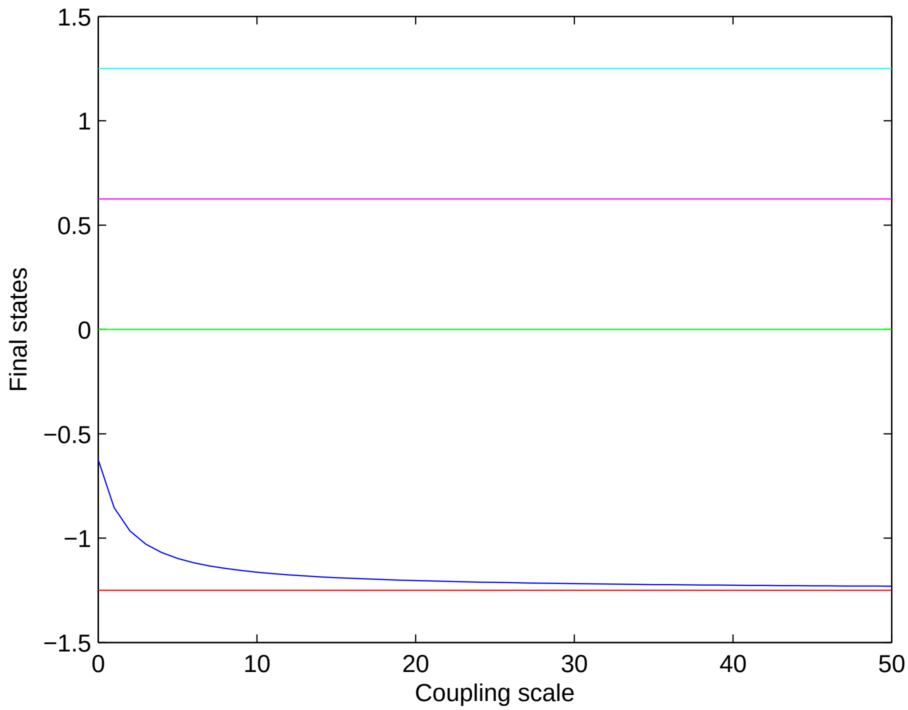

Figure 13c: Plot of final states as a function of coupling scale for a 5-node chain graph, $\Delta_\mu = 2.5$, and all speaking durations = 10 secs. The communication sequence is $[A, B, C, D, E]$, where A is the node with maximum positive bias, B is the node with moderate positive bias, C is the central node with zero bias, D is the node with moderate negative bias and E is the node with maximum negative bias.

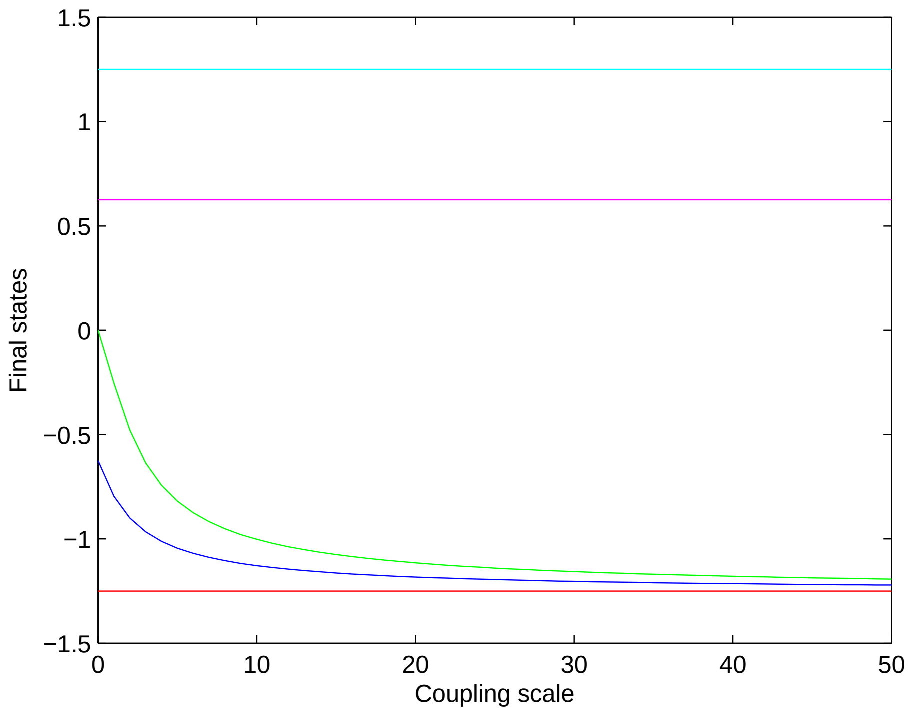

Figure 13d: Plot of final states as a function of coupling scale for a 5-node broker-A graph, $\Delta_\mu = 2.5$, and all speaking durations = 10 secs. The communication sequence is $[A, B, C, D, E]$, where A is the node with maximum positive bias, B is the node with moderate positive bias, C is the broker node with zero bias, D is the node with moderate negative bias and E is the node with maximum negative bias.

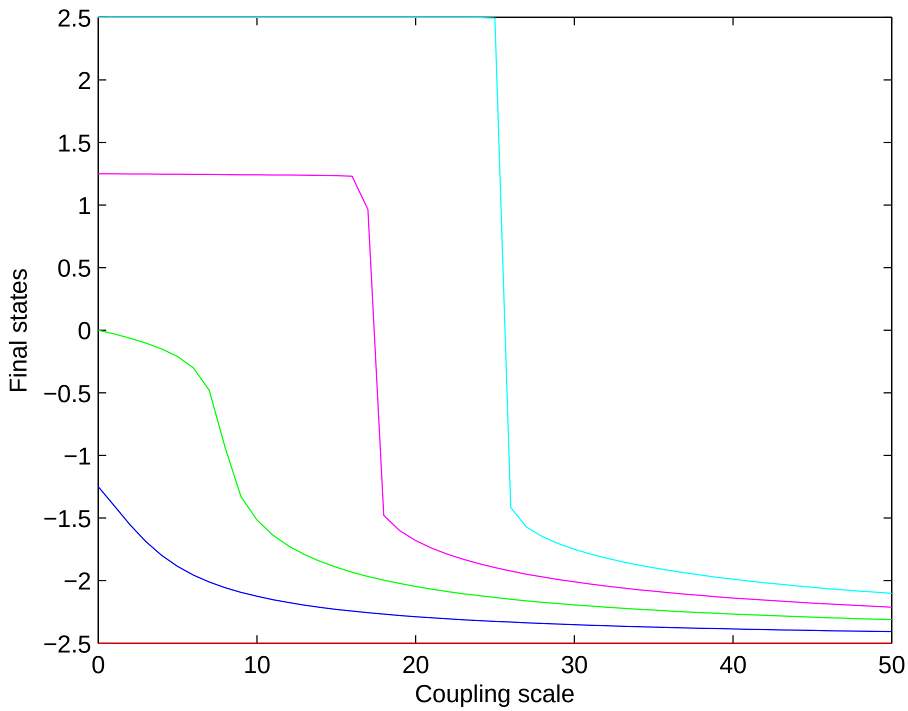

Figure 14a: Plot of final states as a function of coupling scale for a 5-node complete graph, $\Delta_\mu = 5.0$, and all speaking durations $= 10$ secs. The communication sequence is $[A, B, C, D, E]$, where A is the node with maximum positive bias, B is the node with moderate positive bias, C is the node with zero bias, D is the node with moderate negative bias and E is the node with maximum negative bias.

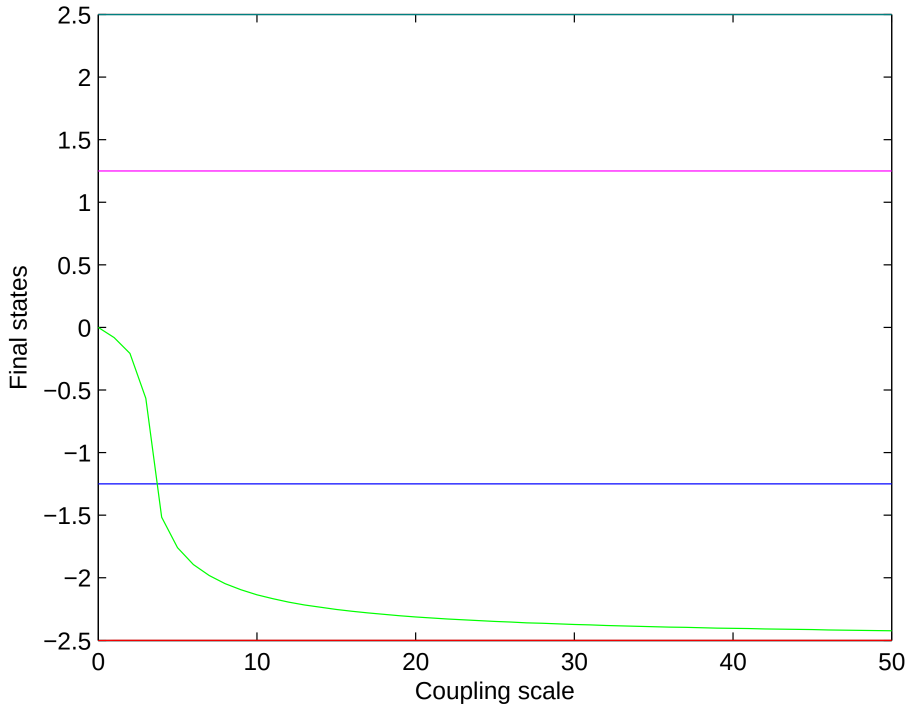

Figure 14b: Plot of final states as a function of coupling scale for a 5-node star graph, $\Delta_\mu = 5.0$, and all speaking durations = 10 secs. The communication sequence is $[A,\ B,\ C,\ D,\ E]$, where A is the node with maximum positive bias, B is the node with moderate positive bias, C is the hub node with zero bias, D is the node with moderate negative bias and E is the node with maximum negative bias.

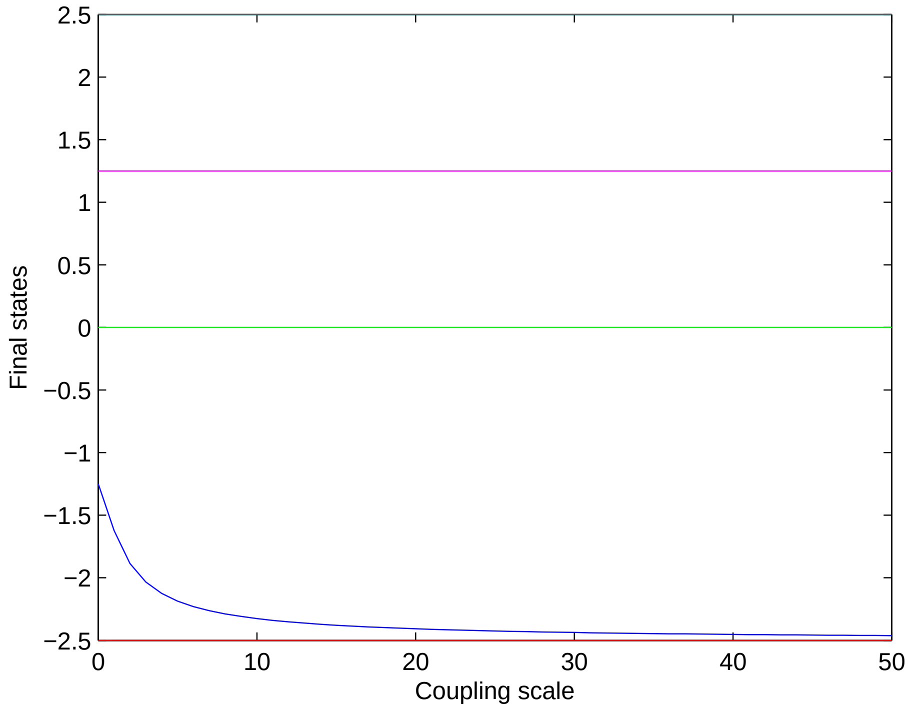

Figure 14c: Plot of final states as a function of coupling scale for a 5-node chain graph, $\Delta_\mu = 5.0$, and all speaking durations = 10 secs. The communication sequence is [A, B, C, D, E], where A is the node with maximum positive bias, B is the node with moderate positive bias, C is the central node with zero bias, D is the node with moderate negative bias and E is the node with maximum negative bias.

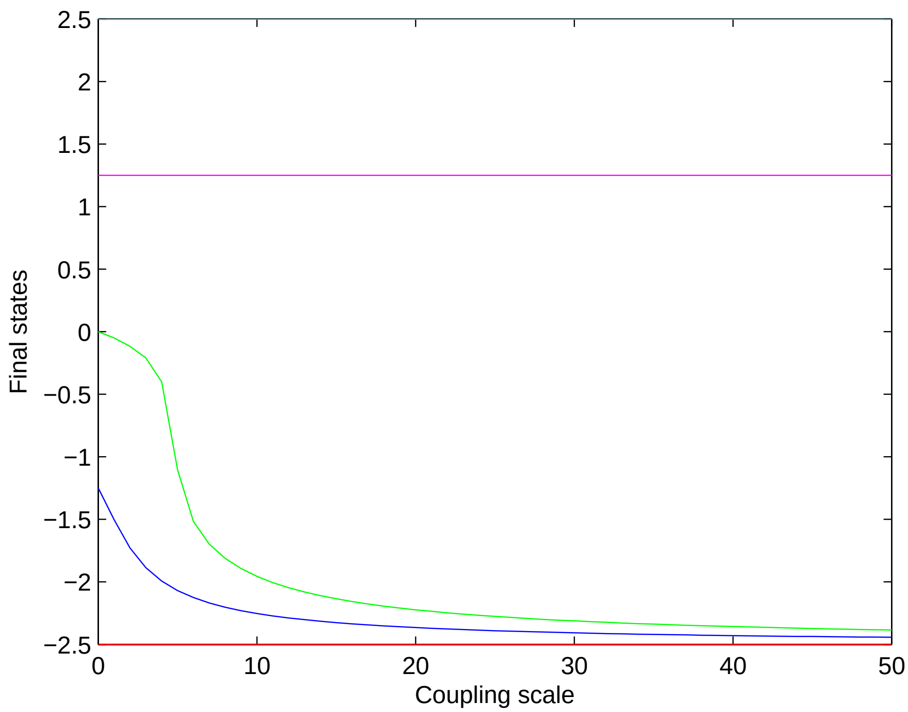

Figure 14d: Plot of final states as a function of coupling scale for a 5-node broker-A graph, $\Delta_\mu = 5.0$, and all speaking durations = 10 secs. The communication sequence is $[A, B, C, D, E]$, where A is the node with maximum positive bias, B is the node with moderate positive bias, C is the broker node with zero bias, D is the node with moderate negative bias and E is the node with maximum negative bias.

Appendix 3

Majority Rule as Spontaneous Symmetry Breaking in Small Opinion Networks

Majority Rule as Spontaneous Symmetry Breaking in Small Opinion Networks

Michael Gabbay* and Arindam K. Das†

Applied Physics Laboratory, University of Washington, Seattle, Washington 98105, USA

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Abstract

We study a model of opinion dynamics on networks for a continuous opinion axis which is nonlinear in the level of disagreement between dyads. For triad networks, asymmetric majority rule solutions are observed for initial opinions symmetrically distributed around that of the center node for complete and chain topologies. This occurs for both symmetric and asymmetric coupling between the center node and the extreme nodes and arises due to a symmetry-breaking pitchfork bifurcation. Analytical approximations for bifurcation boundaries are derived which agree well with numerically-obtained boundaries. Bifurcation-induced symmetry breaking represents a novel mechanism for generating majority rule outcomes, providing a route to extreme decisions without pre-existing structural or dynamical asymmetries, one in which, however, the policy outcome is fundamentally unpredictable.

* gabbay@uw.edu

† Current address: Eastern Washington University, Dept. of Engineering and Design, Cheney, WA 99004

I. INTRODUCTION

Modeling the dynamics of opinion change in social networks has received considerable attention recently from physicists and the broader network science community [1–6]. While the overwhelming focus of this work has been on large networks, within social science however, there is a rich tradition of studying small group opinion dynamics [7, 8]. Phenomena such as groupthink, group polarization, conformity, and minority influence have been the subject of extensive theoretical and experimental research [9] and have important implications for decision making by political leaders, corporate management, judicial councils, juries, and military, medical, and other professional teams. Mathematical models of small group decision making have been proposed in a variety of the social sciences [10–14]. In this paper, we provide an example of how a straightforward application of nonlinear dynamical systems theory can reveal a novel mechanism that may operate in group attitude change processes and decision making: the generation of *asymmetric* outcomes of majority rule along a continuous opinion axis for groups with *symmetric* initial opinion distributions and network structure, occurring under conditions of high initial disagreement between the opposite ends of the distribution. For a triad composed of a centrist and two opposed extremists, the centrist essentially splits the difference with one of the extremists to form the majority rule pair (see Fig. 1(b)).

Theories of group decision making based on psychological mechanisms of attitude change typically emphasize a convergence process whereby group member opinions converge toward each other and the mean, whereas, on the other hand, theories rooted in sociology tend to emphasize status inequalities which may skew opinions toward influential group members. The majority rule mechanism that we put forth in this paper is not readily anticipated from either of these perspectives, alone or in tandem. In fact, the psychological convergence perspective and the sociological focus on status and social networks have been combined in the "social influence network theory" of Friedkin and Johnsen, a highly influential network-based model of small group dynamics, which over two decades of research has been used to fruitfully address a range of small group phenomena and has received experimental support [14]. Yet, because the Friedkin-Johnsen model is linear in the disagreement between member opinions, final states in which the member opinions are asymmetrically distributed around the mean must arise from structural asymmetries, either skewed initial opinion distributions

or lopsided coupling weights in favor of one extreme. However, by making member interactions nonlinear in disagreement so that the interaction force eventually wanes past a certain disagreement level — an entirely reasonable assumption, one supported by attitude change research — highly asymmetric outcomes can result from symmetric conditions as a result of spontaneous symmetry breaking induced by the onset of a pitchfork bifurcation at high disagreement levels.

An important implication of our result concerns the spread of ideological extremism. This has been a common motivating context in many recent models of opinion networks in which extremists are assumed to be more resistant to persuasion than moderates [15–18]. Significantly, given its occurrence at high disagreement, the bifurcation-induced majority rule mechanism proposed herein can result in extreme majority opinions and thereby provides for an alternative route to extreme opinions and associated policy decisions that is not due to a built-in persuasional asymmetry. The routes to extreme opinions and decisions that have been identified in the psychological literature on group polarization also involve a pre-existing asymmetry such as the presence of a shared norm or a collection of like-minded individuals initially inclined toward a given attitude [9, 19]. In contrast, extreme disagreement, rather than shared predispositions, is crucial to the symmetry-breaking mechanism, which flows directly from the opinion model dynamics when subject to inevitable perturbations. In addition, the fact that this route to majority rule is induced by a bifurcation in which the symmetric equilibrium becomes unstable points to the fundamentally unpredictable nature of the direction of the outcome itself — whether majority opinions or policies are softline or hardline, conciliatory or confrontational, reactionary or radical.

This paper proceeds as follows: Section II describes the nonlinear opinion dynamics model we study and relates it to other models. Section III presents simulation results showing the behavior of triads with chain and complete topologies. Section IV demonstrates the pitchfork nature of the bifurcation to majority rule, derives analytical approximations for boundaries between different solution regimes, and compares these with boundaries obtained via numerical analysis. Concluding remarks are made in Sec V.

II. NONLINEAR OPINION DYNAMICS MODEL

We use the nonlinear opinion dynamics model presented in Refs. [20, 21] which evolves group member opinions along a one-dimensional continuum x in response to forces which arise due to group influence and member ideological predispositions. The opinion x_i of the i^{th} member in an N-person group evolves as

$$\frac{dx_i}{dt} = -\gamma_i(x_i - \mu_i) + \sum_{j=1}^{N} \kappa_{ij} h(x_j - x_i). \tag{1}$$

The first term on the right is a linear "self-bias force" which expresses the psychological tension or cognitive dissonance that a person feels if her opinion is displaced from her *natural bias* μ_i; it is proportional to her *commitment* γ_i. The second term is the "group influence force" on i where κ_{ij} is the *coupling strength* and $h(x_j - x_i)$ is the *coupling function.* The coupling strength, which we take to be non-negative, represents the components of influence of j upon i arising from their relationship; it depends on factors such as how often j communicates with i and the regard with which i holds j for the issue at hand due to j's expertise, wisdom, status, or friendship. The coupling function represents how the influence of j upon i depends on the difference between their opinions. We use a dependence in which the force grows for $|x_j - x_i| < \lambda_i$, where λ_i is i's *latitude of acceptance*, but wanes for differences in excess of λ_i:

$$h(x_j - x_i) = (x_j - x_i) \exp\left[-\frac{1}{2}\frac{(x_j - x_i)^2}{\lambda_i^2}\right]. \tag{2}$$

Experimental support for an eventual turndown in influence like that assumed in (2) comes from the attitude change research associated with social judgment theory [9]. Substituting (2) into (1) yields the nonlinear model,

$$\frac{dx_i}{dt} = -\gamma_i(x_i - \mu_i) + \sum_{j=1}^{N} \kappa_{ij}(x_j - x_i) e^{-\frac{1}{2}\frac{(x_j - x_i)^2}{\lambda_i^2}}. \tag{3}$$

The above model is most directly related to that of Friedkin and Johnsen. The Friedkin-Johnsen model describes the temporal evolution of a linear discrete time influence process in a group of N people as a weighted average of their previous opinions and their initial opinions [14]:

$$x_i(t_{k+1}) = (1 - w_{ii}) \sum_{j=1}^{N} w_{ij} x_j(t_k) + w_{ii} x_i(0), \tag{4}$$

where the w_{ij} are non-negative interpersonal coupling weights with $\sum_{j=1}^{N} w_{ij} = 1$, w_{ii} is a self-weighting, and $x_i(0)$ is i's initial opinion.

We now show that the Friedkin-Johnsen model can be made equivalent to the linear limit of the nonlinear model (3). Equation (4) can be cast as a difference equation by subtracting $x_i(t_k) = (1 - w_{ii} + w_{ii})x_i(t_k)$ from both sides and rearranging to yield

$$\begin{aligned} x_i(t_{k+1}) - x_i(t_k) = &-w_{ii}\left(x_i(t_k) - x_i(0)\right) \\ &+(1 - w_{ii})\sum_{j=1}^{N} w_{ij}(x_j(t_k) - x_i(t_k)), \end{aligned} \tag{5}$$

where we have also made use of the unit sum of the coupling weights. The continuous time analog of the Friedkin-Johnsen model can accordingly be written as

$$\begin{aligned} \frac{dx_i}{dt} = &-w_{ii}\left(x_i(t_k) - x_i(0)\right) \\ &+(1 - w_{ii})\sum_{j=1}^{N} w_{ij}(x_j(t_k) - x_i(t_k)). \end{aligned} \tag{6}$$

Comparing this with the linear limit, $\lambda_i \to \infty\ \forall i$, of Eq. (3), it is seen that the nonlinear opinion dynamics model reduces to the continuum version of the Friedkin-Johnsen model where μ_i is identified with $x_i(0)$ and up to the constraint on the sum of the w_{ij} not demanded of γ_i and κ_{ij}.

If we set $w_{ii} = 0$, then Eq. (6) becomes

$$\frac{dx_i}{dt} = \sum_{j=1}^{N} w_{ij}(x_j(t) - x_i(t)), \tag{7}$$

which has been referred to as the consensus or agreement protocol in the literature on distributed network control. Its convergence properties have been intensively studied and it has been applied to the small network context [22–24]. Nonlinear variants have been proposed as well [25]. When applied to the opinion context, however, the consensus protocol suffers from the problem that member opinions will all converge to exactly the same value for a (bidirectionally) connected network. This runs counter to intuitive expectations that disagreements need not be completely extinguished among a set of communicating individuals as well as empirical evidence that opinion diversity does indeed survive in connected political discussion networks [26]. The persistence of the initial opinions $x_i(0)$ in the Friedkin-Johnsen model inhibits such complete convergence of connected members upon a single point along the opinion axis as does the self-bias force in our nonlinear model.

Rather than demanding precise agreement, consensus can be said to be reached in the nonlinear model if the final opinions of the group members are sufficiently close. This is reasonable in the context of group decision making in which group members can agree to embark upon a common policy or course of action if their opinions are close enough together even if individuals have some reservations about that policy. The latitude of acceptance provides a length scale with which the specification of "sufficiently close" can be referred to meaningfully — for instance if all final opinions fall within the λ_i of all the group members — whereas such a specification would be more arbitrary in the Friedkin-Johnsen model.

Other models of continuous opinion dynamics in which the dyadic interaction changes nonlinearly with distance have been proposed. These employ hard threshold nonlinearities in which the interaction force vanishes beyond a certain critical distance as well as continuously varying ones [15, 27–29]. As these models were developed to address large network questions, they do not display interesting dynamics for the small networks of concern herein. Furthermore, they lack an analog to the self-bias force and so exhibit the same problematic behavior as the consensus protocol in the generation of exact consensus or, as is also observed, exact agreement within node clusters located at discrete locations along the opinion axis in which the clusters no longer interact.

III. TRIAD NETWORK SIMULATIONS

We consider a three-person group in which the natural biases are symmetrically distributed around zero: $\mu_1 = -\Delta\mu/2$, $\mu_2 = 0$, and $\mu_3 = \Delta\mu/2$. We compare two topologies. One is a chain whose ends — nodes 1 and 3 with opposite natural biases — are not connected so that the symmetric, binary adjacency matrix elements are $A_{12} = A_{21} = A_{23} = A_{32} = 1$ and $A_{13} = A_{31} = 0$ (and we set $A_{ii} = 0$). The other is a complete network in which all members are connected, $A_{ij} = 1 - \delta_{ij}$ where δ_{ij} is Kronecker's delta. We introduce the parameter ν to allow for the possibility of asymmetric coupling between the center node 2 and the end nodes so that $\kappa_{12} = \kappa_{32} = \kappa + \nu$ and $\kappa_{21} = \kappa_{23} = \kappa - \nu$ where $|\nu| < \kappa$. We set the latitudes of acceptance and the commitments to unity: $\lambda_i = 1$, $\gamma_i = 1$ $\forall i$. The equations

of motion for the triad are then:

$$\begin{aligned}
\frac{dx_1}{dt} &= -\left(x_1 + \frac{\Delta\mu}{2}\right) + (\kappa+\nu)h(x_2 - x_1) \\
&\quad +\kappa A_{31} h(x_3 - x_1), \\
\frac{dx_2}{dt} &= -x_2 + (\kappa-\nu)(h(x_1 - x_2) + h(x_3 - x_2)), \\
\frac{dx_3}{dt} &= -\left(x_3 - \frac{\Delta\mu}{2}\right) + (\kappa+\nu)h(x_2 - x_3) \\
&\quad +\kappa A_{31} h(x_1 - x_3).
\end{aligned} \tag{8}$$

It will be useful to define the following pair of variables: the *discord*, $r = x_3 - x_1$, the opinion difference between the outer nodes; and the *asymmetry*, $s = (x_3 - x_2) - (x_2 - x_1)$, the difference in distances from the outer nodes to the middle node.

Figure 1 shows simulations of the chain network for three values of the coupling strength κ and with symmetric coupling between all nodes ($\nu = 0$). The difference in the natural biases of the end nodes is $\Delta\mu = 5$ and the initial opinions are set equal to the natural biases although we perturb the center node by a tiny displacement, $x_2(0) = 10^{-6}$ (this is done so that, for asymmetric solutions, x_2 always moves in the same direction for visualization purposes and because numerical error which typically perturbs numerical solutions off of unstable equilibria need not do so for the special case of $x_2(0) = \mu_2 = 0$). Three qualitatively distinct equilibria are observed. At low coupling, Fig. 1(a) shows a state of Symmetric High Discord (SHD) in which the end nodes barely move from their natural biases and the center node remains at zero. At intermediate coupling, Fig. 1(b) shows the Majority Rule (MR) state in which the center node moves toward one of the end nodes to form a majority rule pair. At high coupling, the outer nodes move considerably toward the center to form a Symmetric Low Discord (SLD) state as shown in Fig. 1(c). The SHD state corresponds to a deadlock situation in which all group members are far apart and no acceptable mutual decision can be made. In the MR state, the majority pair can likely agree on a common policy which will become the policy of the group as long as majority rule is sufficient for reaching a decision. In the SLD state, the distance between the outer nodes is much reduced and the basis for a compromise around the centrist's position could be set. Based on intuitive expectations of a psychological convergence process, only the deadlock and compromise outcomes of the SHD and SLD states should result and not the asymmetric MR state.

The behavior is now compared at low and high initial disagreement for both the chain

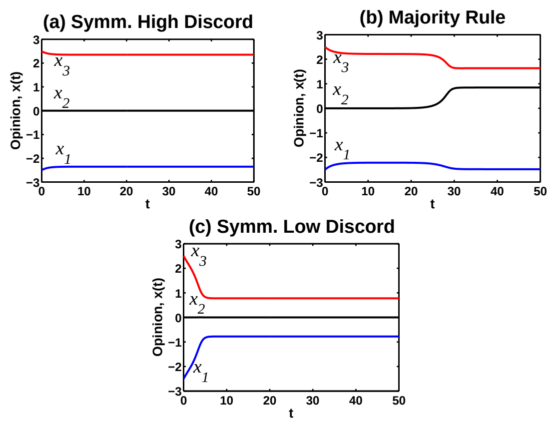

FIG. 1. (Color online) Node opinions vs. time showing equilibrium outcomes in symmetrically-coupled ($\nu = 0$) triad chain network with high initial disagreement, $\Delta\mu = 5$, at different coupling strengths: (a) $\kappa = 1$, Symmetric High Discord; (b) $\kappa = 1.5$, Majority Rule; (c) $\kappa = 3$, Symmetric Low Discord. Initial conditions: $x_1(0) = -2.5,\ x_2(0) = 10^{-6},\ x_3(0) = 2.5$.

and complete topologies. We define the coupling scale α as the average of the summed couplings impacting upon each node,

$$\alpha = \frac{1}{N}\sum_{i,j=1}^{N} \kappa_{ij}, \tag{9}$$

in order to provide a basis for fixed cost comparison across different network topologies under the assumption that the factors underpinning the coupling strengths such as communications, expertise, and friendship have associated costs [20]. Figures 2 and 3 show simulations of the chain and complete triads for the symmetric coupling case ($\nu = 0$) in which the final equilibrium node positions are plotted as a function of the coupling scale. The node initial positions are set to their natural biases (with the perturbation $x_2(0) = 10^{-6}$ as above). Figure 2 shows a case of a relatively low initial disagreement of $\Delta\mu = 1.5$. We observe that the final node opinions vary smoothly with coupling scale and that the outer nodes end up symmetrically situated about the center node in accordance with expectations of a psychological convergence process toward the mean. From Fig. 2(c), we also see that the difference between the final opinions of the outer nodes in the complete network is less than in the chain throughout the coupling scale range as would be expected from the intuitive association of higher social cohesion with a greater density of network ties [30].

The high initial disagreement case, $\Delta\mu = 5$, presents a striking contrast. As seen in Figs. 3(a) and (b), smooth variation is replaced by discontinuous transitions which demar-

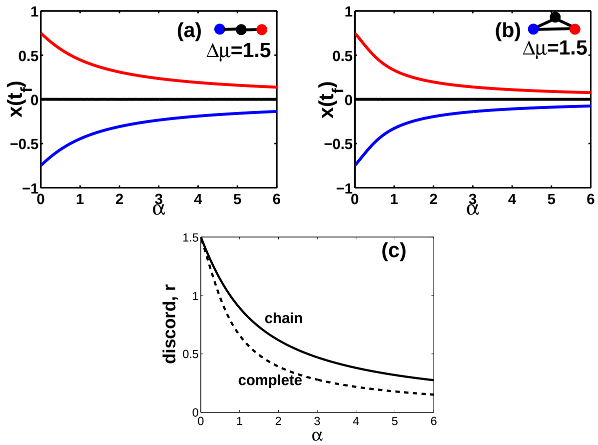

FIG. 2. (Color online) Low natural bias difference behavior for symmetrically-coupled triad, $\Delta\mu = 1.5$. Dependence upon coupling scale α of: (a) chain network final opinions,(b) complete network final opinions; (c) discord for chain (solid) and complete (dashed) networks (asymmetries are 0 and therefore not plotted). Initial conditions: $x_1(0) = -0.75,\ x_2(0) = 10^{-6},\ x_3(0) = 0.75$. Each simulation was run for a duration $t_f = 200$.

cate three qualitatively distinct zones of behavior: the SHD state at low values of α, the asymmetric MR state at intermediate α values, and the SLD state at high α.

The comparison of discord and asymmetry in Figs. 3(c) and (d) shows that the chain network undergoes transitions at lower coupling scales to the MR and SLD states than the complete network despite having fewer ties. The earlier transition to the MR state indicates that lower tie density may have advantages for reaching a decision at high initial disagreement, albeit a non-consensus one. The earlier transition of the chain to SLD, however, shows that for high natural bias differences, it can be beneficial with respect to group cohesion (as assessed by discord) to have a lower tie density, counter to the behavior at low differences. The intuitive reason for this is that communications between extremely divergent group members is essentially wasted and so greater communication via a moderate member is more effective [20]. Consequently, if we simply neglect the terms involving A_{31} in Eqs. (8), then the chain and complete network equations become identical implying that the transitions occur for the same κ value. Given the definition of coupling scale (9), the ratio of the complete (CO) to chain (CH) coupling scales at the transitions should be approximately equal to the ratio of their mean degrees: $\alpha_{CO}/\alpha_{CH} = \bar{d}_{CO}/\bar{d}_{CH} = 2/(4/3) = 1.5$. This is in excellent agreement with the numerically observed ratio of $2.36/1.57 = 1.50$ for the transi-

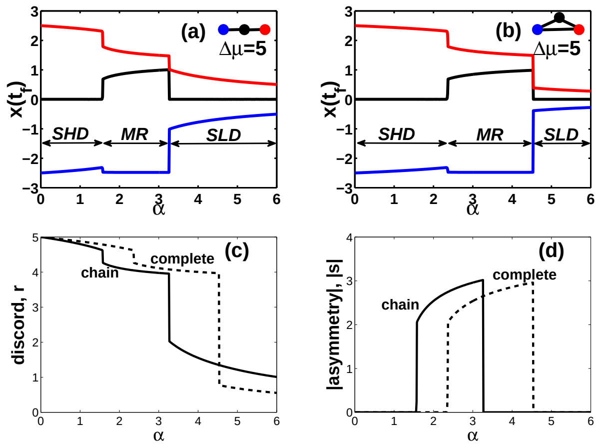

FIG. 3. (Color online) High natural bias difference behavior for symmetrically-coupled triad, $\Delta\mu = 5$. Dependence upon coupling scale α of: (a) chain network final opinions; (b) complete network final opinions; (c) discord for chain (solid) and complete (dashed) networks; (d) absolute value of asymmetry for chain (solid) and complete (dashed). Initial conditions: $x_1(0) = -2.5$, $x_2(0) = 10^{-6}$, $x_3(0) = 2.5$. Simulation duration: $t_f = 200$. Equilibria labels: SHD = Symmetric High Discord; MR = Majority Rule; SLD = Symmetric Low Discord.

tion between the SHD and MR states which directly corresponds to a bifurcation. It is also in rough agreement with the value of $4.54/3.26 = 1.39$ for the MR to SLD transition which corresponds to the natural bias initial conditions moving from the MR basin of attraction to the SLD one.

IV. BIFURCATION ANALYSIS

In this section, we demonstrate that the majority rule outcome is the result of a symmetry-breaking pitchfork bifurcation, we calculate analytical approximations for three of the bifurcation boundaries separating distinct solution regimes, and we plot numerically-determined boundaries for symmetric and asymmetric ($\nu \neq 0$) coupling cases.

A. Onset of Majority Rule due to Pitchfork Bifurcation

We transform the individual node opinions into the discord r and the asymmetry s. Additionally, we form the mean $\bar{x} = \frac{1}{3}\sum_{i=1}^{3} x_i$. This results in the equations:

$$\frac{dr}{dt} = -(r - \Delta\mu) - (\kappa + \nu)\left(h(\frac{r+s}{2}) + h(\frac{r-s}{2})\right)$$
$$-2\kappa A_{31} h(r), \tag{10}$$
$$\frac{ds}{dt} = -s - (3\kappa - \nu)\left(h(\frac{r+s}{2}) - h(\frac{r-s}{2})\right), \tag{11}$$
$$\frac{d\bar{x}}{dt} = -\bar{x} - \frac{2}{3}\nu\left(h(\frac{r+s}{2}) - h(\frac{r-s}{2})\right), \tag{12}$$

where we have made use of the coupling function being odd, $h(-x) = -h(x)$. Denoting the equilibrium discord and asymmetry by r^* and s^* respectively, the last equation yields for the mean opinion at equilibrium,

$$\bar{x}^* = -\frac{2}{3}\nu\left(h(\frac{r^*+s^*}{2}) - h(\frac{r^*-s^*}{2})\right), \tag{13}$$

which shows that for nonzero ν in the MR state ($s^* \neq 0$), the mean equilibrium opinion will be shifted from the natural bias mean of zero.

For the equilibrium SHD state, the asymmetry is $s^* = 0$. Rather than calculating the Jacobian of the full system, it suffices to consider small perturbations s around zero in Eq. (11) while assuming fixed $r = r^*$. The Taylor expansion is

$$\frac{ds}{dt} \approx -\left(1 + (3\kappa - \nu)h'(\frac{r^*}{2})\right)s - \frac{1}{24}(3\kappa - \nu)h'''(\frac{r^*}{2})s^3, \tag{14}$$

where only the odd power terms survive. This shows that the symmetric state will be unstable when $1 + (3\kappa - \nu)h'(r^*/2) < 0$. When $h'''(\frac{r^*}{2}) > 0$, the cubic term damps the instability and the transition to the MR state is seen to be a supercritical pitchfork bifurcation as Eq. (14) can be readily rescaled to the associated normal form: $dy/dt = Ry - y^3$, where R is a control parameter [31]. When $h'''(\frac{r^*}{2}) < 0$, the pitchfork bifurcation is subcritical. The relevant zero crossing of $h'''(x) = (-x^4 + 6x^2 - 3)e^{-\frac{1}{2}x^2}$, marking the boundary between supercritical and subcritical bifurcations, occurs at $x = (3 + \sqrt{6})^{1/2}$ corresponding to a discord of $r^* = 4.66$. As seen in Fig. 4, the supercritical bifurcation displays a smooth loss of stability whereas the subcritical one exhibits a discontinuous loss of stability, multistability, and hysteresis.

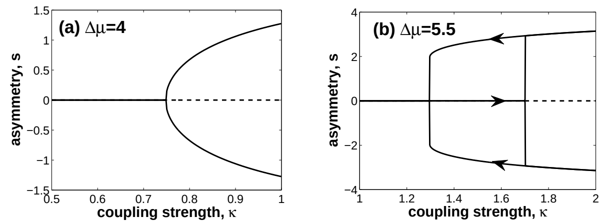

FIG. 4. Bifurcation diagrams showing asymmetry vs. coupling strength: (a) supercritical pitchfork bifurcation for $\Delta\mu = 4$; (b) subcritical pitchfork bifurcation for $\Delta\mu = 5.5$ — in the multistable region, right and left arrows mark branches for natural bias and majority rule initial conditions respectively, showing hysteresis as κ is increased and then decreased. Dashed line indicates unstable symmetric solution.

B. Calculation of Solution Regime Boundaries for Chain Network

We proceed to present derivations of approximate analytical expressions for the following boundaries in $\Delta\mu$-κ parameter space: κ_1 which forms the upper boundary of the SHD state; κ_2, the lower boundary of the MR state in the subcritical regime; and κ_3, the lower boundary of the SLD state. These approximations are calculated for the chain topology, $A_{31} = 0$ in Eq. (10), and are valid for large $\Delta\mu$.

1. *SHD Upper Boundary: κ_1*

From Eq. (14), the critical value of the coupling strength κ_1 at which the SHD state becomes linearly unstable is

$$\kappa_1 = -\frac{1}{3h'(\frac{r^*}{2})} + \frac{\nu}{3}. \tag{15}$$

Although the equilibrium discord r^* in the SHD state is close to $\Delta\mu$, a better approximation for κ_1 can be derived if we account for the decrease of r^* with κ as can be seen in Fig. 3(c). To first order at large $\Delta\mu$ this can be obtained by taking $r^* = \Delta\mu + \theta$ where $\theta \ll 1$. Since $s = 0$, Eq. (10) becomes

$$\frac{dr}{dt} = -(r - \Delta\mu) - 2(\kappa + \nu)h(\frac{r}{2}), \tag{16}$$

which upon substituting the above form for r^* yields

$$0 = \theta + 2(\kappa + \nu)h(\frac{\Delta\mu + \theta}{2}). \tag{17}$$

Expanding the coupling function as $h(\frac{\Delta\mu+\theta}{2}) \approx h(\frac{\Delta\mu}{2}) + h'(\frac{\Delta\mu}{2})\frac{\theta}{2}$ and substituting into (17) enables us to solve for θ as

$$\theta = -\frac{2(\kappa+\nu)h(\frac{\Delta\mu}{2})}{1+(\kappa+\nu)h'(\frac{\Delta\mu}{2})}, \tag{18}$$

where $h'(\frac{\Delta\mu}{2}) = (1-\frac{\Delta\mu^2}{4})e^{-\frac{\Delta\mu^2}{8}}$ (note that without confusion we write $(\Delta\mu)^n$ as $\Delta\mu^n$ in this paper).

The condition (15) for the critical coupling strength is expanded as

$$\kappa_1 \approx -\frac{1}{3}\left\{\frac{1}{h'(\frac{\Delta\mu}{2})} - \frac{h''(\frac{\Delta\mu}{2})}{h'^2(\frac{\Delta\mu}{2})}\frac{\theta}{2}\right\} + \frac{\nu}{3}. \tag{19}$$

The expression (18) for θ can then be inserted into the above which, after rearranging, yields the characteristic equation

$$\begin{aligned} 0 = 3h'(\frac{\Delta\mu}{2})\kappa_1^2 + \left(4 + M + 2\nu h'(\frac{\Delta\mu}{2})\right)\kappa_1 \\ +\frac{1}{h'(\frac{\Delta\mu}{2})} + M\nu - \nu^2 h'(\frac{\Delta\mu}{2}), \end{aligned} \tag{20}$$

where $M = \frac{\Delta\mu^4 - 12\Delta\mu^2}{(\Delta\mu^2-4)^2}$. This can be solved to give the following approximation for κ_1:

$$\begin{aligned} \kappa_1 \approx \frac{2}{3}\frac{e^{\frac{\Delta\mu^2}{8}}}{(\Delta\mu^2-4)}\Big\{4 + M + 2\nu h'(\tfrac{\Delta\mu}{2}) - \big[4 + 8M + M^2 \\ +8\nu h'(\tfrac{\Delta\mu}{2})\left(2 - M + 2\nu h'(\tfrac{\Delta\mu}{2})\right)\big]^{\frac{1}{2}}\Big\}. \end{aligned} \tag{21}$$

Equation (21) is valid for large $\Delta\mu$ and increases rapidly as $\Delta\mu$ becomes very large. The appearance of ν as a product with the very small $h'(\frac{\Delta\mu}{2})$ implies that κ_1 will essentially be unchanged from the $\nu = 0$ case as $\Delta\mu$ gets large, showing that coupling asymmetry between the extremes and the center will have negligible effect.

2. MR Lower Boundary in Subcritical Zone: κ_2

Turning now to the disappearance of the Majority Rule solutions in the subcritical bifurcation regime, this corresponds to the transition between the multistable zone where the MR and SHD states coexist to the zone in which only the SHD state exists as the coupling strength is lowered. This transition occurs via a saddle-node bifurcation in which stable and unstable asymmetric equilibria collide. The associated bifurcation boundary κ_2 can be

calculated by analyzing Eq. (11) around the MR equilibrium in which the minority node x_1 stays near its natural bias while the majority pair (x_2, x_3) is very nearly symmetrically positioned around the midpoint between their natural biases, $\Delta\mu/4$, as in Fig. 3(a). Asymmetric coupling, $\nu \neq 0$, will shift the equilibrium mean of the majority rule pair by an amount ε defined by

$$\varepsilon = \frac{(x_2^* + x_3^*)}{2} - \frac{\Delta\mu}{4}. \tag{22}$$

Accordingly, we make the approximations for the outer opinion coordinates:

$$x_1 \approx -\frac{\Delta\mu}{2}, \quad x_3 \approx \frac{\Delta\mu}{2} + 2\varepsilon - x_2. \tag{23}$$

The asymmetry is then $s = (x_3 - x_2) - (x_2 - x_1) = -3x_2 + 2\varepsilon$. Rearranging yields $x_2 = -s/3 + 2\varepsilon/3$ and then $x_3 = s/3 + \Delta\mu/2 + 4\varepsilon/3$ so that the discord can now be written in terms of s as $r = x_3 - x_1 = s/3 + \Delta\mu + 4\varepsilon/3$.

For large $\Delta\mu$, $x_2 - x_1 = (r - s)/2$ is large and we can neglect the term $h((r - s)/2)$ in Eq. (11). The argument of the coupling function term retained from Eq. (11) is $(r + s)/2 = \frac{2}{3}(s + \frac{3}{4}\Delta\mu + \varepsilon)$. Finally, we transform to the variable $\tilde{s} = s + 3\Delta\mu/4 + \varepsilon$ and Eq. (11) becomes

$$\frac{d\tilde{s}}{dt} = -(\tilde{s} - \frac{3}{4}\Delta\mu - \varepsilon) - (3\kappa - \nu)h(\frac{2}{3}\tilde{s}). \tag{24}$$

Equation (13) for the three-node mean $\bar{x}^*$ can be used to calculate the mean of x_2^* and x_3^* as follows:

$$\begin{aligned} \frac{1}{2}(x_2^* + x_3^*) &= \frac{1}{2}(3\bar{x}^* - x_1^*) \\ &= -\nu h(\frac{r^* + s^*}{2}) + \frac{\Delta\mu}{4}, \end{aligned} \tag{25}$$

where we have neglected the $h((r - s)/2)$ term in (13) and used $x_1^* \approx -\Delta\mu/2$. Substituting this into (22) yields

$$\varepsilon = -\nu h(\frac{r^* + s^*}{2}) = -\nu h(\frac{2}{3}\tilde{s}^*). \tag{26}$$

Taking $\nu \ll \kappa$, the first order contribution of ν resulting from the last term in Eq. (24) is given by $\nu h(\frac{2}{3}\tilde{s}^*)$ which therefore cancels out the ε term. Thus, we get

$$\frac{d\tilde{s}}{dt} = -(\tilde{s} - \frac{3}{4}\Delta\mu) - 3\kappa h(\frac{2}{3}\tilde{s}), \tag{27}$$

and we see that the effect of asymmetric coupling between the center and the extremes disappears for small ν and so will not impact the bifurcation boundary.

The equilibrium value for which the saddle-node bifurcation occurs is marked by the vanishing of the right-hand side of the above equation as well as its derivative, yielding upon rearrangement the conditions:

$$\tilde{s}^* - \frac{3}{4}\Delta\mu = -2\kappa_2 \tilde{s}^* e^{-\frac{2}{9}\tilde{s}^{*2}} \tag{28}$$

$$1 = -2\kappa_2(1 - \frac{4}{9}\tilde{s}^{*2})e^{-\frac{2}{9}\tilde{s}^{*2}}, \tag{29}$$

where κ_2 denotes the coupling strength at which the bifurcation occurs. Taking the ratio of (28) to (29) and rearranging yields the cubic equation

$$0 = \tilde{s}^{*3} - \frac{3}{4}\Delta\mu\tilde{s}^{*2} + \frac{27}{16}\Delta\mu. \tag{30}$$

For large $\Delta\mu$, the leading order solution is obtained by retaining the last two terms and is seen to be constant, $\tilde{s}^* \approx \frac{3}{2}$. Employing (28) to solve for κ_2 and then substituting in $\tilde{s}^* = \frac{3}{2}$ yields:

$$\kappa_2 = \frac{\frac{3}{4}\Delta\mu - \tilde{s}^*}{2\tilde{s}^*} e^{\frac{2}{9}\tilde{s}^{*2}} \tag{31}$$

$$\approx \left(\frac{\Delta\mu}{4} - \frac{1}{2}\right) e^{\frac{1}{2}}, \tag{32}$$

which makes evident that κ_2 increases linearly to leading order in $\Delta\mu$. However, a better approximation for κ_2 can be obtained if we use the $O(\frac{1}{\Delta\mu^2})$ solution to Eq. (30), which is given by

$$\tilde{s}^* = \frac{3}{2}\left(1 + \frac{1}{\Delta\mu} + \frac{5}{\Delta\mu^2}\right), \tag{33}$$

as can readily be verified. Using this approximation for $\tilde{s}^*$ in (31) gives

$$\kappa_2 \approx \frac{1}{4}\frac{\Delta\mu^3 - 2(\Delta\mu^2 + \Delta\mu + 5)}{\Delta\mu^2 + \Delta\mu + 5} e^{\frac{1}{2}(1+\frac{1}{\Delta\mu}+\frac{5}{\Delta\mu^2})^2}. \tag{34}$$

While the rapidly increasing κ_1 delineates when the MR state will arise from natural bias initial conditions, the linear dependence of κ_2 upon $\Delta\mu$ shows that the coupling strength for which the MR state becomes available does not outpace $\Delta\mu$. This is important because if we add a stochastic forcing to Eq. (1), due for instance to random incoming external information, then state-switching can take place in which the SHD state jumps to the MR state (and vice versa) as we have observed in simulations.

3. SLD Lower Boundary: κ_3

We now consider the transition in which the SLD state given by ($r^* = r^*, s^* = 0$) becomes absolutely unstable. The boundary itself, κ_3, can be calculated by simply considering Eq. (16) for the discord with respect to perturbations to r (although the MR state and not just the SHD state can be stable on the other side). Equation (16) will undergo a saddle-node bifurcation in which the low discord stable equilibrium present in the SLD state collides with an unstable intermediate discord one, leaving only the high discord equilibrium. In similar fashion to the calculation for κ_2, we set the righthand side of (10) and its derivative to zero to obtain the conditions:

$$r^* - \Delta\mu = -(\kappa_3 + \nu) r^* e^{-\frac{1}{8}r^{*2}} \tag{35}$$

$$1 = -2(\kappa_3 + \nu)(1 - \frac{r^{*2}}{4}). \tag{36}$$

Taking the ratio of the above pair and rearranging gives

$$0 = r^{*3} - \Delta\mu r^{*2} - 2r^* + 4\Delta\mu. \tag{37}$$

Near the bifurcation, the equilibrium discord for the SLD state is $r^* \approx 2$ to leading order, as can be obtained by retaining only the terms with $\Delta\mu$ in the coefficients in (37). Solving (35) for κ_3 and then inserting $r^* = 2$ gives:

$$\kappa_3 = \frac{\Delta\mu - r^*}{r^*} e^{\frac{1}{8}r^{*2}} - \nu \tag{38}$$

$$\approx \left(\frac{\Delta\mu}{2} - 1\right) e^{\frac{1}{2}} - \nu, \tag{39}$$

which exhibits a linear increase with $\Delta\mu$. Again, as with κ_2, a more accurate expression can be found by using the $O(\frac{1}{\Delta\mu^2})$ solution to Eq. (37),

$$r^* \approx 2 + \frac{1}{\Delta\mu} + \frac{9}{4}\frac{1}{\Delta\mu^2}, \tag{40}$$

which when substituted into (38) yields,

$$\kappa_3 \approx \frac{\Delta\mu^3 - 2\Delta\mu^2 - \Delta\mu - \frac{9}{4}}{2\Delta\mu^2 + \Delta\mu + \frac{9}{4}} e^{\frac{1}{8}\left(2 + \frac{1}{\Delta\mu} + \frac{9}{4}\frac{1}{\Delta\mu^2}\right)^2} - \nu. \tag{41}$$

While κ_2 and κ_3 share an approximate linear dependence upon $\Delta\mu$, κ_3 also depends linearly upon ν, thereby making it much more responsive to asymmetries in coupling between the center and extreme nodes as will be seen below.

C. Chain Stability Diagrams

Figure 5(a) shows the bifurcation boundaries and the regions in which the various equilibria are stable for the case of symmetric coupling between the middle and extreme nodes, $\nu = 0$. The open markers show boundaries obtained from a numerical bifurcation analysis using the MATCONT software package for prediction-correction continuation [32]. In addition to κ_1, κ_2, and κ_3, the boundary κ_4, beyond which the MR state disappears is also shown. We observe that the MR state is not present below $\Delta\mu$=3.8, which we refer to as the *critical divergence* of natural biases. The analytical approximations, Eqs. (21), (34), and (41), are in excellent agreement with the numerically-determined values for κ_1, κ_2, and κ_3 except in the immediate vicinity of the critical divergence. We note the existence of a zone in which only the MR state is stable as well as zones of multistability in which the initial conditions determine which equilibrium the system reaches and, if stochastic forcing is allowed, noise-induced transitions between states can take place. The results for the complete network are qualitatively similar [33].

Figures 5(b) and (c) show cases for negative ν, which corresponds to the extreme nodes having more influence on the middle node than vice versa. It is important to recognize that for $\nu < 0$, the mean of the majority rule pair will shift in the direction of the extreme member of the pair thereby making an ensuing MR decision more extreme than in the $\nu = 0$ case. The critical divergence is observed to become smaller so that the MR state arises at lower disagreement levels as is intuitively reasonable. Also, the MR-only zone expands due to the upward shift of κ_3 caused by $\nu < 0$ in (41) while κ_1 and κ_2 have negligible dependence upon ν at large $\Delta\mu$. Figure 5(d) shows a positive ν case corresponding to greater influence of the middle node on the extremes. The effect is to forestall the emergence of the MR state from natural bias initial conditions as the critical divergence shifts to higher $\Delta\mu$ compared to the symmetric coupling case. Also, the MR-only zone has all but disappeared due to the downward shift in κ_3. Yet the fact that the MR state is still present for $\nu > 0$ is significant because it shows that substantially skewed opinions and decisions can result even when the moderate is *more* influential than the extremists.

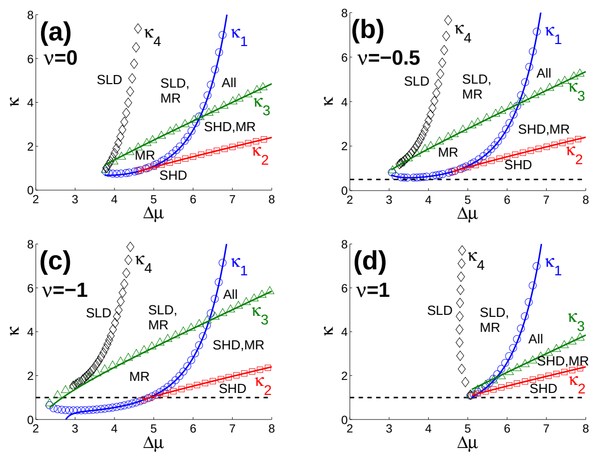

FIG. 5. (color online) Bifurcation boundaries and solution zones for chain: (a) $\nu = 0$; (b) $\nu = -0.5$; (c) $\nu = -1$; (d) $\nu = 1$. Markers are numerically determined bifurcation values and lines are analytical approximations from Eqs. (21), (34), and (41). Regions below dashed lines are disallowed due to negative coupling strengths.

V. CONCLUSION

In this paper, we have shown how asymmetric majority rule outcomes can result from symmetric conditions under the dynamics of our nonlinear model (3). This runs counter to first-blush intuition derived from qualitative consideration of basic psychological mechanisms of attitude change in which member opinions are assumed to converge on dyadic and group levels. To our knowledge, no other mathematical model of opinion change has been used to propose such a bifurcation-induced mechanism for majority rule. Our model is a nonlinear variant of the Friedkin-Johnsen model, a highly influential model of small group dynamics within the sociological and social network analysis communities. By the application of dynamical systems analysis methods, we have shown the possibility of a new route to majority rule that does not rely on built-in structural or dynamical asymmetries. More broadly, our work illustrates the potential of nonlinear dynamical systems theory to shed new insight into social science, even into areas that have long been studied such as small group dynamics.

That the majority rule outcome stems from a bifurcation implies that this phenomenon should be more general than the particular opinion dynamics model of Eq. (3) given the universal nature of bifurcations; for instance, if the coupling function has similar shape but different functional form than (2). Importantly, the behavior can be framed in essentially

qualitative terms in that majority rule should be manifest at high initial disagreement levels (i.e., above the critical divergence), not low or moderate ones. It therefore should be amenable to empirical validation via laboratory experiments using human subjects where precise quantification of opinions is difficult. Although the pitchfork bifurcation itself is not generic with respect to slight asymmetries in the natural bias distribution, the lopsided nature of the majority rule outcome persists; simulations in which the natural biases are shifted by small random amounts still display the same basic behavior as in Fig. 3. And while the value of a slightly off-center μ_2 theoretically determines the side on which the MR state falls, given the imprecise nature of real-world opinion measurement, the majority rule policy itself remains essentially unpredictable, at least at the level of resolution of group discussion processes represented by the model.

The nonlinear model we analyzed was developed for the context of decision making by political elites [21, 34] and our results have important implications for policy making. For highly controversial issues marked by two comparably strong advocates of opposing policies bracketing a moderate person, a deadlock or compromise around the middle — the two outcomes that would be most expected intuitively — may be less likely than a majority rule outcome in which the moderate swings toward one of the extremes; the direction being determined, not by more weighty factors like the fundamental merits of the policies themselves, but by relatively minor factors such as who speaks first or any slight inclination toward one side on the moderate's part. This bifurcation-induced majority rule mechanism represents a potential route to extreme decisions by the majority pair, the extremity being exacerbated the more influence that the extremist advocates have over the moderate ($\nu < 0$) as noted in Sec. IV C.

Our focus in this paper on triads is natural for the application to policy making given the importance of small groups in that context. We have also observed the majority rule outcome in simulations of various 5-node network topologies with natural biases uniformly distributed about zero and symmetric coupling [33]. However, triads are also important to social network processes in the general population as well, for example in the phenomenon of triad closure and the calculation of network clustering [35]. Significantly, core discussion networks in modern Western societies have been found to be very small, and in fact, quite close to triadic; people surveyed regarding with whom they discuss important matters reported an average of 2.03 such confidants in the United States and 2.4 in the Netherlands [36, 37].

While the strong tendency toward triad closure that operates in social networks might lead one to think that only the complete triad is of concern, a recent result reporting that 40% of all core discussion triads are not closed [37] signifies that both complete and chain triads should be considered in opinion dynamics within general social networks. Finally, we remark that although this paper concerns small networks, there is nothing in the formalism of Eq. (1) that prevents it from being used to model large opinion networks and we expect that bifurcation phenomena will occur there as well.

ACKNOWLEDGMENTS

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Appendix 4

Experimental Study of Persuasion and Decision Making in Small Networks

Experimental Study of Persuasion and Decision Making in Small Networks

Justin Reedy
Department of Communication and Center for Risk, Crisis, and Resilience
University of Oklahoma

Michael Gabbay
Applied Physics Laboratory
University of Washington

John Gastil
Department of Communication Arts & Sciences
Pennsylvania State University

We describe an experimental effort to test the predictions of a nonlinear model of opinion dynamics occurring over small social networks. Hypotheses derived from this model involve the efficacy of different network structures in reaching group decisions and producing consensus and the generation of majority rule outcomes. The experiments employ online discussion groups tasked to debate public policy issues and decide upon a particular policy option among three choices. Group member initial policy opinions are determined by a pre-survey and used to populate low and high disagreement level groups. Network structure is manipulated so that: (1) all group members can communicate directly; or (2) group members with opposing policy preferences can only communicate with a central member chosen to have an intermediate policy preference. After a specified minimum discussion time, group members are allowed to vote on policy options; post-discussion individual opinion change is gauged by another survey. We present a preliminary analysis of our experimental results with respect to our hypotheses and other measures of group discussion.

Experimental Study of Persuasion and Decision Making in Small Networks

Justin Reedy, Michael Gabbay, and John Gastil

In this paper, we present a preliminary analysis of an experiment concerning the interaction of network structure and disagreement level in small group opinion dynamics. In particular, we test hypotheses that arise out of a nonlinear model of opinion dynamics on networks (Gabbay, 2007a). The experiment involves online political discussions in three-person groups in which the group members debated about which of three economic policy options to support. Two network topologies were used as conditions: a clique in which all three group members could communicate with each other and a "broker" network in which two of the members could only communicate with a central person (equivalent to a chain topology). Two disagreement levels, low and high, were used. Analysis and simulation of the nonlinear model show that group outcomes in terms of the ability to reach a majority rule or consensus decision interacts with the network structure: cliques are expected to be more effective at bringing about consensus at low disagreement while broker networks are more effective in yielding group decisions – majority rule or consensus – at high disagreement.

The paper proceeds as follows. First, the nonlinear model of group decision making is briefly described. Second, the model dynamics for triad networks, which form the basis of our hypotheses, are presented. The third section describes the design of the experiment and the fourth section presents results.

Nonlinear Model of Group Decision Making

The nonlinear model is similar to the "social influence network theory" of Friedkin and Johnsen, the most prominent network-based model of small group opinion dynamics (Friedkin and Johnsen, 2011). Huckfeldt et al. (2004) used the Friedkin-Johnsen model as a component of their formal argument addressing the persistence of opinion diversity in political communication networks. The Friedkin-Johnsen model is linear in that it assumes that the force moving members of a dyad toward agreement grows linearly in proportion to the difference between their opinions. In contrast, the nonlinear model assumes that this dyadic force wanes past a certain critical disagreement level, known as the latitude of acceptance, eventually tending toward zero. Accordingly, this model has two regimes of behavior: a "linear" one, which essentially corresponds to the intuitive dynamics of the Friedkin-Johnsen model, and a "nonlinear" regime in which behaviors can run counter to initial intuition. The linear regime is characterized by: gradual changes in policy outcomes and the level of equilibrium group discord as parameters such as the coupling scale are varied; only one equilibrium for a given set of parameter values; lower group discord for higher network tie densities; and symmetric conditions of opinions and couplings always lead to symmetric final states. The nonlinear regime can exhibit the opposite behaviors: discontinuous transitions between deadlock and consensus as parameters are varied; multiple equilibria for a given set of parameter values; greater discord reduction in less dense networks; and asymmetric outcomes of majority rule even for symmetric conditions.

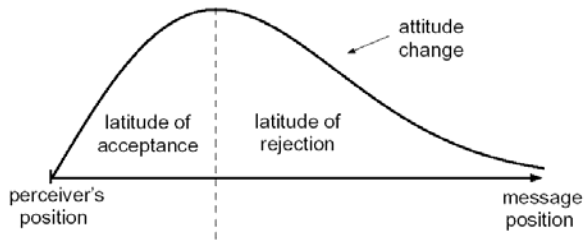

Figure 1. Coupling function in nonlinear model showing the strength of the influence force of one dyad member upon another as a function of the difference between their opinions.

The nonlinear model is formulated as follows: Denoting the opinion of the i^{th} group member by x_i, the mathematical equation which evolves x_i over time t is

$$\frac{dx_i}{dt} = -\gamma_i (x_i - \mu_i) + \sum_{j \neq i} \kappa_{ij} (x_j - x_i) \exp\left(-\frac{1}{2} \frac{(x_j - x_i)^2}{\lambda_i^2}\right),$$

where γ_i is the member's commitment; μ_i is the natural bias which also corresponds to i's initial opinion; κ_{ij}, the coupling strength parameter, scales the influence of member j on member i; and λ_i is i's latitude of acceptance. The first term on the right-hand side corresponds to the "self-bias force" that results when a member's opinion is displace from her natural ideological bias. The second term is the "group influence force" which is a function of the pairwise differences in opinion between i and the other group members to which she is connected. The use of the functional form parameterized by the latitude of acceptance is motivated by social judgment theory (Eagly and Chaiken, 1993). The entire system consists of N coupled nonlinear differential equations corresponding to the N members of the group.

Model Dynamics for Triad

Figure 2 shows simulation results for a triad broker network which is symmetric in both network structure and initial opinion distribution. The network is symmetric in the sense that the matrix of coupling strengths, κ_{ij}, is symmetric. The initial opinion distribution is symmetric because the natural biases of the extreme nodes, μ_1 and μ_3, are equally distant from the central node's natural bias μ_2. The level of disagreement between the two extreme is $\Delta\mu = 5$ which is high in comparison to the latitude of acceptance ,$\lambda = 1$, for all three nodes. Figure 2(a) is for a low value of the coupling strength and we observe that the node opinions do not change much from their initial positions, corresponding to an outcome of Symmetric High Discord (SHD). At intermediate coupling, Fig. 2(b) shows the Majority Rule (MR) state in which the center node moves toward one of the end nodes to form a majority rule pair. At high coupling, the outer nodes move considerably toward the center to form a Symmetric Low Discord (SLD) state as shown in Fig. 2(c). The SHD state corresponds to a deadlock situation in which all group members are far apart and no acceptable mutual decision can be made. In the MR state, the majority pair can likely agree on a common policy which will be the policy of the group if majority rule is sufficient for reaching a decision. In the SLD state, the distance between the outer nodes is much reduced and the basis for a compromise around the centrist's position could be set.

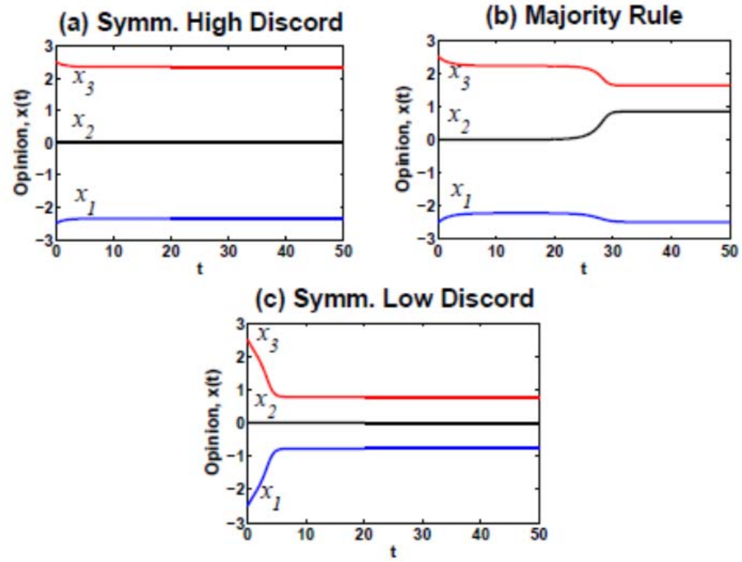

Figure 2. Member opinions vs. time showing equilibrium outcomes in symmetrically-coupled triad broker network with high initial disagreement $\Delta\mu = 5$ at different coupling strengths: (a) $\kappa = 1$, Symmetric High Discord; (b) $\kappa = 1{:}5$, Majority Rule; (c) $\kappa = 3$, Symmetric Low Discord. Initial conditions: $x_1(0) = -2{:}5$; $x_2(0) = 10^{-6}$; $x_3(0) = 2.5$. The central node is perturbed by a tiny amount to evaluate whether the symmetric state is stable.

Figure 3 compares the behavior at both low and high initial disagreement for both the broker and clique networks. We define the coupling scale as the average coupling strength, $\alpha = \sum_{i,j} \kappa_{ij} / N$, in order to provide a basis for fixed cost comparison across different network topologies under the assumption that the factors underpinning the coupling strengths such as communications, expertise, and friendship have associated costs. The upper panels, Fig. 3(a) and (b), show a case of relatively low initial disagreement level, $\Delta\mu = 3$. We observe that the final opinions vary smoothly with coupling scale and that the outer nodes end up symmetrically situated about the center node in accordance with expectations of a psychological convergence process toward the mean. The Friedkin-Johnsen model exhibits behavior similar to this at all disagreement levels. However, this is not the case for the nonlinear model as we now discuss.

The high initial disagreement case, $\Delta\mu = 5$, of the lower panels of Fig. 3 displays markedly different behavior. Smooth variation is replaced by discontinuous transitions which define three qualitatively distinct outcome regimes: the SHD or deadlock state at low coupling scales, the MR state at intermediate coupling scales, and the SLD state at high ones. The origin of the asymmetric majority rule outcome is due to symmetry-breaking arising from a pitchfork bifurcation in which the symmetric SHD state becomes unstable to small perturbations as the coupling scale increases past a critical value (Gabbay and Das, 2012).

Comparison between the broker and clique results shows that the broker network undergoes transitions at lower coupling scales to the MR and SLD states than the complete network despite having fewer ties. The earlier transition to the MR state indicates that lower tie density may have advantages for reaching a decision at high initial disagreement, albeit a non-consensus one. The earlier transition to SLD, however, shows that for high natural bias differences, it can be beneficial with respect to group cohesion to have a lower tie density, counter to the behavior at low differences. The intuitive reason for this is that communications between extremely divergent group members is essentially wasted and so greater communication via a moderate member is more effective (Gabbay, 2007b).

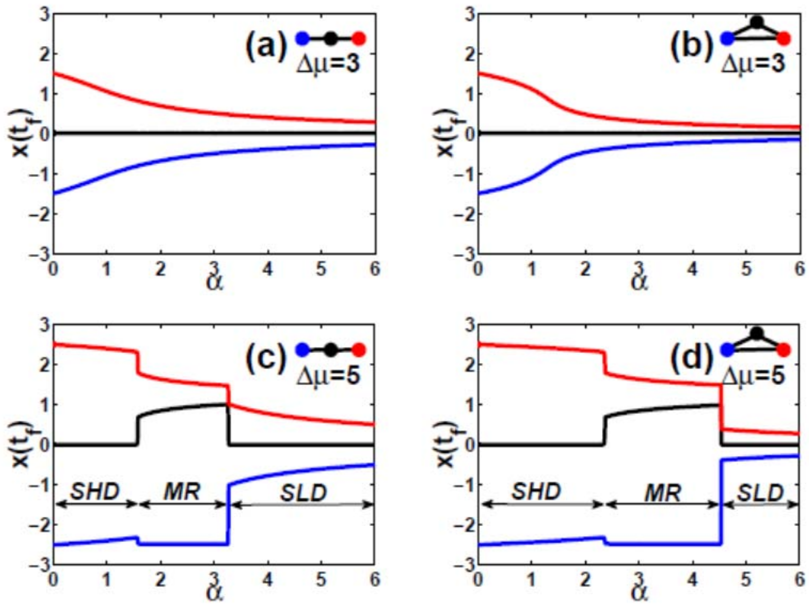

Figure 3. Final member opinions in a symmetrically-coupled triad for broker and clique networks and at low and high disagreement levels: (a) broker, low disagreement; (b) clique, low disagreement; (c) broker, high disagreement; (d) clique, high disagreement. SHD = Symmetric High Discord; MR = Majority Rule; SLD = Symmetric Low Discord.

The triad simulation results are used to motivate the following hypotheses concerning the outcomes of group deliberations:

H1: Majority rule outcomes should be more prevalent at high disagreement than low disagreement levels.

This hypothesis reflects the fact that there is a critical disagreement level below which the majority rule outcome is not present in the nonlinear model dynamics as seen in the upper panels of Figure 3.

H2: At high disagreement, the broker network should be more effective at reaching a group decision – majority rule or consensus – than a clique.

This hypothesis stems from the earlier transition to the MR state for the broker network as compared with the clique.

H3: At high disagreement, the broker network should be more effective at producing consensus.

This hypothesis stems from the earlier transition to the SLD state for the broker network as compared with the clique.

Experiment Design

To test the hypotheses listed above, we conducted a series of experiments involving 324 participants, each taking part in a three-member online discussion in a text-based chat system. Participants were recruited through Amazon Mechanical Turk, an online workplace where users perform small tasks online in exchange for pay. A larger pool of Turk workers were invited to take a pre-discussion survey with questions on political issues, as well as demographic questions. We then selected a portion of those individuals (based on their pre-discussion survey answers) to continue to the second phase of the study, in which they visited a website to login to an online chat system created by the study team and a computer programmer.

Participants then engaged in a three-person group discussion for approximately thirty minutes, with the discussion focusing on the issue of the United States economy and national debt. When they logged in, participants were provided with a set of instructions that tasked them with trying to reach agreement within their group on how best to address the debt and economy issue. They were given three policy choices to choose from: cutting taxes, a government-funded economic stimulus, or a compromise option of

raising taxes while also cutting social spending. Participants were given a stronger motivation to reach a decision by being told that an anonymous donor (in actuality, the study authors) will give a significant contribution to a cause or organization based on the recommendations of the discussion groups.

To determine the impact of varying levels of pre-discussion disagreement on group decision making, groups were created based on the level of initial disagreement on the debt/economy issue among the participants. Based on pre-discussion survey responses, participants were assigned a Political Alignment score, ranging from +2 (strongly supports tax cuts) to -2 (strongly supports economic stimulus), with those supporting the compromise position (raising taxes and cutting social spending) were assigned a 0. Participants were then assigned to either a high- or low-disagreement group. High-disagreement groups consisted of one member with a +2 alignment, one with a -2 alignment, and one with an alignment of 0. Low-disagreement groups, by contrast, had a member at +1 (somewhat supports tax cuts), another at -1 (somewhat supports stimulus), and a member at 0 (the compromise position).

In addition to the high- and low-disagreement conditions, participants were also assigned to one of two conditions related to the communication network that would be used within their group discussion. The online discussion program developed for this experiment allows manipulation of the communication network in a group discussion; that is, when creating discussion sessions, users can change who can speak with whom during the group discussion. In this experiment, participants were assigned to either a ‘clique’ style

of group, in which every group member could interact directly with each other, or a ‘broker’ style of group, in which a central member (in this case the person in the compromise position) acts as a go-between for the two other group members, who otherwise cannot communicate.

Participants were given a maximum of 30 minutes to discuss their topic, and were allowed to begin voting on their choices after 20 minutes had elapsed. Participants were encouraged to reach a majority decision (two out of three members agreeing) or consensus decision (all three members agreeing) in favor of one of the options, with a consensus decision giving the group greater weight in the decision being made by the anonymous donor. Discussions continued until the 30 minutes had elapsed, or until a consensus decision had been reached. Groups that reached a majority decision were allowed to continue discussing the topic in an attempt to reach consensus before the 30 minutes elapsed.

In the chat program interface, a section of the webpage next to the chat window showed participants’ three voting choices, each of which would encourage the anonymous donor to make a donation to an organization working in the area of the US economy and national debt and generally advocating a particular position. The voting options consisted of: “Make a significant donation to an organization advocating Cutting Taxes,” “Make a significant donation to an organization advocating a Government-Funded Economic Stimulus,” and “Make a significant donation to an organization advocating Raising Taxes and Cutting Social Spending.” Once the voting period began, 20 minutes into the

discussion, any group member could call for a vote, and a second was required by another group member to open a voting session. Participants then saw four buttons in the chat interface: Choosing one of the three options listed above, or choosing to abstain from the vote.

Group decision outcomes were measured for each vote taken by the group, as well as by the final decision reached by the group. Groups that were unable to reach a majority decision during a vote were coded as deadlocked; those that reached a majority or consensus were coded as such, and their group choice (tax cuts, economic stimulus, or raising taxes & cutting social spending) was also recorded. In addition, individual choices at each voting point were recorded and included in the data.

When the discussion ended, either due to the group reaching consensus or the 30 minutes of discussion time expiring, participants were directed to an online survey to record their post-discussion opinions and assess their views on the group discussion. Participants were asked their favored option of the three voting choices, as well as whether they agreed with their group decision and how much of an impact they believed their decision would have on the issue they discussed. The survey also asked how seriously other group members took the discussion, which was included to help check for (and later remove) groups that were not earnestly discussing the issue; only a handful of groups were identified as problematic and then excluded from our analysis.

Results

The experiments resulted in data from 108 group discussions with a total of 324 individual participants. The distribution of groups was split roughly equally across the experiment's four conditions: high-disagreement broker groups, high-disagreement clique groups, low-disagreement broker groups, and low-disagreement clique groups. Before presenting the results that bear upon our hypotheses, we discuss some other metrics which help characterize the group discussions.

Message Length and Volume

Two variables calculated from the group interaction data were Average Message Length, the average number of characters in the individual messages sent by each participant during their discussion, and Messages Sent, the total number of messages sent by each participant, which was a proxy measure for speaking turns within a discussion. High-disagreement groups, both broker and clique groups, generated more messages per person (mean value=24.7) than did low disagreement (mean=20.7), a difference that was statistically significant ($p=.021$). Average Message Length was also longer for high-disagreement groups (mean value=71.2 characters) than for low-disagreement groups (mean=64.5), which was also a significant difference ($p=.004$). The results indicate that high-disagreement groups seemed to be generating more discussion overall than low-disagreement groups, regardless of the communication network of the group.

Speaking Turns by Group Member Position

When analyzing differences in speaking turns by participants' position or political alignment within the group, an interesting pattern emerged. There were few differences in the number of speaking turns between group members in clique groups, in either the low- or high-disagreement conditions. However, in low-disagreement broker groups, the broker/moderate member had, on average, more messages sent than their two other group members. This difference was even larger in high-disagreement broker groups, as illustrated in Figure 4 below.

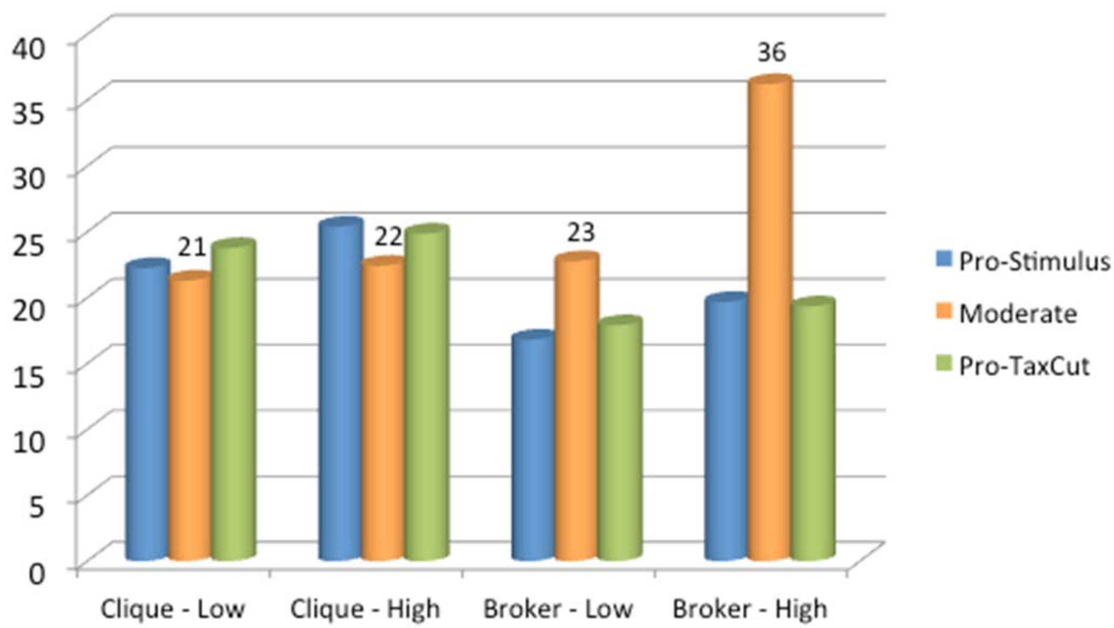

Figure 4: Distribution of speaking turns by political alignment and group type.

The results indicate that broker groups, especially those in the high-disagreement condition, created a situation for the moderate/broker figure that generated a great deal of discussion for that group member.

Shifts in Policy Choice after Discussion

Participants' responses on the post-discussion survey showed that a substantial portion of them did shift their position after participating in the discussion. However, the distribution of opinions revealed that this shift was not distributed equally among the three policy options.

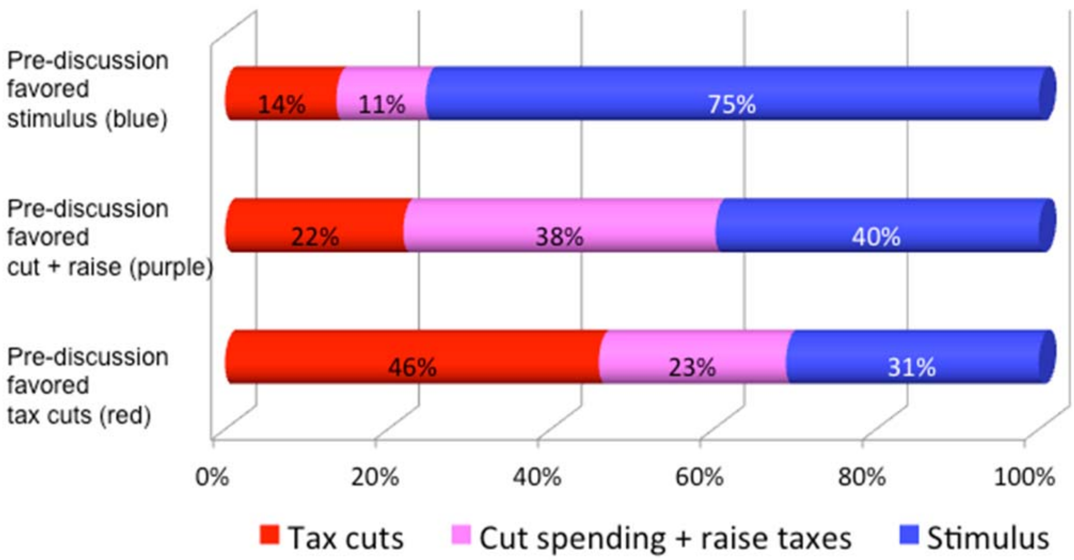

Figure 5: Post-discussion policy choice by pre-discussion policy preference.

Figure 5 shows that the overwhelming majority of those who favored the stimulus option (75 percent) retained that as their position after the discussion, with only a quarter of those participants shifting to either the tax cuts or middle option. Among the other two sets of participants, though—those who favored the middle option or tax cuts option before the discussion—the most common policy for those who changed their position after the discussion was the stimulus option. Though almost half of those favoring the right-most option (tax cuts) retained their view after the discussion, nearly a third of that set of participants changed to the left-most option (economic stimulus), compared with

only 23 percent who shifted to the middle option. Those who initially favored the middle option, who we might have expected to break evenly for either the right- or left-most options, instead went toward the economic stimulus policy choice by about a two-to-one ratio. The differences between the three sets were tested with a Chi-squared analysis, and the sets were found to be significantly different ($p<.001$).

These results suggest that the array of policy choices used for this experiment may have been less balanced than we intended, and that the economic stimulus option was more popular than pre-experiment pilot testing suggested. Future experiments in this project will focus on other policy issues and choices, to help us assess whether the national debt and economy issue in general, and our three policy choices specifically, were skewing our experimental results.

Group Decision and Experimental Condition

An analysis of the groups' final decision outcome showed that nearly all groups reached either a majority or consensus decision—only a small amount of them were deadlocked for the entire 30-minute discussion period and unable to reach a decision. Breaking down those results by experimental condition—that is, by clique or broker network, and by low or high disagreement—shows some interesting differences between group outcomes, as seen in Figure 6. These differences were tested with a Chi-squared test on the two categorical variables (network type and disagreement level), and the overall differences were significant (X^2=6.96, $p = .031$).

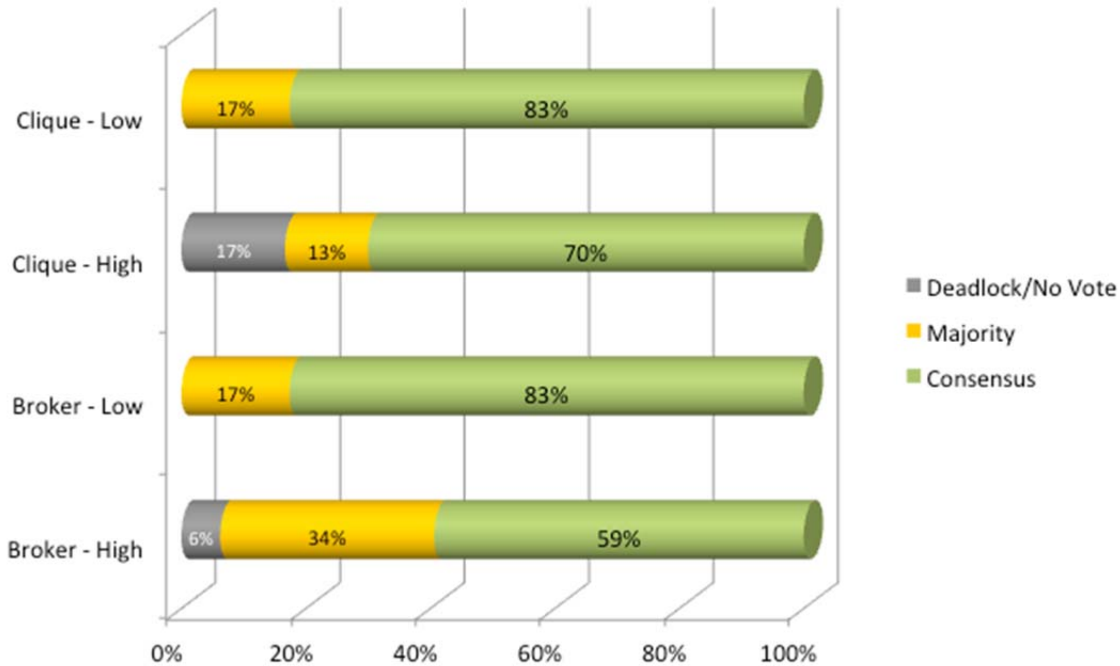

Figure 6: Group decision outcome by experimental condition.

In the high-disagreement condition, clique groups were more likely to remain deadlocked than broker groups (17% vs. 6%) which more readily reached either a majority or consensus decision in accordance with our hypothesis H2. However, there are more consensus outcomes in the high-disagreement clique (70%) than in the high-disagreement broker (59%) contrary to hypothesis H3. For the clique, there are slightly more majority rule outcomes at low disagreement than high disagreement, contrary to hypothesis H1. However, the broker network does provide some support for H1 in that there were more majority rule outcomes at high disagreement (34%) than low (17%).

Satisfaction with Group Decision and Effectiveness of Group

As noted in the methods section, we included in the experiment a motivation tool for group participants: that their discussion would influence a substantial private donation to a political group working in this area. We hoped that this motivation would raise the stakes of the discussion for participants, make them invested in the group discussion, and give them a sense that their discussion would make an impact in the issue domain. To help confirm that this was effective, the post-discussion survey included questions asking participants how satisfied they were with the group discussion and how effective they believe their group was in influencing the issue at hand.

Overall, group members were somewhat satisfied with their group's discussion, but those results varied quite a bit depending on the outcome of their discussion, as seen below in Figure 7. The average ratings shown below were tested with ANOVA, and differences between them were statistically significant ($p<.001$).

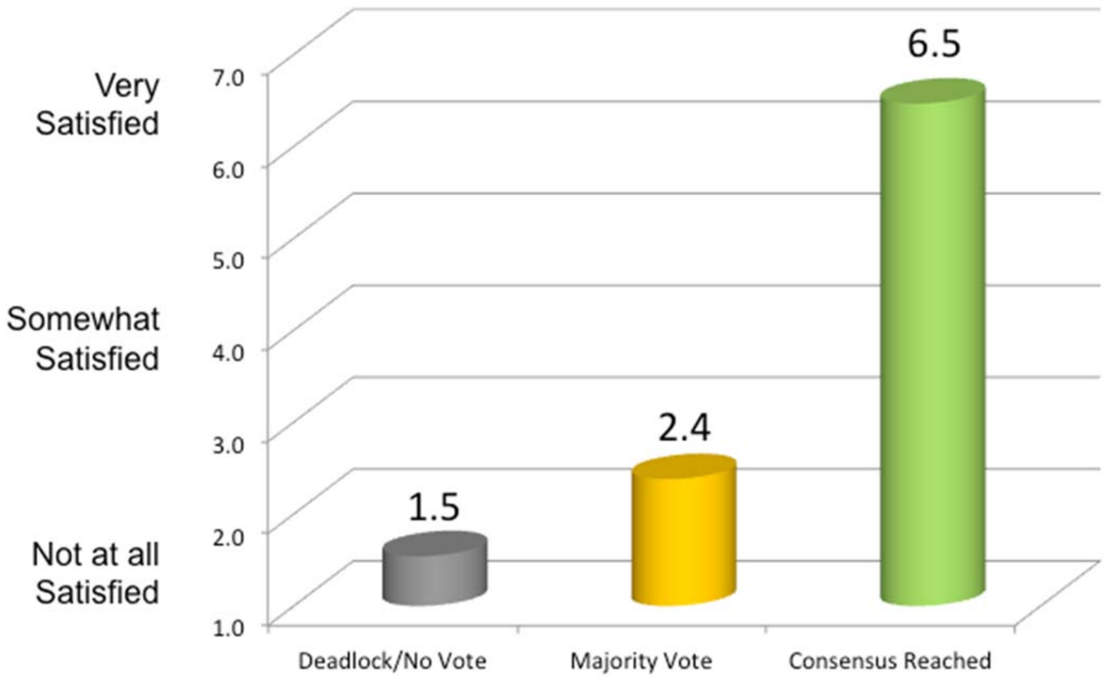

Figure 7: Average decision satisfaction, by group decision outcome.

Members of groups that were deadlocked were, on average, very unsatisfied with their group's decision outcome. Surprisingly, those groups that reached a majority vote had members who were more satisfied, but still below the midpoint of this seven-point scale of decision satisfaction. Reaching consensus, however, meant that group members were, on average, quite satisfied with the group decision—nearly at the top of this seven-point scale. Study participants seemed to be viewing consensus within their group as a very good thing, while those who reached a majority—which consisted of two out of the three group members agreeing—were viewing their decision in a much less favorable light. When asking group members to rate the effectiveness of their final group decision, we found some notable differences between experimental conditions, as shown in Figure 8

below. These figures were also tested with ANOVA, and differences were statistically significant (p=.014).

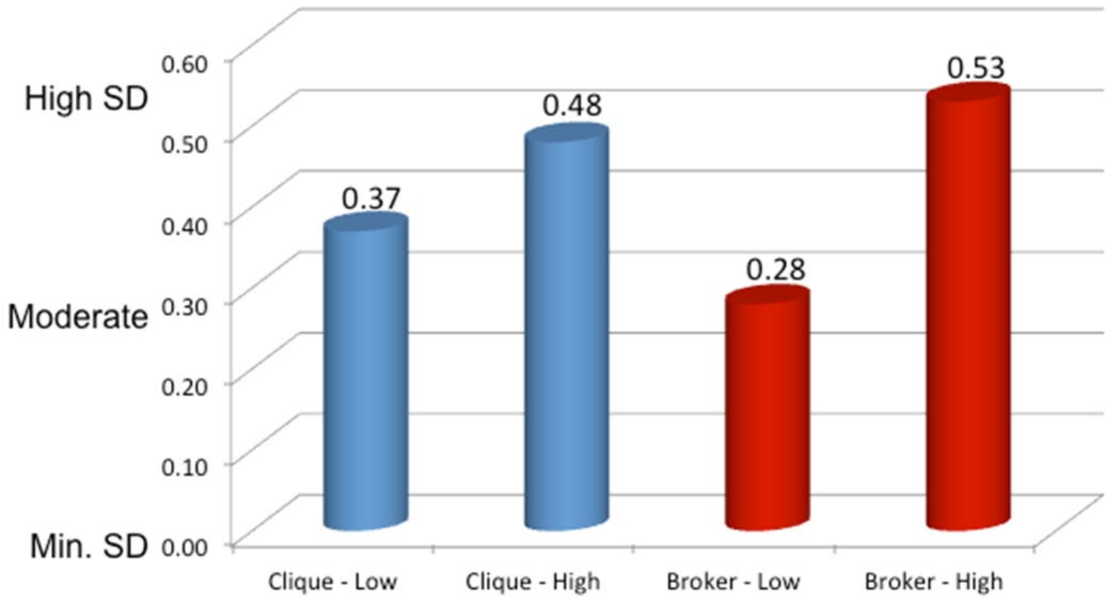

Figure 8: Average standard deviation in post-discussion effectiveness ratings of policy choices, by experimental condition.

As expected, members of low-disagreement groups tended to have less divergence in their opinions about the effectiveness of the group's final decision (that is, have a lower standard deviation in their average effectiveness). Those from high-disagreement groups, however, had more divergence in their opinions. This difference was especially stark in the broker condition, which had a much larger gap between the low- and high-disagreement conditions. This would suggest that the broker discussion network was preventing group members from coming closer together on their opinions about the policy choices.

Conclusion

We presented the results of a nonlinear model of group opinion dynamics which were used as the basis for three hypotheses which we tested experimentally. The experimental results provided partial support for one of our hypotheses, H2, that broker networks should more readily facilitate reaching a group decision at high disagreement as compared with the clique. However, no support was provided for H3 which stated that broker networks should yield more consensus at high disagreement. There was partial support from the broker network for H1 stating that majority rule outcomes should be more prevalent at high disagreement than low. The large percentage of consensus outcomes overall has inhibited our ability to reach conclusions of statistical significance. The apparent skew toward the stimulus discussed above could be a significant problem in that the hypotheses assume a symmetric initial condition in which neither of the extreme options have an a priori advantage. Further analysis of the pre- and post-survey results may reveal the source of this problem. Simulation of the model using parameter values fit from the surveys and the discussion may also allow more direct connection between the data and the model than the hypotheses do.

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Appendix 5

Terrorism and Small Groups: An Analytical Framework for Group Disruption

Terrorism and Small Groups: An Analytical Framework for Group Disruption

Justin Reedy, John Gastil and Michael Gabbay
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Terrorism and Small Groups: An Analytical Framework for Group Disruption

Justin Reedy[1], John Gastil[2], and Michael Gabbay[3]

Abstract

Terrorism scholarship has revealed the importance of small groups—both cells and leadership groups—in the proliferation of violence, yet this field remains only loosely connected to small group theory and research. There exists no systematic consideration of the role that group dynamics play in the disruption of terrorist activities. This article proposes an analytical framework for terrorist group disruption that shows how the goals and methods of counterterrorist intervention intersect with small group behavior. We use this framework to theorize how three intervention types—repression, manipulation, and persuasion—interact with group variables and processes, such as communication networks, social identities, group cohesion, and intragroup conflict. Seven theoretical propositions demonstrate how the framework can show how the direct and indirect effects of group behavior can augment or undermine counterterrorist strategies.

Keywords

cohesion, decision making, virtual groups, conflict, group, formation/dissolution

[1]University of Oklahoma, Norman, OK, USA
[2]Pennsylvania State University, University Park, PA, USA
[3]University of Washington, Seattle, WA, USA

Corresponding Author:
Justin Reedy, Department of Communication and Center for Risk & Crisis Management, University of Oklahoma, Burton Hall, 610 Elm Avenue, Norman, OK 73019, USA.
Email: jreedy@ou.edu

In October 1970, two cells of the Front de Libération du Québec (FLQ) conducted separate kidnappings. The Liberation Cell kidnapped a British diplomat, then 5 days later, the Chenier Cell abducted a Quebec government minister. The diplomat was eventually released, the minister murdered. The two cells had initially been a single group, but they split after a 5 to 4 vote on whether to conduct the operation. The losing side, which became the Chenier Cell, wanted to pursue a patient strategy of building up finances, logistics, and a wider underground movement, whereas those who formed the Liberation Cell wanted immediate action. The Liberation Cell planned its operation carefully and freed their hostage after winning the propaganda coup of having their manifesto broadcasted. The Chenier Cell left Canada to be outside the country during the operation but then suddenly reversed course and carried out an improvised kidnapping. Their demands were unwavering and unacceptable to the government, and when they killed their hostage, they triggered a hostile public reaction that led to the demise of the violent wing of the Quebec secessionist movement (Crelinsten, 2001).

As this example illustrates, understanding terrorists' behavior often requires understanding *group* behavior. The historical record provides many cases of terrorist cells and leadership groups debating, often fractiously, actions ranging from political engagement to property destruction to the indiscriminate murder of civilians (Horgan, 2005; McCormick, 2003). Researchers must theorize these groups not as single-minded actors but as decision-making units shaped by organizational imperatives, individual motivations, and complex group processes. According to the most influential advocates of group-centered study of terrorism (Horgan, 2005; Sageman, 2008), however, terrorism research has produced very limited data—and deployed insufficient systematic theory—concerning terrorist group processes.

In this article, we aim to strengthen the link from small group theory and research to terrorism studies, and we do so for many reasons. Though violent extremist groups differ from the conventional laboratory subjects (Poole, Keyton, & Frey, 1999) or field studies (Frey, 1994), small terrorist groups should fall within the scope of the most sophisticated group theories. Moreover, when viewed as a limiting case in which group cohesion, moral norms, and the gravity of decisions are pushed to extremes, the terrorism context can also test the validity of these theories at the farthest points of their theoretical reach.

Interfacing with terrorism research can advance group research along many dimensions, but to illustrate the power of this linkage, we focus on one particular way it can stimulate significant theoretical innovation. Conventional research focuses on the factors facilitating group success and satisfaction,

with contrary forces blocking or constraining group achievement (Gouran & Hirokawa, 1996). In the context of terrorism, however, we develop an analytical framework for terrorist group *disruption*. We encourage research that can help identify and exploit group vulnerabilities to prevent a terrorist group from functioning smoothly, limit its further radicalization, or at least moderate its reliance on violence.

Disrupting terrorist groups might yield direct social benefits by rendering them ineffective, but such interventions could also backfire, such as when an intervention splinters a larger group and produces an extremist splinter that aims to carry out indiscriminate bombings against civilians. One benefit of a concerted effort to bridge group theory with terrorism studies will be a greater appreciation and anticipation of how counterterrorism can have such indirect, adverse effects.

Toward this end, we present a group disruption analytical framework that clarifies the pathways by which small group phenomena can interact with three counterterrorist interventions—repression to degrade group capabilities, manipulation to induce group dysfunction, and persuasion to pursue more moderate objectives. Within our framework, we advance seven propositions about how group structures and processes amplify, undermine, or complicate counterterrorist interventions. We do not attempt a comprehensive application of small group theory, though we discuss that possibility in the conclusion. Rather, we focus on particular domains of group research and generate novel theoretical propositions regarding the efficacy of counterterrorist interventions. By doing so, we hope to demonstrate the generative power—or positive heuristic value (Lakatos, 1978)—of our framework.

We begin by explaining the value of studying extreme group contexts and providing a conceptualization of terrorist groups as distinct from other small group types. After introducing the group disruption analytical framework, we use it to generate propositions relating to two key structural features of groups—their communication network and membership composition. We then use the framework to generate predictions concerning the relational dimension of terrorist group behavior, with emphasis on group cohesion and interpersonal conflict. In the conclusion, we summarize our propositions, discuss the practical importance of theorizing terrorist groups more systematically, and discuss how to move toward a more comprehensive theory of terrorist groups, with an eye toward future empirical investigation.

Theoretical Presumptions and Definitions

Theory and research can advance considerably through the investigation of novel contexts. There is much to be gained through "normal" research on

groups, which involves straightforward replications and theoretical extensions in experimental and field research designs that gradually accumulate knowledge (Kuhn, 1970). That said, fresh insight can come from exploring new research contexts that put theories under stress and generate both empirical anomalies and original propositions—not through ad hoc explanations but by revisiting the core ideas in existing theories and deploying them in original ways (Lakatos, 1978).

This impulse has precedent in the field of group research. When Gouran (1999) set out an agenda for future research in the landmark volume, *Handbook of Group Communication Theory and Research*, he encouraged researchers to "venture even farther than they already have into the realm of natural and bona fide groups" (p. 25). Though subsequent work would expand the scope of group research (e.g., Frey, 2002), Gouran encouraged scholars to study more extreme and hazardous group forms, such as cults, to learn how, for instance, communication shapes "members' unquestioned commitment and loyalty" (Gouran, 1999, p. 25). It is in that spirit that we bring group theory into contact with terrorism.

Research on terrorist behavior has often neglected the group as a unit of analysis. The more common foci are individuals (McCauley & Moskalenko, 2008; Wiktorowicz & Kaltenthaler, 2006), organizations (Asal & Rethemeyer, 2008; Krebs, 2001), and geopolitical dynamics (Li & Schaub, 2004; Weinberg, 1991). A narrow stream of research on political violence, however, has examined factors at the small group level (Jackson, 2006; McCormick, 2003). In particular, a handful of studies have considered the extreme isolation and intense secrecy that terrorist groups (and their parent organizations) require for operational security, as well as how this isolation distorts terrorists' judgments through groupthink, seeing false dichotomies, and biases toward violent action (Crenshaw, 1988; Horgan, 2005; Post, 1998). Even so, terrorism scholarship has typically treated group research as a static body of work, rather than seeking a more dynamic integration of the group and terrorism literatures.

Before pursuing such an integration, we begin by addressing basic definitional questions. Rigorous social scientific theory can distinguish *terrorism, terrorist organizations*, and *terrorist groups*, even though such terms get used as arbitrary or interchangeable labels in lay discourse. The greater problem is that *terrorist* (and its grammatical variants) carry considerable political value, and the term has been used to decry entities viewed by others as freedom fighters, revolutionaries, or even legitimate states defending themselves against insurgents (Townshend, 2002). Consequently, any definition of the term will have its critics, but it is still possible to provide an operationally

coherent definition that captures much of what is understood to be terrorist in the vernacular.

We define a small group as being part of a terrorist network/organization or as an independent leaderless cell (Sageman, 2008) if it pursues ideological/political aims through violent acts (e.g., assassination, kidnapping, hijacking) designed to instill "shock, horror, fear, or revulsion" in a general public or specific sub-public (Townshend, 2002, p. 8). This includes marginalized social groups and insurgencies that seek to discredit what they perceive as a repressive state, as well as state-sponsored violence designed to repress those same causes.

This broad definition focuses on purposes and methods, not on the institutional position of the actors or the morality of the action (see Wardlaw, 1989). By *small* group, we mean a collection of at least 3 people (and no more than about 30) who are co-present (physically or electronically, even by degrees) and perceive their group as an entity. Generally, the group must have some shared goals or interests, with its members interdependent on one another. We principally use the word *group* to describe such an entity, though in some cases the word *team* applies, and we use the latter descriptor when citing scholarship focused on teams.

Some terrorist acts are planned and carried out by individuals acting alone, but most incidents involve small groups plotting or carrying out violent acts (Horgan, 2005). Sometimes that means a small cell of extremists who spend a great deal of time bonding as a group, plus more time training, planning, and preparing for their operation (Miller & Stone, 2002). In other cases, a small group of organizational leaders, such as a leadership council in Al Qaeda or the high command of the Provisional Irish Republican Army, have to decide as a group on their strategies for waging a campaign of violence or on the best tactics for launching particular attacks (Horgan, 2005; Jackson, 2009; McCormick, 2003). For example, one key strategic question is whether to assassinate political leaders, attack military units, execute civilian collaborators, or indiscriminately cause mass public casualties. Group members may disagree on the morality or strategic efficacy of targeting civilians, and disputes like this can sow dissension within insurgencies, as occurred in Egypt and Algeria in the 1990s (Hafez, 2003).

The sheer gravity of the decisions faced by terrorist groups help differentiate them from most other decision-making groups, but other distinguishing characteristics necessitate the deeper analysis of this context in its own right. Terrorist groups engage in actions that are illegal and often viewed as immoral by the larger society, putting them in stark contrast with, say, a work team or social group. Terrorist groups also operate under heavy secrecy; they face imprisonment or death if captured.

Because of the illicit and covert nature of their activities, terrorist groups share some common ground with criminal gangs (Horgan, 2005). However, terrorists have a (radical) political agenda for their countries or regions which crucially distinguishes them from criminal gangs: They must remain covert while seeking publicity, they must not merely kill but proclaim that they have done so, they must persuade as well as coerce, and their goals and violence must resonate with an aggrieved population not just intimidate local neighborhoods or extract payback. In addition, many terrorists are driven by ideological or religious zeal that gives them an inner compass and sense of mission beyond themselves and the fulfillment of immediate financial and social needs in contrast to criminal gang members. Finally, unlike gangs, terrorists often aspire to an extremely high level of violence, accepting full blown insurgency or civil war as a necessary evolution in their challenge to the state. After all, the pathway to terrorism is routinely labeled a political radicalization (McCauley & Moskalenko, 2008), which sharply distinguishes it from passionate but democratic social movements (Evans & Boyte, 1992). These strong distinctions demand that terrorist groups be considered in a fresh light through a wide lens of group scholarship and not simply as a variant of another context of small group research.

Group Disruption Analytical Framework

There are dozens of factors that influence small groups throughout their lifetime, from their creation to their eventual dissolution or evolution into a new group. The complex feedback loops between terrorist groups, their larger organizational setting, and the wider society in which they exist offer many avenues for research on these groups. We choose to look solely at communication, cohesion, and conflict in established terrorist cells and leadership groups, and this narrow focus permits us to show in more depth one of the many ways the terrorist context can spark theoretical advances in small group research.

Our approach to terrorist groups requires turning the basic approach of group research upside down. Researchers generally investigate the inputs and processes that yield (or obstruct) high-quality group decisions, member satisfaction, and other metrics of functionality. In the case of groups with a darker mission, however, research might more fruitfully focus on the means by which one can deliberately disrupt such groups.

Disruption encompasses a range of outcomes, including dysfunctional decision making, conflict and dissension, or distorting discussions to reach particular decisions. As is often the case, however, unintended consequences (Giddens, 1984) can flow from one's attempts to break a group apart, set it

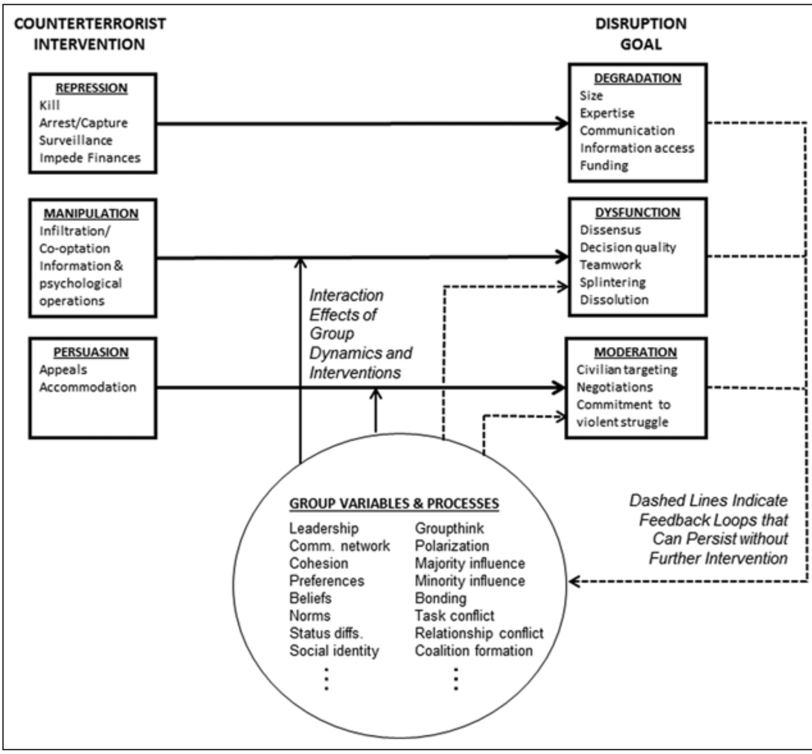

Figure 1. Analytical framework for terrorist group disruption.

adrift, or steer it off course. It is foolhardy to disrupt a group without an appreciation of how such counterterrorism could interplay with group variables and processes.

Thus, we propose a basic analytical framework of group disruption to assess systematically the prospects of different group-based counterterrorist interventions. The essential task in constructing this framework is the parsimonious characterization of government and counterterrorist actions and goals connected with group dynamics.

Though the framework draws on a wide range of theory, its central concepts and relationships are straightforward and are summarized in Figure 1. The framework references only those counterterrorist actions specifically designed to have direct effects on a targeted terrorist group. (Beyond the scope of this article are actions governments take that do not target the terrorist group itself, such as propaganda campaigns or indiscriminate repression.)

We differentiate three main types of interventions, each of which has a corresponding disruption goal: (a) *repression* intended to degrade organizational and group capability; (b) *manipulation* intended to cause group dysfunction (e.g., discord and judgmental errors); and (c) *persuasion* intended to moderate the group away from civilian targets and violent acts in general.

The first intervention type, repression, includes efforts to kill or capture group members, monitor their communications and movements, destroy their training facilities, and/or restrict their access to resources (e.g., money, arms, expertise). Killing or capturing members degrades group capability by reducing its total size and removing individuals with key functional capabilities. Surveillance (e.g., intercepting electronic messages) reduces the group's ability to plan, coordinate, and execute attacks. Destroying training facilities or other instructional mechanisms limits a terror cell's ability to develop its technical capabilities.

Whereas the direct intended goal of repression is to reduce a group's tangible resources and abilities, manipulation aims to cause dysfunction so that the group more poorly utilizes what it has. Manipulation involves efforts to create discord within a group, which may manifest as dissension, decision-making errors, ineffectual teamwork, splintering, or disintegration. For example, manipulation can employ infiltrators, co-opted group members, or media-based psychological operations to widen group faultlines, such as personal rivalries or cultural rifts.

The third intervention type, persuasion, seeks to convince terrorist groups to moderate their behavior: Persuasive appeals may target group attitudes, beliefs, norms, or social identities to move groups as a whole, or particular subgroups, in a less violent direction or, at least, away from more ever-more-violent alternatives. Instead of pursuing indiscriminate attacks against civilians via weapons of mass destruction, persuasive interventions might prod a group into entering negotiations or abandoning violent struggle altogether. In Afghanistan, the United States has made persuasive appeals via covert infiltrators and high-level group members released from detention (Sieff, 2009). Persuasive appeals also can be communicated through the media or by direct talks with terrorist group officials, which may or may not include substantive accommodations of grievances. Even when they fail to sway a terrorist group as a whole, persuasive efforts can deepen internal disputes over the appropriate use of violence.

Looking more closely at these three types of intervention in Figure 1, counterterrorist actions can have both direct and indirect effects by virtue of their interaction with group variables and processes. The vertical arrows in Figure 1 show the two points of interaction between intervention efforts and group dynamics. Dashed lines trace the indirect feedback pathways through

which the intended effects of an intervention inadvertently initiate or heighten group dynamics that impact other disruption goals, positively or negatively.

More precise theoretical propositions concerning the effects of counterterrorist interventions can be traced out in the framework using solid pathways or a mix of solid and dashed pathways. Some of the propositions we will put forth in this article involve group processes interacting with an intervention to influence group outcomes, such as the availability of an inclusive superordinate social identity improving the prospects of persuasion interventions (Proposition 4). Other propositions involve the indirect feedback loops: An intervention, via group dynamics, produces secondary effects on other goals via the dashed arrows, as when repression degrades group capability but enhances a leader's influence on group decisions (Proposition 1). Still other propositions involve two different interventions: The first activates group dynamics (via a dashed path), which in turn limit or enhance the effectiveness of a second intervention (via solid paths). Thus, Proposition 2 holds that easing up on repression to enable group communication can increase the efficacy of persuasive interventions.

For a more narrative illustration involving these feedback processes, consider the case of the Islamist insurgency in Algeria in the 1990s, which involves repression, persuasion, and groupthink (Hafez, 2004). The Armed Islamic Group (GIA) in Algeria was caught in a spiral of encapsulation due to government repression, whereby intragroup ties strengthened and external ties were severed. Isolation led GIA to conduct ill-advised massacres of Muslim civilians in their own strongholds. These extreme and self-destructive decisions fractured the insurgency and diminished its civilian support.

In the terms of our framework, repression successfully degraded the communication capability of the GIA and induced dysfunction by activating a feedback loop whereby groupthink mechanisms led to poorer quality decision making. More indirectly, however, this led to an adverse effect with respect to the goal of group moderation, as another feedback loop through distorted decision making led the GIA to its indiscriminate civilian attacks. This paroxysm of violence may have been an unavoidable cost in the Algerian government's defeat of the GIA, but a framework accounting for group dynamics encourages counterterrorists to anticipate such outcomes and seek out less hazardous alternatives.

There will be times when group factors play a smaller role than in the Algerian example. For instance, small group dynamics do not directly interact with the deployment of repressive measures, the success of which hinges on the balance of counterterrorist military and police resources, competence, and intelligence with respect to their terrorist adversaries. Group dysfunction can lead to a dissatisfied group member informing on his group and yielding

better intelligence for repressive actions, but such *turncoating* is exogenous to the intervention itself.

Finally, we acknowledge that Figure 1 cannot encompass the whole universe of interventions, goals, variables, and linkages. In designing it, we hope to strike the right balance between parsimony and complexity for generating novel propositions concerning the interplay of counterterrorism and terrorist behavior. The remainder of this article illustrates the model's heuristic value, as it has inspired the following propositions about the roles of group structure and composition, cohesion, and intragroup conflict in the disrupting of terrorist groups.

Communication Networks

Government and military repression of terrorist organizations can restructure the leadership groups and operational cells within it. Killing or capturing of terrorist and insurgent leaders, for instance, has been a subject of considerable research and controversy in political science (Jordan, 2009; Pryce, 2012), though such study has neglected the implications for small groups within those organizations. Thus, we begin by considering repressive interventions that affect a group's communication network structure and capabilities.

A group's communication network refers to who communicates with whom. Research on small groups has distinguished several network configurations: the centralized star-shaped network, in which each group member can only talk to a central figure; the chain, which connects a member only to adjacent neighbors; and the comcon or clique network, in which each member can talk to any other (Shaw, 1964).

Contemporary research on group networks may be sparse because the vast majority of small groups studied have comcon networks. Terrorist groups' concern with operational security and covertness, however, often demands that they employ alternatives to reduce the total number of links between group members (Enders & Su, 2007). Little research addresses the network structure within terrorist cells, but one line of research suggests that cells can be made up of subgroups that have little contact with one another, as in the case with the Bali nightclub bombing (Koschade, 2006; see also Jackson, 2006).

Placing this discussion within the group disruption framework, consider how a sharp increase in repression to degrade a group's communication network could have indirect effects. Terrorist groups often respond to intense repression by restructuring their communication network to reduce their detectability and to minimize the damage resulting from a member's capture.

This can result in decision making becoming more concentrated in the hands of the leader or leadership group. From an information processing perspective, this concentration can lead to poorer quality decisions, as a result of the central leader in such groups becoming overwhelmed by information from various group members (Hare, 1976).

In addition, a concentrated flow of messages through the leader may magnify the importance of that leader's policy preferences and leadership style on the group's behavior. If the leader is a crusader who challenges constraints and is closed to information (M. G. Hermann, 2001), this may lead a group to pursue even more radical actions. Thus, stepped up organizational repression (even in the more benign form of more intense surveillance rather than killing or arresting group members) may not only inhibit communications but also have the adverse indirect effect of radicalizing a group toward more violent actions. We can restate this as the following theoretical proposition:

> **Proposition 1:** Significantly increasing a terrorist group's communication costs will increase the congruence between leader preferences and group decisions.

Although Proposition 1 has been stated in language specific to terrorist groups, it may well prove amenable to investigation in other small group contexts. As explained earlier, we aim to advance small group theory and research generally by focusing on this unusual and understudied context.

Communication network structure might also influence the efficacy of counterterrorist persuasion campaigns. Researchers have distinguished centralized structures of knowledge sharing (i.e., critical knowledge is shared with a single member) from decentralized ones that share information equally, a design that tends to be more efficient and effective (Huang & Cummings, 2011). If a decentralized structure helps to ensure that more moderate arguments are shared more broadly, a persuasion campaign's prospective success may be enhanced by deemphasizing repressive measures. Doing so could reduce communication costs and facilitate a network closer to the comcon model. Accordingly, we put forth this second proposition:

> **Proposition 2:** Significantly decreasing group communication costs will enhance the effectiveness of persuasion tactics aimed at moderating terrorist group decisions, if a substantial fraction of group members or key leaders are leaning toward moderation.

The proviso at the end of the proposition assumes that the persuasion campaign is timed with the coexistence of a sizeable or influential subgroup receptive to messages of moderation (e.g., after the group has suffered a series of setbacks after undertaking more violent actions). In essence, Proposition 2 can be thought as a kind of corollary to our first proposition: If intensified repression allowed an extremely hawkish leader to control and allocate costly communications to further his policy preferences, then allowing freer communications could facilitate the coalescence of a critical mass of dissenters favoring moderation. In terms of the group disruption framework, this is an example of a positive indirect effect: While easing up on repression runs contrary to the direct goal of degrading a terrorist group's communication capabilities, it can indirectly facilitate the goal of moderation.

Membership Composition and Group Identity

As important as the network structure of a small group is the composition of its membership. Though findings on heterogeneity remain mixed, researchers have found that membership diversity often aids groups undertaking complex tasks (Bowers, Pharmer, & Salas, 2000). Member differences can also be a source of deep faultlines within a group, especially if some group members favor homogeneity and perceive that group divisions reflect deeper cultural differences (Homan & Greer, 2007; Thatcher & Patel, 2011).

This may seem irrelevant to terrorism, since extremist groups presumably draw from specific cultural, ethnic, or religious pools presently endorsing political violence, as in Ireland, Spain, and Sri Lanka. Nevertheless, some terrorist organizations are becoming more ethnically and culturally diverse. Muslim radical groups in the United States may have members who are immigrants from the Arab world and others who are native-born, such as African Americans affiliated with the Nation of Islam, and overseas jihadist groups can include both lifelong Muslims and recent converts (Miller & Stone, 2002). The international organization Al Qaeda has recruited across a wide range of cultures and national origins (Gunaratna, 2002), which has spawned tensions between Arabs and non-Arabs at the "foot soldier" level (Stenersen, 2010).

The case of the 9/11 hijacking group provides a striking example (National Commission on Terrorist Attacks Upon the United States, 2004). The core hijacking group, composed mainly of members of the Hamburg cell, was ethnically diverse, with members from Egypt, Lebanon, the United Arab Emirates, and other nations. A potential faultline existed between them and the bulk of the hijacking team, which consisted almost entirely of Saudi

Arabian men (i.e., the muscle trained to overpower passengers and crew who were never told of the mission's details).

When substantive disagreement, demographics, and group status align (Lau & Murnighan, 1998, 2005; Thatcher & Patel, 2011), such faultlines could be made more prominent and exacerbated to yield significant group dysfunction. Anticipating our discussion of intragroup conflict below, the conjunction of relationship conflict (perhaps caused by cultural differences) and task conflict over the performance of group functions is particularly damaging to group effectiveness (De Dreu & Weingart, 2003). If subgroups with different functions also correspond to differences in culture, then repression that makes it more difficult for one subgroup to perform its function will increase task conflict within the group, thereby making it more susceptible to cultural conflict and ensuing dysfunction. Consequently, the especially debilitating impact of simultaneous task and relationship conflict yields the following proposition:

> **Proposition 3:** Manipulations intended to heighten the salience of cultural faultlines within a terrorist group will be most effective when paired with repressive measures that activate task conflict along those same faultlines.

With respect to our analytical framework, group dynamics contribute to both direct and indirect effects. The direct effect comes via manipulations (e.g., information operations, psychological operations, and infiltrator incitement) intended to excite relationship conflict along cultural faultlines, and the indirect impact comes by way of activating task conflict within the group due to repression hampering subgroup task execution.

To minimize conflict based on cultural differences, groups routinely define their in-group identity during a social bonding process, which develops members' sense of what it means to belong to their group and strengthen their connection to it as an entity (Abrams, Hogg, Hinkle, & Otten, 2005). Terrorist organizations do likewise by providing an overarching narrative and shared identity that help motivate both their subgroups and individual members. Like so many other organizations and social movements, they offer a sense of belonging and their own particular understanding of what can be a bewildering social world (Hogg & Terry, 2000; Tajfel & Turner, 1986).

Terrorist groups rely heavily on in-group/out-group development—building up the in-group helps create the extreme dedication to the group required in this situation, whereas out-groups are dehumanized to allow them to be more easily targeted for violence (Horgan, 2005). In addition, members are

encouraged to follow the path of the organization and their operational group without deviating, and follow along with the rest of the group even if a disagreement arises. Again, Al Qaeda provides an example, as some scholars believe that the organization's strength comes less from its actual planning and more from its ability to inspire violent jihad with its dual commitment to in-group solidarity and out-group dehumanization (Sageman, 2008).

One among many potential avenues for counterterrorists is infiltrating or co-opting members to orient toward a superordinate identity that could override the messages instilled by lower-level in-group identities (Abrams et al., 2005; Kramer & Brewer, 2006). For this purpose, the ideal superordinate identity connects oneself to humanity (Nickerson & Louis, 2008); however, even a larger in-group, such as a nationality or a religion, exists at a higher level than the small group (terrorist cell) or organizational identity (Al Qaeda member) and could facilitate persuasion toward a more moderate course of action in line with those more inclusive identities.

> **Proposition 4:** Persuasive messages intended to yield more moderate group decisions will prove more effective in those terrorist groups with at least some members oriented toward superordinate identities that are strong enough to compete with or override the in-group identity of their group or larger movement.

Fostering a superordinate identity is not always a straightforward enterprise, since the behavioral implications of such an identity can be ambiguous. For example, while the U.S. military was engaged in Iraq, both Sunni insurgents and the Shiite-dominated Iraqi government could claim that they were acting in a way consistent with an Iraqi national identity—the former by resisting a foreign occupier and the latter by protecting Iraq from social disintegration. With the departure of U.S. forces, the anti-occupation resistance frame become harder to reconcile with Iraqi national identity, so fostering an Iraqi identity among Sunni insurgents could more clearly serve to mitigate sectarian tensions and restrain anti-Shiite terrorism (although Sunni insurgents can argue they are fighting the Iranian occupation; see Gabbay, 2008).

Cohesion and Intragroup Conflict

In addition to network structure and composition, another factor that can prove critical to the maturation of a terrorist cell is the time and effort that members spend becoming a group. Social bonding, team building, identity formation, and creation of shared purpose help individual members become

a coherent entity (Beal, Cohen, Burke, & McLendon, 2003). These are important to nearly any group that will be spending time together or taking on a significant task, but cohesion may be vital to terrorist groups, which require exceptional dedication to the group and its destructive purposes (Horgan, 2005). Therefore, authorities trying to undermine a terrorist group from the outside need to understand how to diminish cohesion to break the group apart or render it inert.

Social Cohesion Versus Task Cohesion

Carron and Brawley (2000) define cohesion as “a dynamic process that is reflected in the tendency for a group to stick together and remain united in the pursuit of its instrumental objectives and/or for the satisfaction of member affective needs” (p. 119). Thus, equally cohesive groups can differ with respect to whether their primary glue is task-based cohesion or social bonds. Either task or social bonds could aid a terrorist group because, unlike conventional militaries, their larger organizations often lack an enduring institutional unity.

The relative emphasis placed on these two forms of cohesion could result in different outcomes for disruptive manipulation. For a terrorist group whose cohesion stems predominantly from commitment to a common objective, as in the Iraqi insurgency that united diverse ethnic and political factions (Hafez, 2007), repressive measures that thwart terrorist attacks may lower the group’s perception of its own efficacy as a vehicle for achieving political goals. By contrast, groups whose cohesion flows from affective relationships among its members, as for those Taliban who fought together against the Soviets (Zaeef, 2010), should prove more resilient to repression and tactical setbacks. Socially cohesive groups can even escalate their commitment in response to failures, as has been witnessed in socially cohesive laboratory groups (Dietz-Uhler, 1996). Thus, our fifth proposition addresses how task-cohesive groups respond differently to successful repression than socially cohesive groups:

> **Proposition 5:** Repressive interventions that succeed in degrading group capabilities (a) will spur dissolution more frequently in task-cohesive groups than in socially cohesive ones, but (b) may increase members’ commitment to socially cohesive groups.

In the disruption framework, this is a case where an intervention achieves its direct goal (repression degrades group capabilities) but can either spur or

avoid group dysfunction depending on the prevalence of two different forms of cohesion.

Though we focus on theoretical issues in this essay, a methodological aside shows other challenges in studying terrorist groups. In this case, one might wonder if it is possible to measure the task and social components of cohesion in such groups, for which questionnaires and direct observation are unavailable. With respect to terrorist leadership circles, one approach could employ the variable of "primary group identity" put forth in the literature on foreign policy group decision making. This variable assesses whether a group member's loyalty on a policy issue lies in the group itself or an external constituency (C. F. Hermann, Stein, Sundelius, & Walker, 2001). For instance, a leader may be more loyal to the faction she or he leads than the terrorist organization as a whole. Such factions can be based on ideological, geographical, tribal, or clan subgroups. If the leadership is an amalgamation of factions—as is the case for the Pakistani Taliban (Franco, 2009)—the primary group identities of factional leaders reside outside the group; therefore, cohesion of the leadership group is based on a common commitment to the task at hand, rather than from interpersonal bonds and affection.

In other groups, leaders' primary identities are with the group itself and social cohesion trumps task cohesion. The nature and history of the bonds between members resulting from kinship, friendship, education, and shared experiences are crucial in assessing the nature of a particular group's cohesion. For example, the top leader of the Afghan Taliban, Mullah Omar, and his principal deputy, Mullah Baradar, hail from different Pashtun tribes but they fought together against the Soviets in the 1980s. The two men cooperated in the genesis and rise of the Taliban in the 1990s; Omar even fled the approach of U.S. forces in November 2001 on the back of Baradar's motorcycle (Moreau, Hirsh, Barry, & Hosenball, 2009). Consequently, they have a deep bond of trust. Thus, though Baradar was arrested by Pakistani authorities in early 2010, his position within the Taliban has officially remained vacant in his absence.

Intragroup Conflict

In the popular imagination, terrorist groups are highly cohesive and act as unified entities, but in reality, they can experience detrimental interpersonal strife. For instance, Mullah Dadullah, a controversial Taliban leader known for extreme violence and a prominent media profile, had a contentious relationship with the more moderate and low-key Akhtar Osmani. At one meeting, the animosity led to a fist fight. A 2006 airstrike killed Osmani after Dadullah allegedly tipped off NATO as to his comrade's location. In turn,

senior Taliban leaders may have betrayed Dadullah, who was killed by U.S. forces a few months later (Coghlan, 2009).

Relational clashes like these may not always lead to such extreme outcomes, but they can undermine effective group performance (De Dreu & Weingart, 2003; de Wit, Jehn, & Greer, 2012). Substantive task conflict, by contrast, has been theorized to improve group performance on nonroutine tasks by spurring evaluation of alternatives (Jehn, 1995; Orlitzky & Hirokawa, 2001). A recent meta-analysis, however, found no clear effect of task conflict on group performance, except for studies of senior management teams (de Wit et al., 2012). Group performance suffers most when relationship and task conflict are highly correlated (De Dreu & Weingart, 2003; de Wit et al., 2012).

Thus, counterterrorist manipulations that generate *relational* intragroup conflict will be more effective in inducing dysfunction than those aimed at producing task conflict. These interventions would differ greatly from the ones discussed in the previous section. In this case, authorities would not conduct a repressive crackdown on a group to break apart a coalition; instead, counterterrorists would manipulate a group (via information operations or infiltrators) to exacerbate personal conflicts. Such relational conflict-inducing manipulations could exploit rivalries, status inequalities, and the aforementioned cultural faultlines. (Clumsy efforts to sow dissension, however, could yield a norm against intragroup conflict, given the possibility that such discord stemmed from enemy machinations.) The more damaging nature of relational conflict on group performance yields our sixth following proposition:

> **Proposition 6:** Counterterrorist manipulations that spur intragroup relational conflict will prove more effective at engendering group dysfunction than those designed to generate task-related intragroup conflict.

This proposition would not be of much practical value if counterterrorists were free to target both relational and task conflict independently, without any need to consider tradeoffs between them. In reality, counterterrorists have few opportunities to conduct manipulative interventions and must use them carefully; an infiltrator will not survive long if used to both spread malicious rumors and transmit disinformation that undermines missions. Thus, assuming a terrorist group is equally susceptible to both task and relationship conflict, scarce opportunities aimed at long-term disruption should target the relational dimension of groups.

Because of the incendiary quality of relational conflict, there may be circumstances in which terrorist groups experiencing such turmoil may not be ideal targets for repression. Actions that inhibit intragroup communication may have the undesirable effect of reducing group discord. Intense repression raises the cost of communicating, which could give a consensus-oriented leader both the impetus and justification to restructure a group's network so that opposing factions cannot communicate directly with one another. The severing of direct links between clashing members would end counterproductive communication (like fist fights) and thereby ease relationship conflict and associated dysfunction. Thus, we propose the following:

Proposition 7: For groups with high relational conflict and a consensus-oriented leader, increasing communication costs will reduce relationship conflict and, thereby, improve group performance efficiency.

This proposition provides another example of how a counterterrorist intervention (repression aimed at network degradation) can have a counterproductive secondary effect on another goal (group dysfunction). Terrorists may be aware of such a dynamic and use it to their benefit, as might have been the case with the prior example involving the Taliban and Mullah Dadullah. If the Taliban leadership was indeed culpable in Dadullah's death, then they essentially outsourced the termination of a disruptive senior executive to the U.S. military. In effect, they turned counterterrorist repression toward their own purpose of easing in-house dysfunction, with the added benefit of blaming enemy forces for the murder. Once again, the point is to draw out the adverse consequences that may flow from underlying group dynamics.

Conclusion

To summarize, we have introduced an analytical framework for addressing the interactions of small group phenomena with counterterrorist interventions, and we used the framework to generate seven novel theoretical propositions. In Table 1, we summarize those propositions, along with their relationship to the framework's main elements. For each proposition, the table identifies the intervention type (repression, manipulation, and/or persuasion) and its corresponding primary goal (degradation, dysfunction, and/or moderation). The table also shows the particular group process occurring within the proposition, the goal affected by that process, the effect pathway (direct facilitation of the intervention's primary goal, indirect feedback of one type of intervention upon a different goal area, and/or indirect facilitation of

Table 1. List of Propositions and Their Relation to Group Disruption Analytical Framework.

Proposition	Intervention type/goal	Group process	Goal influenced	Effect path and utility
P1: Significantly increasing a terrorist group's communication costs will increase the congruence between leader preferences and group decisions.	Repression/ degradation	Restructuring of group communication network to enhance leader's decision-making power	Moderation	Indirect feedback; potentially negative (heightened extremism)
P2: Significantly decreasing group communication costs will enhance the effectiveness of persuasion tactics aimed at moderating terrorist group decisions, if a substantial fraction of group members or key leaders are leaning toward moderation.	Repression/ degradation	More open communication network allows freer exchange of information, opinions	Moderation	Indirect facilitation of persuasion; positive
P3: Manipulations intended to heighten the salience of cultural faultlines within a terrorist group will be most effective when paired with repressive measures that activate task conflict along those same faultlines.	Manipulation/ dysfunction and Repression/ degradation	Synergistic damage to group performance efficiency by simultaneous relationship and task conflict	Dysfunction	Direct and indirect facilitation of manipulation; positive
P4: Persuasive messages intended to yield more moderate group decisions will prove more effective in those terrorist groups with at least some members oriented toward superordinate identities that are strong enough to compete with or override the in-group identity of their group or larger movement.	Persuasion/ moderation	Less hostility to out-groups by promotion of social identity which includes them and lessens salience of terrorist group as in-group	Moderation	Direct facilitation of persuasion; positive

(continued)

Table 1. (continued)

Proposition	Intervention type/goal	Group process	Goal influenced	Effect path and utility
P5: Repressive interventions that succeed in degrading group capabilities: (a) will spur dissolution more frequently in task-cohesive groups than in socially cohesive groups; but (b) may increase members' commitment to socially cohesive groups.	Repression/ degradation	Task-cohesive group members lose commitment to group if it does not serve instrumental goals; failures, losses of socially cohesive groups motivate desire to prove efficacy and justify sacrifices	Dysfunction	Indirect feedback; both positive and negative
P6: Counterterrorist manipulations that spur intragroup relational conflict will prove more effective at engendering group dysfunction than those designed to generate task-related intragroup conflict.	Manipulation/ dysfunction	Relationship conflict more consistently impairs group performance	Dysfunction	Direct facilitation of manipulation; positive
P7: For groups with high relational conflict and a consensus-oriented leader, increasing communication costs will reduce relationship conflict and, thereby, improve group performance efficiency.	Repression/ degradation	Leader restructures communication network to minimize counterproductive interactions between conflicting group members	Dysfunction	Indirect feedback; negative

another type of intervention), and the positive or negative utility of the net effects from the counterterrorism perspective.

Though the disruption of terrorist groups can be considered from many perspectives, most previous studies have primarily focused on various aspects of the repression-degradation linkage, such as leadership decapitation (Pryce, 2012). We take a different approach by placing small group behavior front-and-center in how we model the disruption of terrorist groups. Furthermore, the disruption analytical framework provides a guide for systematically applying a broad range of small group theory to the different types of counterterrorist interventions, through both direct interaction and indirect feedback. In addition to its use in academic research, the framework can provide a conceptual architecture for counterterrorists to use in the practical assessment of small group effects in real-world terrorism situations.

Our seven propositions include relatively straightforward insertions of group theory into the terrorism context, some warnings as to potential adverse side effects of counterterrorism interventions, and reconsiderations of group theories spurred by an interest in promoting group dysfunction, dissolution, and moderation. In addition to providing insight into the effects of counterterrorist actions themselves, the disruption framework poses questions and associated propositions that push the boundaries of small group research itself beyond the terrorist context. For example, the effects of a marked increase in communication costs and the associated propositions involving the interaction of leadership and communication networks (Propositions 1 and 7), although activated by intense repression for terrorism, can be investigated in other group contexts where communication among members can be curtailed sharply by exogenous forces.

Toward a Comprehensive Theory of Terrorist Groups

Though the framework has utility, it is not a single theory of terrorist group disruption, and it does not rely on any particular small group theory. Given the diverse types of group processes involved, we do not believe a single grand terrorist disruption theory is any more feasible than a grand unified theory of small group dynamics. It is, however, possible to develop well-defined theories that yield specific behavioral propositions for particular aspects of the disruption framework, such as the effects of large changes in communication costs on network restructuring or the routes by which repression can lead terrorist groups to more extreme violence via groupthink or other processes.

Moreover, a more comprehensive treatment of the broad range of group phenomena which play roles in the disruption of terrorist groups is certainly

possible. Within the framework itself, more hypotheses can be generated regarding group polarization, minority influence, and groupthink. Further research aimed at systematic application of our disruption framework to a wide sweep of group behaviors would benefit by integrating the framework with a general theoretical model that has sufficient abstractness to encompass even terrorist groups.

An example would be the embedded system theoretical (EST) framework (Gastil, 2010). Like other system models (Arrow, McGrath, & Berdahl, 2000), this framework emphasizes the embedded character of groups and complex feedback relationships between the groups, their organizational setting, and the wider society in which they exist. The larger organizational context is critical for the formation, characteristics, membership, and resources of smaller terrorist groups (Asal & Rethemeyer, 2008; Jackson, 2009; Sageman, 2008). Beyond training and resources, organizations provide a grand narrative and purpose to individual members (Post, Sprinzak, & Denny, 2003). In an equally important sense, terrorist groups are embedded in far wider social structures, such that they can transform the political landscape of a nation where they carry out attacks, or even the wider global community. Though Fine (2012) uses the term *tiny publics* generally to refer to prosocial (or at least innocuous) groups, terrorist groups share the same ability as those micro-publics to weave the larger social reality of their members, but they also have an even greater capacity for reshaping mass cultural identities and political agendas far beyond their group boundaries.

Designing Empirical Research

On the more practical question of research methods, we acknowledge that empirically testing propositions involving terrorist groups is difficult. Some guidance can be drawn from the research on foreign policy decision making. The most straightforward approach involves retrospective case studies as has been done with respect to groupthink in foreign policy decision making (Janis, 1982; 't Hart, Stern, & Sundelius, 1997). Methods devised to systematically test groupthink and related propositions across a number of historical cases may also be transferable to the terrorism context (Schafer & Chrichlow, 2010; Tetlock, Peterson, McGuire, Chang, & Feld, 1992). However, terrorist group internal dynamics are usually much more opaque than the U.S. and Western cases analyzed in these studies. Yet terrorist public statements are readily observable and it may be possible to use these to empirically assess propositions as has been done for groupthink among political leaders (Tetlock, 1979). The Soviet-era Kremlin was a famously closed leadership group but public statements by its members have been used to analyze its group

decision-making processes (Stewart, Hermann, & Hermann, 1989). Terrorist rhetoric has been used to assess the leadership styles of terrorist leaders (M. G. Hermann & Sakiev, 2011) and in cases where multiple individuals within terrorist groups make statements and give interviews, it is possible that group differences and processes may be revealed.

Another distinctive group context presents similar observational problems for group researchers. Courts have shielded jurors' deliberations from public scrutiny, so researchers routinely have turned to mock juries—small groups of people set up in an experimental setting, tasked with hearing a made-up court case and reaching a decision (e.g., Hastie, Penrod, & Pennington, 2002).Though ecological validity questions persist for such simulated group deliberations, the research toolkit for terrorist groups could include experimental studies of group decision making on options with widely divergent benefits and consequences. (Obviously, such studies would need to focus on topics that are controversial but not so intense as to cause personal or political strife among study subjects.) For example, an experimental group created from an online convenience sample could be given a controversial subject and asked to decide between an array of (legal) options ranging from moderate to more extreme. Varying the shape of the group's communication network could simulate the consequences of outside repression and test the impact of network structure on the ability to achieve consensus.

Such an innocuous experimental setting should also show the wider theoretical relevance of findings inspired by questions particularly salient in the terrorism context. More generally, the framework for group disruption described herein could be useful for scholars and practitioners thinking about how to disrupt other malevolent groups—ones whose covert and illegal behavior poses a sufficiently grave threat as to demand a multipronged government response which coordinates repressive, manipulative, and persuasive interventions.

Whether using historical case studies or conducting experiments on mock cells, much work remains if we are to connect research on small groups with terrorism scholarship. Toward this end, we have introduced a theoretical framework and advanced seven propositions, but this represents an early stage of a more systematic research program. Only by advancing our knowledge can we hope to learn what makes these groups persistent in their memberships and deadly in their decision making, as well as what leads them to dysfunction or disintegration. By applying existing theories and conducting further research on terrorist groups, we may gain additional and timely insights that could make it even harder for such groups to formulate and carry out their most destructive ambitions.

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Author Biographies

Justin Reedy, PhD, is an assistant professor in the Department of Communication and a research associate in the Center for Risk, Crisis and Resilience at the University of Oklahoma, USA. He studies political and small group communication, including topics such as political discussion and conversation, deliberation and civic engagement, and group behavior and political violence.

John Gastil, PhD, is a professor in the Department of Communication Arts and Sciences at the Pennsylvania State University, USA. With support from the National Science Foundation, his research has focused on political and group behavior, including studies of jury deliberation, cultural cognition, and the Oregon Citizens’ Initiative Review.

Michael Gabbay, PhD, is a senior principal physicist at the Applied Physics Laboratory at the University of Washington, USA. His research involves the study of political networks, particularly insurgent and terrorist groups, from a quantitative perspective including the development of mathematical representations of rhetoric and computational simulations of network dynamics.

Appendix 6

Modeling Decision-Making Outcomes in Political Elite Networks

Modeling Decision-Making Outcomes in Political Elite Networks

Michael Gabbay

Applied Physics Laboratory, University of Washington, Seattle, WA 98105, USA
gabbay@uw.edu

Abstract. A methodology for modeling group decision making by political elites is described and its application to real-world contexts is illustrated for the case of Afghanistan. The methodology relies on the judgments of multiple experts as input and can improve analysis of political decision making by elucidating the factional structure of the group of elites and simulating their interaction in a policy debate. This simulation is performed using a model of small group decision making which integrates actor policy preferences and their inter-relationship network within a nonlinear dynamical systems theory framework. In addition to the basic nonlinear model, various components required to implement the methodology are described such as the analyst survey, structural analysis, and simulation. Implementation and analysis results are discussed for both the government and insurgent sides of the current conflict in Afghanistan.

Keywords: political networks, social networks, computational social science, nonlinear dynamics, Afghanistan.

1 Introduction

This paper describes a methodology for quantitatively modeling group decision making by political elites. The methodology involves the use of expert judgment as input, structural analysis, and computational simulation using a nonlinear model of small group decision making which can address questions involving the outcome and level of dissent in a given policy debate. The methodology can aid analysis of group decision making by providing both a quantitative and qualitative framework. Quantitative implementation affords a systematic framework for assessing the interaction of member policy preferences and inter-relationships. This is difficult to do on a purely qualitative level as the structure of the group's social network and distribution of policy preferences may be complex — a difficulty that is compounded by the nonlinear nature of the interaction between group members. As a qualitative framework, the model of group decision-making dynamics can provide guidance as to when one should be on guard for the possibility of "nonlinear behaviors" that can lead to sudden and dramatic changes in policy or group discord or to unanticipated, perhaps counterintuitive dynamics.

This paper proceeds as follows: Section 2 presents the nonlinear model of group decision-making dynamics. In Sec. 3, the implementation methodology is

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described. Section 4 illustrates the application of the methodology for the current conflict in Afghanistan for both Afghan government and insurgent leadership groups.

2 Nonlinear Model of Group Decision Making

This section describes the nonlinear model of small group decision making which is used to simulate the evolution of group member policy or ideological positions [8,7]. The theoretical basis of the model draws from social psychology theories of attitude change and small group dynamics and theories of foreign policy decision making [1,17,15]. The model is concerned with the evolution of group member positions for a given policy issue or broader ideological axis. The group member policy positions are arrayed along a one-dimensional continuum known as the *position spectrum.* A group member's position along the position spectrum is subject to change under the influence of three separate forces: (i) the self-bias force; (ii) the group influence force; and (iii) the information flow force. Only the first two forces will be discussed in this paper but information flow force has been used to model interactions between two rival decision-making groups and as a stochastic forcing representing random flow of incoming information.

2.1 Self-Bias Force

For a given policy decision episode, each member comes to the debate with his own preferred position called the *natural bias.* It is a reflection of the member's underlying beliefs, attitudes, and worldview of relevance to the matter at hand. If a member's position is shifted from his natural bias due to group pressures, he will experience a psychological force that resists this change. This self-bias force can be viewed as a form of cognitive dissonance [1]. Denoting the i^{th} member's current policy position by x_i and his natural bias as μ_i, then i's self-bias force $S_i(x_i)$ is proportional to the difference between his current position and natural bias,

$$S_i(x_i) = -\gamma_i(x_i - \mu_i). \tag{1}$$

The proportionality constant γ_i is called the *commitment.*

2.2 Group Influence Force

The group influence force is the total force acting to change a member's position due to the other members of the group. The influence of member j upon member i is assumed to be a function of the difference in their current positions, denoted by $H_{ij}(x_j - x_i)$ and called the *coupling force.* In general, the reciprocal coupling forces between two members will not be of equivalent strength, $|H_{ij}| \neq |H_{ji}|$. If there are N members in the group, the total group influence force on member i, denoted by $G_i(x_i)$, is given by the sum

$$G_i(x_i) = \sum_{j=1}^{N} H_{ij}(x_j - x_i). \tag{2}$$

The coupling force, depicted in Fig. 1, is taken to have the form,

$$H_{ij}(x_j - x_i) = \kappa_{ij}(x_j - x_i) \exp\left(-\frac{(x_j - x_i)^2}{2\lambda_i^2}\right), \tag{3}$$

where κ_{ij} is the *coupling strength* and λ_i is i's *latitude of acceptance.* κ_{ij} gives the strength of the influence of j upon i given their personal relationship and is equivalent to a tie strength in a weighted adjacency matrix ($\kappa_{ij} \geq 0$, $\kappa_{ii} = 0$). It is useful to define a *coupling scale* α which is equal to the average coupling strength, $\alpha = \sum_{i,j} \kappa_{ij}/N$. The coupling scale can be used to represent the overall group cohesion stemming from factors such as the frequency of communications between members, their camaraderie and dedication to the group, and the overall threat to the group.

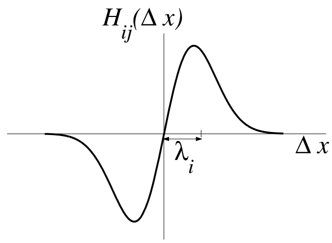

Fig. 1. Plot showing the nonlinear dependence of the coupling force on the inter-member opinion difference. $\Delta x = x_j - x_i$.

2.3 Equation of Motion

The sum of the self-bias and group influence forces determines the rate of change of the i^{th} member's opinion so that $dx_i/dt = S_i(x_i) + G_i(x_i)$. Using the expressions (1)–(3) then yields the following equation of motion for each of the group members:

$$\frac{dx_i}{dt} = -\gamma_i(x_i - \mu_i) + \sum_{j=1}^{N} \kappa_{ij}(x_j - x_i) \exp\left(-\frac{(x_j - x_i)^2}{2\lambda_i^2}\right). \tag{4}$$

With regard to formal models of group decision making, this model is most similar to "social influence network theory," a linear model in which the force producing opinion change in a dyad is always proportional to the level of disagreement [5]. The nonlinear model of Eq. (4), however, has both a "linear" regime at low disagreement levels in which the behavior is intuitive and a "nonlinear" regime at high disagreement levels in which behaviors can run counter to initial intuition. The linear regime is characterized by: gradual changes in policy outcomes and the level of equilibrium group discord as parameters such

as the coupling scale are varied; only one equilibrium for a given set of parameter values; lower group discord for higher network tie densities; and symmetric conditions of opinions and couplings always lead to symmetric final states. The nonlinear regime can exhibit the opposite behaviors: discontinuous transitions between deadlock and consensus as parameters are varied; multiple equilibria for a given set of parameter values; greater discord reduction in less dense networks; and asymmetric outcomes of majority rule even for symmetric conditions [8,7,13].

3 Implementation

This section describes the methodology for implementing the model on real-world, ongoing political contexts based on input obtained from analysts with expertise on the situation of concern. An overview of this methodology is depicted in Figure 2.

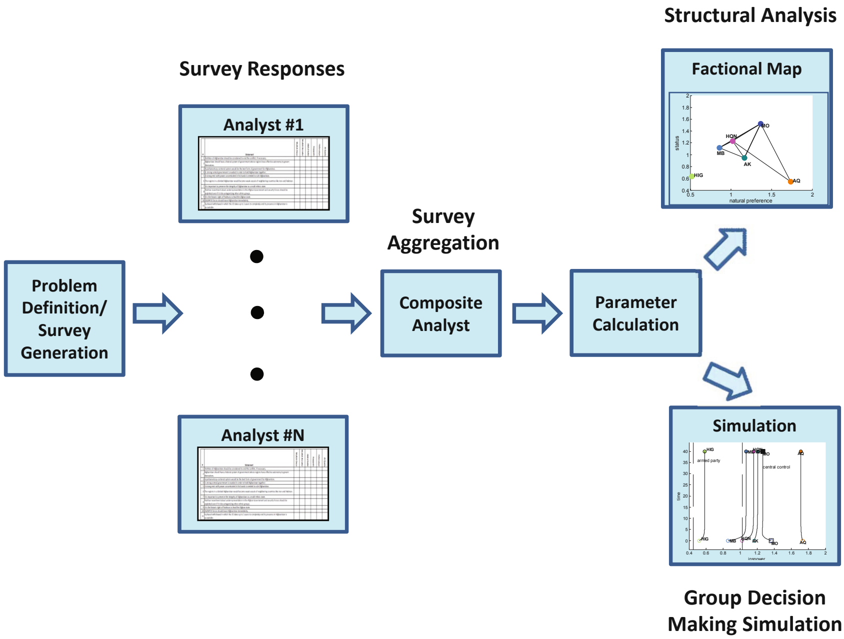

Fig. 2. Overview of Methodology

3.1 Problem Definition and Actor Selection

Problem definition concerns identifying the policy issue(s) of concern and the actors who will comprise the members of the decision-making group that will be modeled. The model assumes that relationships are stable during the course of the decision-making episode. It also assumes that group members are on the

same “team” in that they have important common goals which can be furthered by joint, coordinated action and that their fates are tied together by the success or failure of that action. Accordingly, the policy issue to be modeled should be one in which the achievement of common goals is at stake. This speaks to choosing issues which are core to the success of the group. This can also be achieved by using a broader ideological axis which represents a combination of multiple issues for the position spectrum.

Typically, the members of the decision-making group are individual elites whose policy stances and relationships are critical to the decision-making process. The use of individuals is consistent with the basis of the model in social psychology, although there is no reason based purely on the model formalism which precludes the use of groups or organizations as actors.

Selecting the political elites to include in the model is often difficult given the need to limit the number of actors. This limit does not stem from computational demands of the model but rather practical demands on analyst time for survey completion. A limit of twenty actors seems reasonable based on having a survey that can be completed within a few hours. Another practical factor limiting the size of the group is that it appears to be rare for analysts to have knowledge of a large number of actors at the resolution required by the survey. In addition, a large number of actors can also excessively complicate model interpretation and visualization without significantly improving the analytical value.

Actor selection is most straightforward in situations where there is a formally-constituted small group for making decisions such as the Politburo Standing Committee in China or the General Secretariat of the FARC rebel group in Colombia. In cases where there is no such group, it may be helpful to include actors on some common basis such as having an independent power base external to the group, e.g., bureaucracies, political parties, militias, religious institutions, and tribes (see Sec.4.1).

3.2 Analyst Survey

This section describes the components of the survey which elicits expert judgment on the political group under study. Not all of the components below need to be included in every survey but the Ideologies and Strategic Attitudes and Influence Matrix components are essential.

Ideologies and Strategic Attitudes: This component of the survey is designed to assess the attitudes of the group members relevant to the policy issues of concern. It is used to calculate member natural biases and latitudes of acceptance and to set the intervals along the position spectrum corresponding to different policies. For each member, analysts are asked to estimate the member’s level of agreement/disagreement with a series of statements on a scale ranging from 1 (Strongly Disagrees) to 5 (Strongly Agrees). The instructions direct analysts to evaluate agreement with the statements on the basis of the private beliefs of the members if thought to be at odds with their public rhetoric. The statements cover a range of issues, goals, identities, and specific policies. Examples are shown in Table 3.

Influence Matrix: For the influence matrix, analysts are asked to estimate the strength of each person's direct influence, i.e., that resulting from direct verbal or written communications (perhaps via trusted intermediaries), upon each of the other members in the group. The influence strength depends on factors such as the frequency of communications, status within the group, common or rival factional membership, and personal relationships of friendship or animosity. The influence strength is scaled on a range from 0 (None) to 4 (Very Strong). Each pair of members is represented by two cells in the matrix: one corresponding to influence of i upon j and one for j upon i. The influence matrix values are used to calculate the coupling strengths and commitments.

Status: Analysts are asked to rate the "status" of each group member on a scale from 1 to 10. Status is an estimate of the power of the elite in terms of his ability to influence others within the group. It depends on factors such as his formal rank within the group, the strength and nature of his power base, the amount of resources he controls, and the respect accorded to him. It is used in calculating the policy that emerges from the weighted majority and consensus decision rules and in the factional maps.

Group Affinity: A member's group affinity refers to the extent to which his allegiance resides within the leadership group as opposed to something outside the group such as the organization that he commands or to his ideology. It gives a measure of the degree to which the member will put aside his own personal policy preferences for the sake of preserving group unity. The group affinity is akin to the concept of "primary group identity" used in the decision units framework for foreign policy analysis [16]. The group affinity is scaled from 0 to 1 where 0 signifies total disregard for the opinions of the other group members and 1 signifies that the member is completely concerned with the positions of the others and ignores his own natural bias. Group affinity can be used to calculate the coupling scale.

Decision Rule: The decision rule is the way in which the final positions of the group are combined into a policy decision. Three possible choices are used:

– *Leader Choice:* The chosen policy is the final position of the group leader.
– *Weighted Majority:* The policy supported by the highest status subset of group members wins.
– *Consensus:* All group members must support the final policy. If no consensus policy exists, the status quo policy is the default.

Confidence Level: This component asks the analysts to assess their level of confidence in their knowledge of each of the actors with respect to the information solicited by the survey. A scale of 1 to 4 is used where 1 is "minimal confidence" and 4 is "high confidence." These scores are used in aggregating the analyst surveys to form the composite analyst.

3.3 Survey Aggregation

A composite analyst can be formed by averaging the survey responses of the individual analysts. If desired this can be done in a weighted fashion so that an analyst's answers are weighted by her confidence level for each actor. The aggregation of individual surveys allows for analyst judgments to be synthesized independently of each other, thereby minimizing the chances of social pressures altering individual judgment as can happen if the modeler elicits inputs in an oral discussion with a group of analysts. Note also that results can be generated on the basis of individual surveys as well. This allows for the comparison of the results from individual analysts with the composite analyst and with each other, thereby providing a way of stimulating debate about differences between analyst viewpoints.

3.4 Parameter Calculation

Some parameters can be essentially taken straight from the survey whereas others involve more elaborate calculation. Only the natural bias and latitude of acceptance calculation are noted here.

The natural bias for a given issue is the overall attitude score of a member for that issue which is obtained by averaging the member's responses to the relevant statements for that issue (after flipping those statements phrased to indicate a negative attitude). The attitude scores are put on a scale from -2 (strongly unfavorable) to +2 (strongly favorable). If a linear combination of a number of different issues is used as the position spectrum (e.g., via PCA, see Sec. 3.5), then the natural bias is the linear combination of the attitude scores for the different issues. It is important to remark that this method of placing group members on a position spectrum does not demand of the analyst the task of directly abstracting the range of policy options into a mathematical axis as do some spatial models of group decision making [3,18] — a task for which they may be ill-suited to perform. Rather, it asks for analyst assessments of the level of member agreement/disagreement on the more elemental and concrete aspects of the situation presented in the individual attitude statements.

The latitude of acceptance is calculated as the standard deviation of the natural biases obtained from the individual analysts. This makes the assumption that analyst differences with respect to the member's natural bias reflects genuine ambiguity or uncertainty in his position which in turn affects how open he is to different opinions. Other techniques are possible as well.

3.5 Structural Analysis

Independently of the ultimate simulation of the group interaction dynamics, the survey data can be analyzed to glean insight into the structure of the group of actors with respect to issues, the network of relationships, and actor power.

Structure of Issue Space: Actor positions on the attitude statements and issues can be investigated to understand relationships between different issues

and factional divisions among actors as defined by their positions on the issues. Matrix decomposition techniques such as Principal Component Analysis (PCA) can be used to investigate correlations between issues and actors and the effective dimensionality of the system [2]. PCA decomposes the matrix of actor attitudes on issues into orthogonal principal components. These principal components are ranked in descending order according to the variance of the data along each component. If the first principal component carries the bulk of the variance, then the system is effectively one dimensional. This would be the case, for instance, if one faction of actors consistently takes similar positions on distinct issues whereas another faction takes opposing positions on those issues. In such a situation, the differences between actors on a number of issues can be approximately reduced to a one-dimensional axis in accordance with the assumption of the nonlinear model. The position spectrum can be constructed in such a manner although interpretation is complicated by the fact that it is now a linear combination of a number of issues, rather than a single issue.

Network Structure: The network structure and actor roles as defined by the influence matrix can be analyzed using standard social network analysis methods. This is a distinct picture from that provided by the issue space structure although one would expect there to be similarities in the factional structures exhibited by both under the assumption that birds of a feather flock together, i.e., homophily. As with the issue space, PCA can be used to analyze and visualize the network [4]. If there is a strong factional breakdown in the network, this should be evident in the PCA visualization; those actors with a similar set of relationships should be found near each other in the visualization. For assessing individual roles and influence, metrics such as degree and betweenness centralities can be calculated. Weighted out and in-degree centralities reflect, respectively, the influence going out from and coming into the actor. These can be compared with the direct assessment of actor status from the survey; typically, the correlation between them is high. While the correlation between high status and high out-degree centrality would be expected for a leader, the correlation between high power and high in-degree centrality might be less expected. This stems from the larger number of actors that leaders are connected to and to whom they must be responsive if they seek to maintain the cohesion of the group; one would particularly expect leaders who are interested in consensus-building to have high in-degree.

Factional Maps: Actor issue positions, relationships, and power can be jointly visualized using a "factional map." The actor natural biases for the issue of concern are plotted on the horizontal axis, actor status on the vertical axis, and the relationships are plotted as links between the actors. Examples are shown in Fig. 3. The factional map provides an integrated representation of issue and network-based factional structure. Potential alliances can be identified as well as actors who could play key roles such as brokers or swing players. As an example, factional maps of Iraqi insurgent groups constructed directly from their rhetoric (rather than analyst judgments) reflected alliances that eventually

formed and showed the role of the Islamic Army in Iraq as a bridge between different ideological wings of the insurgency [10].

Another way of integrating ideologies and relationships is via the use of an ideology-weighted centrality metric. Here the tie strengths from the influence matrix are further weighted by a function that decreases with ideological distance, a gaussian for instance. This metric was used to analyze potential successors to Putin in 2007 [6].

3.6 Model Simulation

Model simulation is used to investigate potential results of the group decision-making process with respect to the policy outcome, the level of discord associated with that policy, and which group members sign on to the policy and which dissent. Group members are typically initialized at their natural biases and the model is run until equilibrium. (Currently, the time units are arbitrary given the difficulty of estimating the actual rates implicit in the commitment and coupling parameters.) The decision rule is used to aggregate the final member positions along the policy axis and the members of the winning coalition and dissenters are calculated. Sensitivity and scenario analyses can then be conducted to more fully assess the implications of the model.

The decision rule used to aggregate the group member final positions can be taken as the one chosen by the majority of analysts or it can be varied as well. For leader choice, the leader's final position is the policy. For weighted majority, the policy that has the most status-weighted support is the outcome; the support that each member provides to a prospective policy position decreases as a gaussian function of the distance between the prospective policy and his final position. This method allows for the policy outcome in a case of majority rule to reside within the range of positions of the majority. Otherwise, if a simple status-weighted linear combination of member positions were used then the chosen policy could lie somewhere between the majority and minority positions and, hence, would not correspond to majority rule at all. All those within their latitude of acceptance of the final policy are said to be in the winning coalition and those further away are deemed dissenters. The policy for a consensus decision rule is calculated in the same way as for a weighted majority but there can be no dissenters in order for the policy to be chosen.

Both sensitivity and scenario analysis involve varying parameters but their goals and the manner in which they are accomplished can differ. Sensitivity analysis involves running the simulation while sweeping over a parameter(s). This is used to judge the range of potential outcomes that can result due to uncertainty in model parameters. The selected parameter might be: (1) an intrinsically important one such as the coupling scale which is hard to pin down precisely and could significantly affect the results; or (2) one for which there is a large variance in analyst estimates indicating that there is substantial uncertainty in its value. The ability of an individual to sway the simulation outcome by changing his natural bias can be assessed using an "outcome centrality" metric

which can serve as a sensitivity analysis measure for addressing the importance of uncertainties in the preferences of individual group members [6].

Scenario analysis entails changing parameters to correspond with a hypothesized change in the situation, e.g., a particular member(s) dramatically shifts his position, a member's status increases, a member leaves the group or dies, or a tie between two members is severed. The scenario analysis can be run using natural bias initial conditions or from the equilibrium positions that resulted prior to the changes effected in the scenario.

4 Afghanistan Application

This section illustrates application of the methodology for the case of the ongoing conflict in Afghanistan. Both Afghan government elites and insurgent leaders were included as separate decision-making groups in the analyst survey. Analytical questions focused on the prospects of a negotiated solution between the two sides, continued U.S. presence and influence, the degree of centralization of the Afghan state, and ethnic tensions. Survey responses from analysts with expertise on Afghan politics and the insurgency were obtained in the spring of 2011. Analysis and simulation were conducted in Fall 2011. Some of the implications of this modeling exercise were incorporated into the analysis of Taliban strategy and Afghan government vulnerability presented in Ref. [12].

4.1 Elite Actors

The set of Afghan Government elite actors is listed in Table 1 and the Insurgent elite actors in Table 2. For actor selection purposes, an elite actor was considered to be an independently powerful individual who has communication with other members of his group and should have a power-base independent of his title or position. An actor's power base can be tribal, ethnic, regional, military, religious, or organizational in nature and the constituent members of the power base should hold more allegiance to the individual actor than to the elite actor group (Government or Insurgent) to which he belongs. For inclusion in the Afghan Government group, an elite had to (1) generally support the concept of an Afghanistan arranged along the lines of the current constitution; and (2) not use his influence or constituents to incite large-scale violence against Afghan government or Coalition forces. Insurgent elites had to be marked by the opposites of (1) and (2).

After the surveys were completed but prior to the analysis of results, two major events affected the composition of the actors in these groups: (1) Osama Bin Laden was killed by U.S. commandos in May 2011; (2) Burhanuddin Rabbani was killed in September 2011 by a suicide bomber posing as a Taliban peace emissary meeting with him in his capacity as chairman of the High Peace Council. The use of the Al Qaida core leadership as an actor rather than Bin Laden himself meant that the actor was not lost but his death clearly would be expected to have an impact on Al Qaida's status and relations with the other insurgent actors not

Table 1. Afghan government actors included in survey

Actor	Symbol	Ethnicity	Position
Hamid Karzai	KRZ	Pashtun	President
Mohammed Qasim Fahim	FHM	Tajik	Vice President
Karim Khalili	KAL	Hazara	Vice President
Burhanuddin Rabbani	RAB	Tajik	Chairman, High Peace Council; Head, Jamiat-e-Islami party
Abdul Rashid Dostum	DOS	Uzbek	Founder, Junbesh party; Armed Forces Chief of Staff (ceremonial)
Atta Mohammed Nur	NUR	Tajik	Governor, Balkh
Gul Agha Sherzai	SHZ	Pashtun	Governor, Nangarhar
Mohammed Mohaqiq	MOQ	Hazara	Head, Wahdat-e-Mardum party; Member of Parliament
Ismail Khan	IK	Tajik	Energy Minister
Abdul Rasul Sayyaf	SAY	Pashtun	Member of Parliament

Table 2. Afghan insurgent actors included in survey

Actor	Symbol	Organization	Role/Notes
Mullah Omar	MO	Afghan Taliban	Supreme Leader
Mullah Baradar	MB	Afghan Taliban	Former First Deputy (detained)
Mawlawi Abdul Kabir	AK	Afghan Taliban	Military Commander, Eastern Region
Haqqanis	HQN	Haqqani Network; Afghan Taliban	Amalgam of leaders Jaluluddin & Sirajuddin; also in Taliban Leadership Council
Gulbuddin Hekmatyar	HIG	Hezb-e-Islami	Leader
Al Qaida Leadership	AQ	Al Qaida	Amalgam of core leaders, e.g., Bin Laden, Zawahiri, Abu Yahya al-Libi

reflected in the original survey. However, simulations in which the corresponding parameters were reduced had little impact on the results. Rabbani's death meant the total loss of an actor. Simulations were conducted mostly with him removed from the data but the analysis helps reveal the potential motive behind his assassination as discussed in Sec. 4.4.

4.2 Survey Attitude Statements

The Ideology and Strategic Attitudes component of the survey contained 40 statements for the Insurgent side and 37 for the Government side. The statements explored a number of actor policy issues, ideological attitudes, and social identities such as insurgent political power, state centralization, U.S. influence, Pakistani influence, and Afghan vs. ethnic identities. Table 3 shows a selection of statements for Afghan Government actors bearing on the issues of state centralization and accommodation of insurgent political power.

4.3 Structural Analysis

Factional maps for both sides are displayed in Fig. 3. The policy issue concerns insurgent political power, which entails, on the Insurgent side, how much political

Table 3. Selected attitude statements for Afghan Government actors

1. Partition of Afghanistan should be considered to end the conflict, if necessary.
2. Afghanistan should have a federal system of government where regions have effective autonomy to govern themselves.
3. Karzai's efforts to concentrate power in the presidency show that the Afghan Constitution should be changed to institute a parliamentary-centered system of government.
4. A strong central government is needed in order to hold Afghanistan together.
5. The insurgents are criminals, terrorists and rebels who must be put down militarily, not negotiated with.
6. If the insurgents were to halt their armed struggle and disarm, they could legitimately represent their constituents as a political party.
7. It would be acceptable for the insurgents to openly join the political process without disarming if a permanent ceasefire is agreed to.
8. A coalition government with members including insurgent leaders would be the best way to represent the Afghan population and end the conflict.
9. The best way to achieve peace is to cede effective control of some parts of Afghanistan to the insurgents.

power they are striving for and on the Government side the degree of political power they should be accommodated. It is plotted on the same scale for both sides so that the Government actors mostly have negative scores indicating less accommodation of insurgent power and the Insurgents have positive scores (specific policy labels are noted in Sec. 4.4).

The Afghan Government map shows the non-Pashtun ethnic groups on the hawkish side of the spectrum and Pashtuns on the dovish side. Karzai is the most powerful actor and his network ties show him as a bridge between Pashtuns and non-Pashtuns. Importantly, Rabbani is seen to occupy a pivotal position as the least hawkish of the non-Pashtuns and having strong ties with Karzai and most of the other non-Pashtun actors. This indicates why Karzai may have selected him as chairman of the High Peace Council — to help bring non-Pashtuns onboard with the process of reconciliation with insurgents. For the Insurgents, Mullah Omar is the most powerful and is on the hawkish side of the spectrum. The other Taliban-affiliated actors are less hawkish. Al Qaida is seen to be on the extreme hawkish end of the spectrum but having the least status. Hekmatyar is on the dovish extreme of the spectrum but has relatively little power and has poor relationships with the other insurgent actors.

4.4 Simulations

Simulations of the nonlinear decision-making model of Eq. (4) are shown in Fig. 4 for the insurgent political power issue. The intervals along the position spectrum corresponding to different qualitative policies are indicated: "no power" — no insurgent political power is to be accommodated; "unarmed party" — insurgents can participate in politics after disarming; "coalition" — insurgent leaders should be brought into a national coalition government; "armed party" — insurgents can retain their arms, control some territory, and participate as a political party if they end their violence against the government; "central control" — insurgents seek to conquer the central state and control Afghanistan. The dashed lines which bracket the policies serve as rough guides rather than hard boundaries.

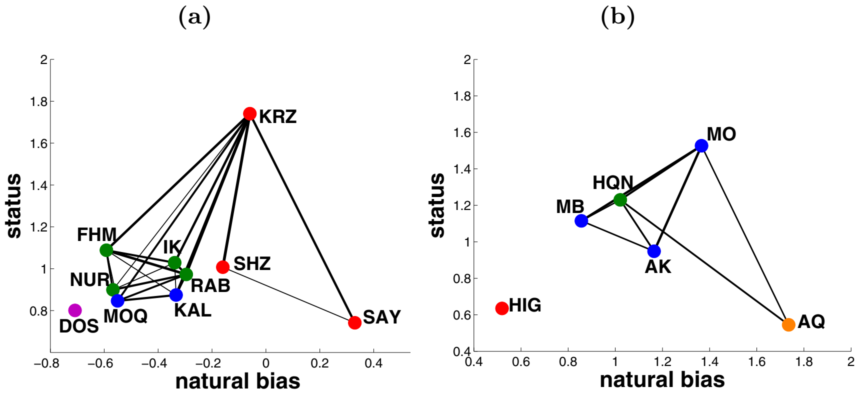

Fig. 3. Factional maps for insurgent political power issue. (a) Afghan Government actors, ethnicities — Pashtun (red), Tajik (green), Hazara (blue), Uzbek (purple). (b) Insurgent actors — those formally part of the Taliban organization in blue or green. Tie strength is proportional to link thickness; weak ties have been thresholded.

For the Afghan Government, the policy under a leader choice decision rule is seen to be "unarmed party," a policy which would support negotiations with insurgents and attempts to bring them into the political process. Dissenters include Nur, Mohaqiq, and Dostum on the hawkish side and Sayyaf on the dovish side. Note that although Rabbani is left out of the simulation shown in Fig. 4(a), none of the other model parameters were changed to account for the effects of his death, but scenario analyses aimed at doing so were conducted. For instance, to model hardened stances of anti-Taliban hawks in response to his killing, the above non-Pashtun dissenters had their commitments set to one, i.e., their positions are fixed, which has the effect of bringing Karzai to the "no power" policy interval. In addition, a sensitivity analysis shows that if the coupling scale were increased, due perhaps to an increased sense of threat to the government, then Karzai also would swing toward a more hardline policy closer to that favored by non-Pashtuns. In the immediate aftermath of Rabbani's assassination, Karzai did indeed become more hardline although since then he appears to have drifted back to a more dovish position, at least on a rhetorical level. In general, scenario analyses show that it is extremely difficult to forge a consensus policy on this issue and that if Karzai moves significantly to the left or right he will lose either Pashtun or non-Pashtun support respectively. This indicates his vulnerability to being isolated from one of these two key constituencies.

The Taliban are seen to coalesce around a "central control" policy which is Mullah Omar's choice. The only Insurgent dissenter is Hekmatyar who does not move significantly from his "armed party" natural bias given his weak links with the other actors. This solid support for a policy of seizing the central state indicates that Taliban negotiations overtures toward the United States in late 2011/early 2012 did not reflect a sincere desire to seek a peace deal with the Afghan government, as argued in Ref. [12]. In perhaps a confirmation of this conclusion, a recent article states that the U.S. government, previously hopeful,

has largely given up on negotiations with the Taliban [19]. Both sensitivity and scenario analyses indicate great difficulty in moving Mullah Omar from the "central control" policy to the "armed party" policy. For example, no matter how much Mullah Baradar were to move toward a dovish position (which might be a condition of his release), it would still not be sufficient to shift Mullah Omar into the "armed party" zone.

These simulations along with insight from the Afghan Government factional map suggest why the Taliban may have assassinated Rabbani and also their broader strategy toward the Afghan government [12]. The conclusion that the Taliban are dedicated to the goal of "central control" implies that they must pursue a military solution vis a vis the Afghan government rather than a negotiated one. Rabbani's pivotal position within the network of Afghan Government elites noted above suggests that his killing would serve to exacerbate ethnic tensions between Pashtun and non-Pashtun government elites and heighten the divide over how to deal with the Taliban; both through the loss of his direct influence as well as the shock of the act itself. This in turn would make it more difficult for Karzai to effectively act as a bridge between Pashtuns and non-Pashtuns as seen in Fig. 3(a) and increases his potential to be isolated from one of those groups. An isolated Karzai decreases the sense of national unity among Afghan government elites and the population at large. This weakened national unity and drop in cohesion within the Afghan government would in turn decrease support for the Afghan National Security Forces — the primary obstacle to a Taliban military victory given the planned U.S. force drawdown.

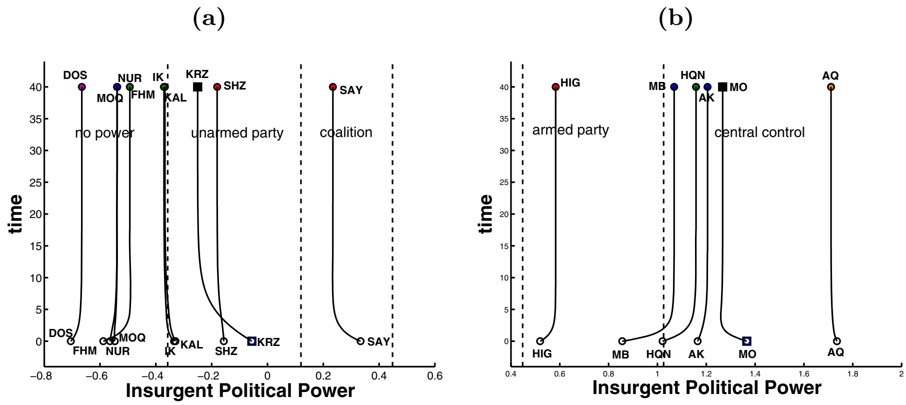

Fig. 4. Simulations of insurgent political power issue. (a) Afghan Government (w/o Rabbani), Karzai choice decision rule. (b) Insurgents, Mullah Omar choice decision rule. Open circles are actor initial positions, solid circles are final positions. Lines are actor position trajectories. Solid square indicates the final policy position; open square would be decision in absence of debate.

5 Conclusion

The nonlinear model employed to simulate decision-making outcomes synthesizes attitude change theory, social network structure, and nonlinear dynamical systems mathematics and so represents an innovative approach to the formal modeling of political decision making. The combination of the policy preference distribution in the group and its social network can form a complex structure whose complexity is further compounded by the nonlinear nature of the interactions between members in which member opinions need not move in simple proportion to their differences. The model provides a framework wherein these elements are integrated in a self-consistent manner that is not readily done by qualitative analysis alone, and allows for the controlled testing of the effects of changes or uncertainties in group variables. The nonlinear aspect of the model gives rise to the fact that the group dynamics can change *qualitatively* — and not merely as a matter of degree — as a function of the level of disagreement.

The associated analyst survey provides a systematic way of obtaining analyst judgment on the substantive aspects of the decision making group that enter into the model. The survey's use of attitude scale methodology to assess and calculate the ideological and policy positions of group members is natural given the nonlinear model's foundations in attitude change theory. This combination of attitude scaling and a formal model of elite decision making is another innovative aspect of the methodology outlined in this paper. It elicits analyst expertise on actor policy preferences without demanding that they perform the abstraction needed to create a policy axis or space itself, — a task which instead is left to the modeler.

As an alternative to implementation with analyst input, the use of rhetoric-based methods of obtaining actor ideologies and networks has been explored at the individual and organizational levels and used to inform policy analysis of ongoing situations [10,9,11,14]. A comparison of rhetoric-based Afghan Government and Insurgent actor ideologies with analyst assessments from the survey yielded good correlations for major issue dimensions. Other potential items for further research include: modeling multi-dimensional issue space dynamics; incorporating stochastic modeling and forecasting; a co-evolution model in which policy positions and actor relationships can evolve simultaneously; and integration with game-theoretic approaches.

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Appendix 7

Majority Rule in Nonlinear Opinion Dynamics

Majority Rule in Nonlinear Opinion Dynamics

Michael Gabbay and Arindam K. Das

Abstract Using a nonlinear model of opinion dynamics on networks, we show the existence of asymmetric majority rule solutions for symmetric initial opinion distributions and symmetric network structure. We show that this occurs in triads as the result of a pitchfork bifurcation and arises in both chain and complete topologies with symmetric as well as asymmetric coupling. Analytical approximations for bifurcation boundaries are derived which closely match numerically-obtained boundaries. Bifurcation-induced symmetry breaking represents a novel mechanism for generating majority rule outcomes without built-in structural or dynamical asymmetries; however, the policy outcome is fundamentally unpredictable.

1 Introduction

Small group opinion change has long been a subject of intense study in social science with implications for decision making by a range of groups such as political leaders, judicial panels, corporate committees, and juries [8, 4]. Mathematical models of small group decision making have been proposed in social science disciplines such as psychology, sociology, political science, economics, and law [2, 9, 5]. In this paper, we put forth a novel mechanism for the generation of majority rule outcomes in small groups via a symmetry-breaking pitchfork bifurcation. This mechanism allows for asymmetric outcomes to appear for symmetric initial opinion distributions even when group members are symmetrically coupled. It occurs in the nonlinear opinion dynamics model of Refs. [7, 6] under conditions of high disagreement be-

Michael Gabbay
Applied Physics Laboratory, University of Washington, Seattle, Washington, 98105 e-mail: gabbay@uw.edu

Arindam K. Das
Applied Physics Laboratory, University of Washington, Seattle, Washington, 98105 e-mail: arindam@uw.edu

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tween the ends of the distribution along a continuous opinion axis. For example, in a triad network consisting of a centrist bracketed by two opposed extremists, the centrist will form a majority pair with one of the extremists. This runs counter to intuition rooted in basic psychological mechanisms of attitude change which emphasize a convergence process of group member attitudes, and so would anticipate either deadlock or various degrees of compromise around the centrist's position, but not majority rule. In particular, it is not predicted by the most prominent network-based model of small group opinion dynamics, the Friedkin-Johnsen model, which is linear in the disagreement between group members [5].

The Friedkin-Johnsen and nonlinear opinion dynamics models are described in the next section. The majority rule outcome for a triad is demonstrated in simulation (Sec. 3) and via bifurcation analysis (Sec. 4). Majority rule in five-node networks is presented in Sec. 5.

2 Opinion Dynamics Models

Most recent work on opinion network dynamics in the physics community has focused on large networks motivated by an interest in population scale dynamics [1]. Consensus in small networks has been studied in the literature on distributed network control with sensor networks as a primary motivation [10, 11]. However, our nonlinear model is most closely related to that of Friedkin and Johnsen, which was explicitly developed for the social influence context and has been subjected to empirical investigation [5].

Friedkin-Johnsen Model

The Friedkin-Johnsen model describes the temporal evolution of a linear discrete time influence process in a group of N people (nodes) as a weighted average of their previous opinions and their initial opinions [5]:

$$x_i(k+1) = a_i \sum_{j=1}^{N} w_{ij} x_j(k) + (1-a_i) x_i(0); \quad i = 1,2,\ldots N,\ k \geq 0, \tag{1}$$

where: $x_i(k)$ is the opinion of node i at time k; $x_i(0)$ is the initial opinion; a_i is the *susceptibility* of node i; and w_{ij} is the coupling weight scaling node j's influence upon i. The w_{ij} are non-negative and satisfy $\sum_{j=1}^{N} w_{ij} = 1$. In addition, the susceptibility is given by $a_i = 1 - w_{ii}$.

Equation (1) can be cast as a difference equation by subtracting $x_i(k) = (1 + a_i - a_i)x_i(k)$ from both sides and rearranging to yield

$$x_i(k+1) - x_i(k) = a_i \sum_{j=1}^{N} w_{ij}(x_j(k) - x_i(k)) - (1-a_i)(x_i(k) - x_i(0)). \tag{2}$$

If $a_i = 1 \; \forall i$ in Eq. 2, then the node opinions will all converge to exactly the same value for a (bidirectionally) connected network. The presence of $x_i(0)$ in the dynamics of the Friedkin-Johnsen model prevents such a collapse onto an exact consensus which would signify the unintuitive complete extinction of disagreement. However, because of the linear coupling in the Friedkin-Johnsen model, equilibria in which the member opinions are asymmetrically distributed around the mean must arise from pre-existing asymmetries; either skewed initial opinion distributions or lopsided coupling weights in favor of one extreme. This is not the case for the nonlinear model which we turn to next.

Nonlinear Model

We use the following model for the evolution of the opinion x_i [7]:

$$\frac{dx_i}{dt} = -\gamma_i(x_i - \mu_i) + \sum_{j=1}^{N} \kappa_{ij} h(x_j - x_i). \tag{3}$$

The first term on the right is a linear "self-bias force" which expresses the psychological tension that a person feels if her opinion is displaced from her *natural bias* μ_i and is proportional to her *commitment* γ_i. The second term is the "group influence force" on i where κ_{ij} is the *coupling strength* and $h(x_j - x_i)$ is the *coupling function*. The coupling strength, which we take to be non-negative, represents the components of influence of j upon i arising from their relationship; it depends on factors such as how often j communicates with i and the regard with which i holds j. The coupling function represents how the influence of j upon i depends on the difference between their opinions. We use a dependence motivated by social judgment theory [4] in which the force grows for $|x_j - x_i| < \lambda_i$, where λ_i is i's *latitude of acceptance*, but wanes for differences in excess of λ_i:

$$h(x_j - x_i) = (x_j - x_i) \exp\left[-\frac{1}{2}\frac{(x_j - x_i)^2}{\lambda_i^2}\right]. \tag{4}$$

For situations in which a group first starts discussing an issue it is appropriate to use natural bias initial conditions, $x_i(0) = \mu_i$.

In the linear limit, $\lambda_i \to \infty$, it can readily be seen that the (discretized) nonlinear model reduces to the form (2) of the Friedkin-Johnsen model, apart from parameter constraints. The natural bias μ_i plays the same role in preventing the collapse onto exact agreement in (3) as the initial opinion does in (1). When applied to group decision-making, we assume that a common decision can be reached between group members if their final opinions $x_i(t_f)$ are sufficiently close. This is in accord with the intuition that people need not precisely agree in order to reach a compromise decision on a common course of action.

3 Triad Simulations

We simulate a triad network in which the natural biases are symmetrically distributed around zero: $\mu_1 = -\Delta\mu/2,\ \mu_2 = 0$, and $\mu_3 = \Delta\mu/2$. We use a chain topology whose ends, nodes 1 and 3, are not connected so that the symmetric, binary adjacency matrix elements are $A_{12} = A_{21} = A_{23} = A_{32} = 1$ and $A_{13} = A_{31} = 0$ (and also $A_{ii} = 0$). However, the complete network in which all members are connected, $A_{ij} = 1 - \delta_{ij}$ where δ_{ij} is Kronecker's delta, has similar behavior as will be seen in Sec. 4. We use the parameter ν to allow for the possibility of asymmetric coupling between the center node 2 and the end nodes so that $\kappa_{12} = \kappa_{32} = \kappa + \nu$ and $\kappa_{21} = \kappa_{23} = \kappa - \nu$ where $|\nu| < \kappa$. A positive value of ν signifies that the center node has greater influence on each of the end nodes than vice versa whereas negative ν signifies that the ends have more influence. The equations of motion for the triad are then:

$$
\begin{aligned}
\frac{dx_1}{dt} &= -\left(x_1 + \frac{\Delta\mu}{2}\right) + (\kappa+\nu)h(x_2 - x_1) + \kappa A_{31} h(x_3 - x_1), \\
\frac{dx_2}{dt} &= -x_2 + (\kappa - \nu)(h(x_1 - x_2) + h(x_3 - x_2)), \\
\frac{dx_3}{dt} &= -\left(x_3 - \frac{\Delta\mu}{2}\right) + (\kappa+\nu)h(x_2 - x_3) + \kappa A_{31} h(x_1 - x_3).
\end{aligned} \tag{5}
$$

It will be useful to define the following pair of variables: the *discord* $r = x_3 - x_1$, the opinion difference between the outer nodes and the *asymmetry* $s = (x_3 - x_2) - (x_2 - x_1)$, the difference in distances from the outer nodes to the middle node.

Figure 1 shows simulations of the chain network for three values of the coupling strength κ and with symmetric coupling between all nodes. The difference in the natural biases of the end nodes is $\Delta\mu = 5$ and the initial opinions are set equal to the natural biases (except for a tiny displacement to the center node as an initial perturbation which always moves x_2 in the same direction for the asymmetric solutions). Three qualitatively distinct equilibria are observed. At low coupling, Fig. 1(a) shows a state of Symmetric High Discord (SHD) in which the end nodes barely move from their natural biases and the center node remains at zero. At intermediate coupling, Fig. 1(b) shows the Majority Rule (MR) state in which the center node moves toward one of the end nodes to form a majority rule pair. At high coupling, the outer nodes move considerably toward the center to form a Symmetric Low Discord (SLD) state as shown in Fig. 1(c). The SHD state corresponds to a deadlock situation in which all group members are far apart and no acceptable mutual decision can be made. In the MR state, the majority pair can likely agree on a common policy which will be the policy of the group if majority rule is sufficient for reaching a decision. In the SLD state, the distance between the outer nodes is much reduced and the basis for a compromise around the centrist's position could be set. Simulations in which μ_1, μ_2, and μ_3 are randomly shifted by a small amount still display all three outcome types.

Figure 2 plots the asymmetry and discord of the symmetrically-coupled chain network with natural bias initial conditions. The emergence of the MR state only occurs past a critical value of the natural bias difference $\Delta\mu_c = 3.8$ which we call the *critical divergence*. Also, note the sharp discontinuities in at the boundaries between the various outcome states. Below the critical divergence, asymmetric solutions do not exist and the discord is smoothly and symmetrically reduced as the coupling strength is raised as would occur in the equivalent case for the Friedkin-Johnsen model, for which the transition from deadlock to compromise to consensus is gradual with no possibility of an MR state.

4 Bifurcation Analysis for Triad

In this section, we show that the majority rule state is the result of spontaneous symmetry-breaking induced by a pitchfork bifurcation and we calculate bifurcation boundaries. We do this for the chain topology in which $A_{31} = 0$. We transform the system (5) into the discord and asymmetry variables, r and s, as well as the

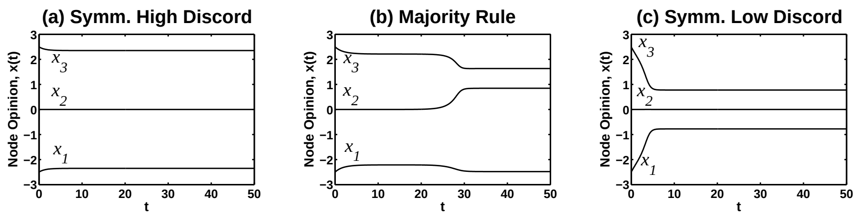

Fig. 1 Equilibrium outcomes in symmetrically-coupled ($\nu = 0$) triad chain network with high initial disagreement, $\Delta\mu = 5$, at different coupling strengths: (a) $\kappa = 1$, Symmetric High Discord; (b) $\kappa = 1.5$, Majority Rule; (c) $\kappa = 3$, Symmetric Low Discord. Initial conditions: $x_1(0) = -2.5$, $x_2(0) = 10^{-6}$, $x_3(0) = 2.5$.

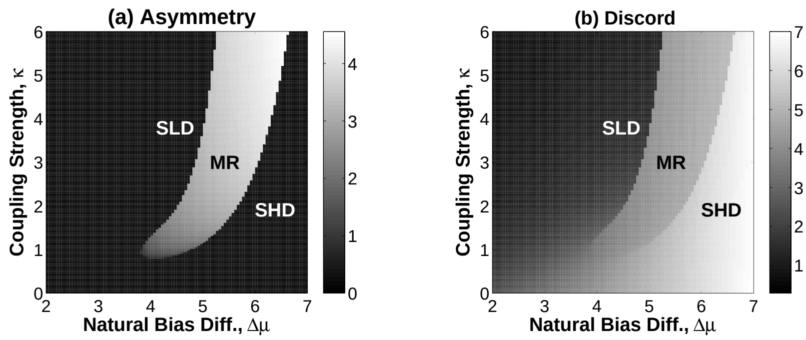

Fig. 2 Simulation of a symmetrically-coupled triad chain network over $\Delta\mu$-κ parameter space showing final: (a) discord and (b) asymmetry (absolute value). Simulation duration is $t_f = 200$.

mean node opinion, $\bar{x} = \frac{1}{3}\sum_{i=1}^{3} x_i$. Using the fact that the coupling function is odd, $h(-x) = -h(x)$, results in the system:

$$\frac{dr}{dt} = -(r-\Delta\mu) - (\kappa+\nu)\left(h(\frac{r+s}{2}) + h(\frac{r-s}{2})\right), \tag{6}$$

$$\frac{ds}{dt} = -s - (3\kappa-\nu)\left(h(\frac{r+s}{2}) - h(\frac{r-s}{2})\right), \tag{7}$$

$$\frac{d\bar{x}}{dt} = -\bar{x} - \frac{2}{3}\nu\left(h(\frac{r+s}{2}) - h(\frac{r-s}{2})\right). \tag{8}$$

For symmetric coupling, Eq. (8) implies that the mean equilibrium opinion is zero, the mean of the natural biases; this will not be the case for $\nu \neq 0$ in the MR state in which $s \neq 0$.

For the equilibrium SHD state, denoted by (r^*, s^*), the asymmetry is by definition $s^* = 0$. For large $\Delta\mu$ we take the discord to be $r^* \approx \Delta\mu + \theta$ where $\theta \ll 1$. Before showing the existence of the pitchfork bifurcation, it will be useful below to calculate θ. When $s = 0$, Eq. (6) becomes

$$\frac{dr}{dt} = -(r-\Delta\mu) - 2(\kappa+\nu)h(\frac{r}{2}), \tag{9}$$

which upon substituting the above form for r^* yields

$$0 = \theta + 2(\kappa+\nu)h(\frac{\Delta\mu+\theta}{2}). \tag{10}$$

Expanding the coupling function as $h(\frac{\Delta\mu+\theta}{2}) \approx h(\frac{\Delta\mu}{2}) + h'(\frac{\Delta\mu}{2})\frac{\theta}{2}$ and substituting into (10) enables us to solve for θ

$$\theta = -\frac{2(\kappa+\nu)h(\frac{\Delta\mu}{2})}{1+(\kappa+\nu)h'(\frac{\Delta\mu}{2})}, \tag{11}$$

where $h(\frac{\Delta\mu}{2}) = \frac{\Delta\mu}{2}e^{-\frac{\Delta\mu^2}{8}}$ and $h'(\frac{\Delta\mu}{2}) = (1-\frac{\Delta\mu^2}{4})e^{-\frac{\Delta\mu^2}{8}}$.

To show the bifurcation, we consider small perturbations s around $s^* = 0$ in Eq. (7). This results in the Taylor expansion,

$$\frac{ds}{dt} \approx -\left(1+(3\kappa-\nu)h'(\frac{r^*}{2})\right)s - \frac{1}{24}(3\kappa-\nu)h'''(\frac{r^*}{2})s^3, \tag{12}$$

where only the odd power terms survive. When the coefficient of the linear term is positive, the symmetric state will be unstable. When $h'''(\frac{r^*}{2}) > 0$, we can rescale as follows:

$$\tau = \left[\frac{1}{24}(3\kappa-\nu)h'''(\frac{r^*}{2})\right]t \tag{13}$$

$$R = -\frac{1+(3\kappa-\nu)h'(\frac{r^*}{2})}{\frac{1}{24}(3\kappa-\nu)h'''(\frac{r^*}{2})} \tag{14}$$

which transforms (12) into the normal form of a supercritical pitchfork bifurcation, $ds/d\tau = Rs - s^3$, where the bifurcation occurs for $R = 0$, beyond which the symmetric $s^* = 0$ equilibrium is absolutely unstable and two stable asymmetric branches emerge [12].

When $h'''(\frac{r^*}{2}) < 0$, the pitchfork bifurcation is subcritical, exhibiting a hard loss of stability, multistability, and hysteresis. The relevant zero crossing of $h'''(x) = (-x^4+6x^2-3)e^{-\frac{1}{2}x^2}$ in marking the boundary between supercritical and subcritical bifurcations occurs at $x = (3+\sqrt{6})^{1/2}$ corresponding to a discord of $r^* = 4.66$.

SHD Upper Boundary: κ_1

We now calculate the boundary in $\Delta\mu$-κ parameter space given by the critical value of the coupling strength κ_1 at which the SHD state becomes absolutely unstable. Setting the coefficient of the first term on the righthand side of (12) equal to zero yields

$$\kappa_1 = -\frac{1}{3h'(\frac{r^*}{2})} + \frac{\nu}{3}. \tag{15}$$

Substituting $r^* \approx \Delta\mu + \theta$, and expanding (15) to first order in θ gives

$$\kappa_1 \approx -\frac{1}{3}\left\{\frac{1}{h'(\frac{\Delta\mu}{2})} - \frac{h''(\frac{\Delta\mu}{2})}{h'^2(\frac{\Delta\mu}{2})}\frac{\theta}{2}\right\} + \frac{\nu}{3}. \tag{16}$$

The expression (11) for θ can be inserted into the above which, after rearranging, yields the characteristic equation

$$0 = 3h'(\frac{\Delta\mu}{2})\kappa_1^2 + \left(4+M+2\nu h'(\frac{\Delta\mu}{2})\right)\kappa_1 + \frac{1}{h'(\frac{\Delta\mu}{2})} + M\nu - \nu^2 h'(\frac{\Delta\mu}{2}), \tag{17}$$

where $M = \frac{\Delta\mu^4 - 12\Delta\mu^2}{(\Delta\mu^2-4)^2}$. This can be solved to give the following approximation for κ_1:

$$\kappa_1 \approx \frac{2}{3}\frac{e^{\frac{\Delta\mu^2}{8}}}{(\Delta\mu^2-4)}\Big\{4+M+2\nu h'(\tfrac{\Delta\mu}{2}) - \big[4+8M+M^2 + 8\nu h'(\tfrac{\Delta\mu}{2})\big(2-M+2\nu h'(\tfrac{\Delta\mu}{2})\big)\big]^{\frac{1}{2}}\Big\}. \tag{18}$$

This increases rapidly as $\Delta\mu$ becomes very large. The appearance of ν as a product with the very small $h'(\frac{\Delta\mu}{2})$ implies that κ_1 will be nearly identical to the $\nu = 0$ case as $\Delta\mu$ gets large.

MR Lower Boundary in Subcritical Zone: κ_2

Turning now to the disappearance of the asymmetric solutions in the subcritical bifurcation regime, this corresponds to the transition between the multistable zone where the MR and SHD states coexist to the zone in which only the SHD state exists as the coupling strength is lowered. This transition occurs via a saddle-node bifurcation in which stable and unstable asymmetric equilibria collide. The associated bifurcation boundary κ_2 can be calculated by analyzing Eq. (7) around the MR equilibrium in which the minority node x_1 stays near its natural bias while the majority pair (x_2,x_3) is very nearly symmetrically positioned around the midpoint between their natural biases, $\Delta\mu/4$. Asymmetric coupling, $\nu \neq 0$, will shift the equilibrium mean of the majority rule pair by an amount given by $\varepsilon = (x_2^*+x_3^*)/2-\Delta\mu/4$. For large $\Delta\mu$, $x_2-x_1=(r-s)/2$ is large and we can neglect the term $h((r-s)/2)$ in Eq. (7). Accordingly, we make the approximations for the outer opinion coordinates: $x_1 \approx -\Delta\mu/2$ and $x_3 \approx \Delta\mu/2+2\varepsilon-x_2$. The asymmetry is then $s = x_3-2x_2+x_3 = -3x_2+2\varepsilon$. Rearranging yields $x_2 = -s/3+2\varepsilon/3$ and then $x_3 = s/3+\Delta\mu/2+4\varepsilon/3$ so that the discord can now be written in terms of s as $r = x_3-x_1 = s/3+\Delta\mu+4\varepsilon/3$. The argument of the coupling function term retained from Eq. (7) is $(r+s)/2 = \frac{2}{3}(s+\frac{3}{4}\Delta\mu+\varepsilon)$. Finally, we transform to the variable $\tilde{s} = s+3\Delta\mu/4+\varepsilon$ and Eq. (7) becomes

$$\frac{d\tilde{s}}{dt} = -(\tilde{s}-\frac{3}{4}\Delta\mu-\varepsilon)-(3\kappa-\nu)h(\frac{2}{3}\tilde{s}). \tag{19}$$

Equation (8) can be used to calculate the shift ε in the mean of x_2^* and x_3^* (neglecting the $h((r-s)/2)$ term and using $x_1^* = -\Delta\mu/2$) yielding $\varepsilon = -\nu h(\frac{r^*+s^*}{2}) = -\nu h(\frac{2}{3}\tilde{s}^*)$. Taking $\nu \ll \kappa$, the first order contribution of ν resulting from the last term in Eq. (19) is given by $\nu h(\frac{2}{3}\tilde{s}^*)$ which cancels out the ε term. Thus, we get

$$\frac{d\tilde{s}}{dt} = -(\tilde{s}-\frac{3}{4}\Delta\mu)-3\kappa h(\frac{2}{3}\tilde{s}), \tag{20}$$

and we see that the effect of asymmetric coupling between the center and the extremes disappears for small ν and so will not impact the bifurcation boundary.

The equilibrium value for which the saddle-node bifurcation occurs is marked by the vanishing of the right-hand side of the above equation as well as its derivative, yielding upon rearrangement the conditions:

$$\tilde{s}^* - \frac{3}{4}\Delta\mu = -2\kappa_2\tilde{s}^* e^{-\frac{2}{9}\tilde{s}^{*2}} \tag{21}$$

$$1 = -2\kappa_2(1-\frac{4}{9}\tilde{s}^{*2})e^{-\frac{2}{9}\tilde{s}^{*2}}, \tag{22}$$

where κ_2 denotes the coupling strength at which the bifurcation occurs. Taking the ratio of (21) to (22) and rearranging yields the cubic equation

$$0 = \tilde{s}^{*3} - \frac{3}{4}\Delta\mu\tilde{s}^{*2} + \frac{27}{16}\Delta\mu. \tag{23}$$

For large $\Delta\mu$, it can be readily verified that to $O(\frac{1}{\Delta\mu})$, the solution to this equation is given by $\tilde{s}^* = \frac{3}{2}(1+\frac{1}{\Delta\mu})$. Employing (21) to solve for κ_2 and then substituting in this approximation for $\tilde{s}^*$ yields

$$\begin{aligned}\kappa_2 &= \frac{\frac{3}{4}\Delta\mu - \tilde{s}^*}{2\tilde{s}^*} e^{\frac{2}{9}\tilde{s}^{*2}} \\ &\approx \frac{1}{4}\frac{\Delta\mu^2 - 2\Delta\mu - 2}{\Delta\mu + 1} e^{\frac{1}{2}(1+\frac{1}{\Delta\mu})^2},\end{aligned} \tag{24}$$

which increases linearly to leading order in $\Delta\mu$. While the rapidly increasing κ_1 marks when the MR state will ensue from natural bias initial conditions, the linear dependence of κ_2 shows that the coupling strength for which a stable MR state is available does keep pace with $\Delta\mu$. This is significant because if a stochastic forcing is added to Eq. (3) — to simulate random incoming external information for instance — then transitions between states can occur in which the SHD state jumps to the MR state (and vice versa) as we have observed in simulations.

SLD Lower Boundary: κ_3

We now calculate the boundary κ_3 below which the SLD state given by $(r, s^* = 0)$ becomes absolutely unstable. The boundary can be calculated by using Eq. (9) and the coefficient of s in Eq. (12) to solve for r for which the system undergoes a pitchfork bifurcation from the SLD equilibrium to the MR state. We obtain the conditions:

$$r - \Delta\mu = -(\kappa_3 + \nu) r e^{-r^2/8} \tag{25}$$

$$1 = -(3\kappa_3 - \nu)(1 - \frac{r^2}{4}) e^{-r^2/8}. \tag{26}$$

Neglecting small ν in the above pair and eliminating κ_3 gives

$$0 = r^3 - \Delta\mu r^2 - \frac{8}{3} r + 4\Delta\mu. \tag{27}$$

Near the bifurcation, the equilibrium discord for the SLD state is $r \approx 2$ and the solution to (27) to $O(1/\Delta\mu)$ is $r \approx 2 + 2/(3\Delta\mu)$. Using this in (25) enables us to calculate κ_3

$$\kappa_3 + \nu = -\frac{r - \Delta\mu}{r} e^{r^2/8} \tag{28}$$

$$\kappa_3 \approx \frac{\Delta\mu^2 - 2\Delta\mu + \frac{2}{3}}{2\Delta\mu + \frac{2}{3}} e^{\frac{1}{8}\left(2 + \frac{2}{3}\frac{1}{\Delta\mu}\right)^2} - \nu. \tag{29}$$

κ_3 shows a linear dependence for large $\Delta\mu$ as did κ_2 but, significantly, it also has a linear dependence upon small ν.

Chain and Complete Stability Diagrams

Figure 3(a) displays the stability diagram of the chain network showing the regimes in $\Delta\mu$-κ parameter space where the different outcomes are stable and the boundaries between them. The open markers represent numerically-obtained bifurcation boundaries as found using the MATCONT software package for prediction-correction continuation [3]. The numerical curves agree very well with the analytical approximations (18), (24), and (29) for κ_1, κ_2, and κ_3 respectively, except in the immediate vicinity of the critical divergence. Also shown is the boundary κ_4 beyond which the MR state is no longer present. Note the presence of a substantial zone where only the MR state is stable. There are also multistable zones in which two or all three states are stable.

The stability diagram for the complete network is shown in Fig. 3(b). For κ_1 and κ_2 the approximations derived for the chain network, (18) and (24), agree very well with the numerically-determined boundaries. This indicates that the coupling between the two outer nodes can be safely neglected due to their extremely disparate opinions in the SHD and MR states. However, the chain approximation for κ_3 is substantially higher than the complete network's κ_3. This is due to the significantly lower discord of the SLD state in the complete network, thereby making that state more robust. This reduces the size of the MR-only zone as compared with the chain. In addition, κ_4 shifts to the right in the complete network which has the effect of expanding the SLD-only zone.

For the asymmetric coupling case of $\nu < 0$, i.e., when the end nodes are more influential than the center node, κ_3 shifts upward as evident from (29) whereas κ_1 and κ_2 are nearly unchanged for large $\Delta\mu$. This decreases the size of the zone where the SLD state is stable and increases the size of the MR-only and MR-SHD zones as observed in simulations; in addition, the critical divergence shifts to lower values of $\Delta\mu$. For $\nu > 0$, κ_3 shifts downward and the critical divergence shifts to the right so that the MR-only and MR-SHD zones decrease in size. However, it is significant

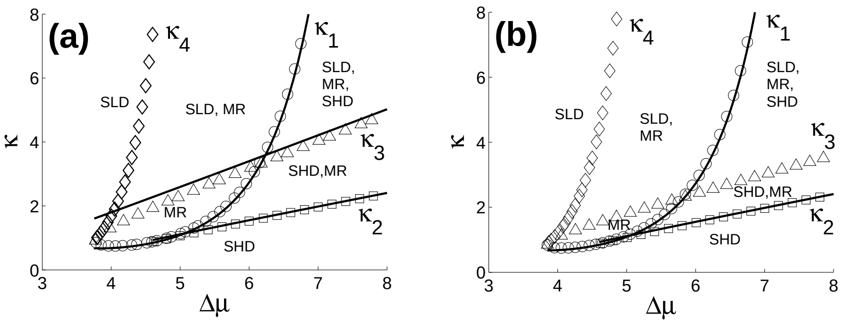

Fig. 3 Stability diagram of triad with symmetric coupling for: (a) chain network and (b) complete network. Open markers are numerically obtained boundaries. Solid lines are chain analytical approximations (18), (24), and (29) for κ_1, κ_2, and κ_3 respectively.

that skewed majority rule outcomes can arise even when the center node has greater influence than the end nodes.

5 Five-Node Networks

We have also observed majority rule outcomes in five node topologies as shown in Figure 4. In the simulations, the natural biases are distributed uniformly over the range $\Delta\mu = 6$ and ordered so that $(\mu_1, \ldots, \mu_5) = (-3, -1.5, 0, 1.5, 3)$. Three different topologies are used: (1) the chain in which each node is connected only to its nearest neighbor along the opinion axis, $A_{ij} = \delta_{i,j\pm 1}$; (2) the complete network where all nodes are connected to each other; and (3) the star in which the off-center nodes are only connected to the center node having $\mu_3 = 0$ so that $A_{i3} = A_{3i} = 1$ for $i \in \{1,2,4,5\}$ else $A_{ij} = 0$. The coupling strengths are identical for all ties, $\kappa_{ij} = \kappa A_{ij}$. But comparing the topologies for the same κ would allow topologies with more ties to have greater total coupling, thereby affording them a greater communication rate, for instance. Consequently, to compare topologies on a common basis, we relate the coupling strengths to the *coupling scale* α via the relationship $\kappa_{ij} = \alpha A_{ij}/\bar{d}$ where $\bar{d}$ is the mean degree, $\bar{d} = \sum_{i,j} A_{ij}/N$. From this form we see that α is equal to the average coupling strength, $\alpha = \sum_{i,j} \kappa_{ij}/N$. It is observed that in the MR state, the majority is 3-2 in the chain and complete networks whereas it is 4-1 in the star in which the intermediate negative node, x_2, is drawn upward into the positive x majority. We also note that the discontinuous transitions along the α axis occur first for the chain then the complete network and last for the star. The earlier transition to the SLD state for the chain network is striking since they both have the same number of directed edges, 12, and can be attributed to the fact that the couplings between the center and the outermost nodes present in the star are weaker compared with the only nearest-neighbor couplings in the chain; however, once achieved, the SLD state is much tighter in the star.

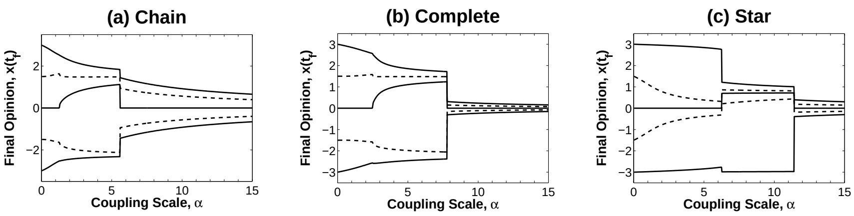

Fig. 4 Final node opinion vs. coupling scale for 5-node networks: (a) chain; (b) complete; (c) star. Simulation duration, $t_f = 200$.

6 Conclusion

We have shown that an asymmetric outcome of majority rule arises from a symmetry-breaking pitchfork bifurcation using a model that is a nonlinear variant of the influential Friedkin-Johnsen model of opinion network dynamics. This symmetry-breaking route to majority rule only occurs for initial disagreements above the critical divergence. For lower disagreement, the more intuitive process of convergence toward the center applies as would be expected from the Friedkin-Johnsen model. This qualitative difference at low and high disagreement suggests that bifurcation-induced majority rule may be observable in laboratory experiments involving group discussion. Finally, we note that although there is a regime in which majority rule is predicted, the actual policy outcome in this regime is fundamentally unpredictable and may depend on relatively minor or random variables such as who speaks first.

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Appendix 8

An Experimental Study of Persuasion, Confidence, and Choice Shift in Small Networks

Zane Kelly
Applied Physics Laboratory
University of Washington

Justin Reedy
Department of Communication and Center for Risk, Crisis, and Resilience
University of Oklahoma

Michael Gabbay
Applied Physics Laboratory
University of Washington

John Gastil
Department of Communication Arts & Sciences
Pennsylvania State University

We present the results of an experiment designed to test the effects of network structure and disagreement level on the risky shift or group polarization effect. We investigate these effects guided by the theoretical predictions of a nonlinear model of opinion dynamics in small social networks. . The nonlinear model predicts that structure, disagreement, and the strength of individual beliefs produce substantively different outcomes than existing linear models. In specific regard to the risky shift, the model predicts that low disagreement groups will show a greater risky shift than high disagreement groups and that clique networks will show a greater shift than broker networks. The experiment involved online discussion by triad groups. The substantive discussion involves participants' own beliefs and the strength with which they hold to them when faced with competing arguments. We first gathered an estimate of public opinion for a fictional foreign policy scenario. Study participants were asked to 1) estimate the percentage of Americans they believed expressed support in the scenario, and 2) how much they were willing to wager that their estimate was correct. Groups were assigned into low vs. high disagreement over the wager amount and clique vs. broker communication network structure. Discussions took place using online software wherein group members are allowed to discuss the scenario and then finalize their own wager. To the extent that groups showed a risky shift effect, the model predictions were verified in that low disagreement, clique groups showed the most risky shift. However, groups also showed an ability to shift their wagers in the correct direction, an issue of group deliberations effectiveness distinct from the risky shift effect. High disagreement, broker networks showed the best performance.

In this paper we present initial results of an experiment concerning the interaction of network structure and disagreement levels in small group opinion dynamics. Specifically, we test hypotheses derived from a nonlinear model of opinion dynamics in networks (Gabbay 2007a, 2007b). The experiment involves online political discussions in three person groups during which the members debated a hypothetical foreign policy scenario and their personal estimates of the broader public's response to it. Two network topologies were used as conditions: a clique in which all members can communicate freely, and a broker network in which two of the members can communicate only through a central node. Model simulations show that group outcomes, in terms of the amount of risk a group accepts and the direction of that decision, are a function of both structure and the amount of disagreement between the extreme positions. Cliques are expected to be more susceptible to risky decision making while broker networks are expected to be better at mitigating risk. Higher levels of disagreement expose the members to more arguments, reduce the chance of premature consensus, and help groups make more accurate decisions.

Are motivations are threefold. First, we seek to address the group behavior that is not accounted for by a linear model of group dynamics. Second, we are interested in assessing structural effects on the "risky shift" effect, known more generally as group polarization, in which group discussion leads to more risky or extreme choices than would occur if individual opinions were aggregated in the absence of discussion (Myers and Lamm 1976; Sunstein 2000). , Third, we aim to expand the scope of experimental social network research by combining individuals, embedded in network structures, with deliberative decision making processes. This research program and our experimental framework have implications of a range of topic including groupthink, policy decisions, group stability and coherence, communication processes, and routes to extreme decisions by governments or terrorist groups. The paper proceeds as follows. The first section describes the nonlinear model of group decision-making. Then we present the model dynamics for triad networks from which we draw our hypotheses. The third section describes the design of the experiment and the fourth section presents results.

Model

The nonlinear model is similar to the "social influence network theory" of Friedkin and Johnsen, the most prominent network-based model of small group opinion dynamics (Friedkin

and Johnsen 2011). Huckfeldt et al. (2004) used the Friedkin-Johnsen model as a component of their formal argument addressing the persistence of opinion diversity in political communication networks. The Friedkin-Johnsen model is linear in that it assumes that the force moving members of a dyad toward agreement grows linearly in proportion to the difference between their opinions. In contrast, the nonlinear model assumes that this dyadic force wanes past a

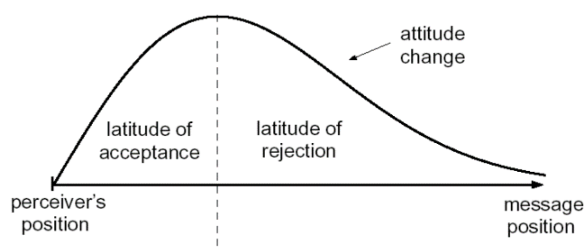

Figure 1. Coupling function in nonlinear model showing the strength of the influence force of one dyad member upon another as a function of the difference between their opinions.

certain critical disagreement level, known as the latitude of acceptance, eventually tending toward zero. Accordingly, this model has two regimes of behavior: a "linear" one, which essentially corresponds to the intuitive dynamics of the Friedkin-Johnsen model, and a "nonlinear" regime in which behaviors can run counter to initial intuition. The linear regime is characterized by: gradual changes in policy outcomes and the level of equilibrium group discord as parameters such as the coupling scale are varied; only one equilibrium for a given set of parameter values; lower group discord for higher network tie densities; and symmetric conditions of opinions and couplings always lead to symmetric final states. The nonlinear regime can exhibit the opposite behaviors: discontinuous transitions between deadlock and consensus as parameters are varied; multiple equilibria for a given set of parameter values; greater discord reduction in less dense networks; and asymmetric outcomes of majority rule even for symmetric conditions.

The nonlinear model is formulated as follows: Denoting the opinion of the i^{th} group member by x_i, the mathematical equation which evolves x_i over time t is:

$$\frac{dx_i}{dt} = f_i(x_i, t) + \sum_j H_{ij}(xj - xi + \eta)$$

Where the self-bias force, $f_i(x_i,t)$, is:

$$-\gamma_i(x_i-\mu_i)$$

and the second term, the group influence force, is:

$$\sum_{j\neq i}\kappa_{ij}(x_j-x_i+\eta)\exp\left\{-\frac{1}{2}\frac{(x_j-x_i+\eta)^2}{\lambda^2}\right\}$$

γ_i is the member's commitment; μ_i is the natural bias which also corresponds to *i*'s initial opinion; κ_{ij}, the coupling strength parameter, which scales the influence of member *j* on member *i*; and λ_i is *i*'s latitude of acceptance. The first term on the right-hand side corresponds to the "self-bias force" that results when a member's opinion is displace from her natural ideological bias. The second term is the "group influence force" which is a function of the pairwise differences in opinion between *i* and the other group members to which she is connected. The use of the functional form parameterized by the latitude of acceptance is motivated by social judgment theory (Eagly and Chaiken 1993). The entire system consists of *N* coupled nonlinear differential equations corresponding to the *N* members of the group.

Summary of model terms:

- κ – coupling strength – the influence of member *j* on member *i*, scales the magnitude of the *coupling force* ($H_{ij}(x_j - x_i - \eta)$) based on the discrepancy between their positions
- γ – the strength of member *i*'s *commitment* to their initial position
- μ – *natural bias* (e.g. *i*'s initial position on a given issue); where $\Delta\mu$ = the level of disagreement
- η – shading – norm-induced position shift
- λ – *i*'s latitude of acceptance, the range of acceptable outcomes beyond which the impact of an argument or idea wanes

Dynamics for Triads

Figure 2 shows simulation results for a triad broker network, which is symmetric in both network structure and initial opinion distribution. The network is symmetric in the sense that the matrix of coupling strengths, κ_{ij}, is symmetric (i.e., no node has a disproportionate amount of

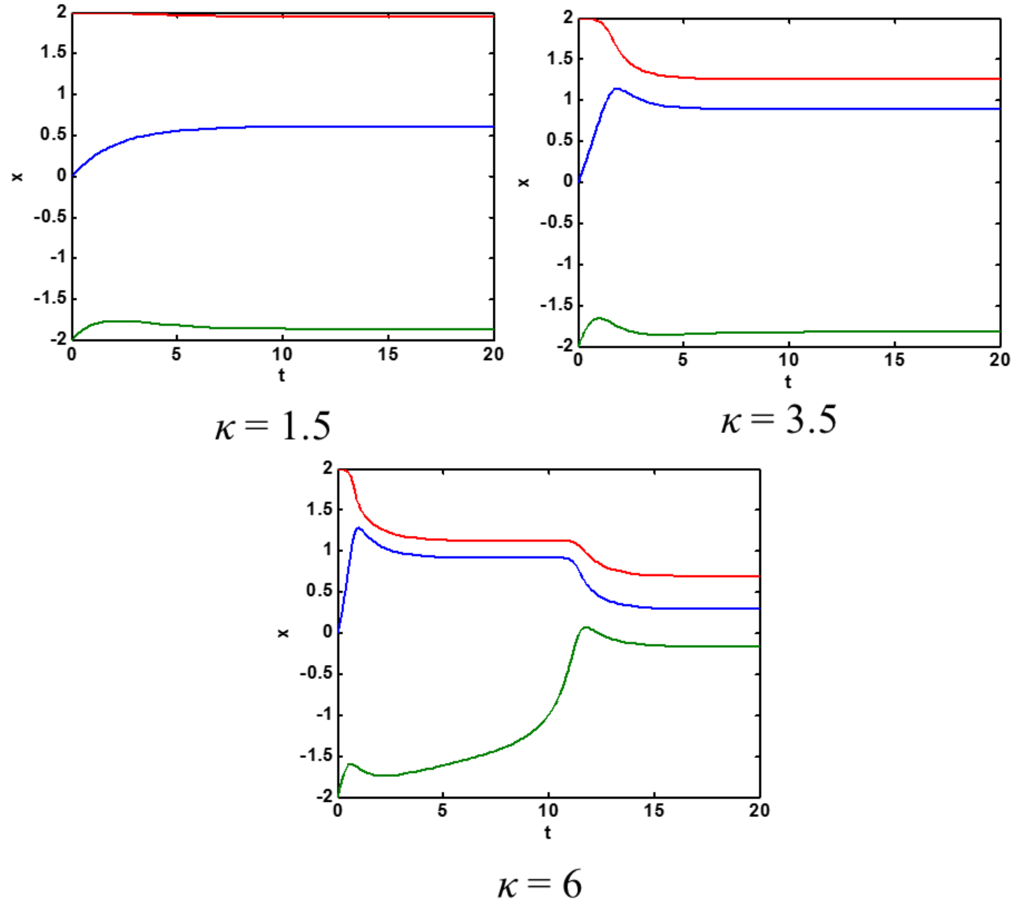

Figure 2. Member opinions vs. time showing equilibrium outcomes in symmetrically-coupled triad broker network with high initial disagreement ($\Delta\mu = 4$) at different coupling strengths. From upper left: $\kappa = 1.125$, Symmetric High Discord (SHD); $\kappa = 2.625$, Majority Rule; $\kappa = 4.5$, Symmetric Low Discord (SLD). Initial conditions: $x_1(0) = -2$; $x_2(0) = 0$; $x_3(0) = 2$. Shading, η, = 0.5. These outcomes correspond to Deadlock, Majority Rule, and Consensus. The shading parameter determines the extent of movement away from the mean position.

influence on any other) and the initial opinion distribution is symmetric around the natural biases of the extreme nodes, μ_1 and μ_3, which are equally distant from the central node's natural bias,

μ_2. The level of disagreement between the two extremes is $\Delta\mu = 5$ which is high relative to the latitude of acceptance, $\lambda = 1$, for all three nodes. Figure 2(a) is for a low value of the coupling strength and we observe that the node opinions do not change much from their initial positions, corresponding to an outcome of Symmetric High Discord (SHD). At intermediate coupling, Fig. 2(b) shows the Majority Rule (MR) state in which the center node moves toward one of the end nodes to form a majority rule pair. At high coupling, the outer nodes move considerably toward the center to form a Symmetric Low Discord (SLD) state as shown in Fig. 2(c). The SHD state corresponds to a deadlock situation in which all group members are far apart and no acceptable mutual decision can be made. In the MR state, the majority pair can likely agree on a common policy that will be the policy of the group if majority rule is sufficient for reaching a decision. In the SLD state, the distance between the outer nodes is much reduced and the basis for a compromise around the centrist's position could be set.

The simulation results motivate the following hypotheses:

H1. Low disagreement groups should exhibit greater risky shift than high disagreement.

H2. Clique networks should exhibit a greater risky shift than broker networks.

These hypotheses taken together imply that low-disagreement cliques are most susceptible to a 'risky shift' while high disagreement broker networks are least susceptible.

Experiment Design

To test the hypotheses listed above, we conducted a series of experiments involving 369 participants, each taking part in a three-member online discussion in a text-based chat system (123 groups). Participants were recruited through Amazon Mechanical Turk, an online workplace where users perform small tasks online in exchange for pay. A large pool of Turk workers was invited to take a pre-discussion survey with questions on political issues, as well as demographic questions. We then selected a portion of those individuals (based on their pre-discussion survey answers) to continue to the second phase of the study, in which they visited a website to login to an online chat system created by the study team and a computer programmer.

Participants then engaged in a three-person group discussion for 20-30 minutes, with the discussion focusing on a hypothetical foreign policy scenario. Participants were asked to both estimate the public's response to a scenario (a 50/50 probability) and to state a wager they would be willing to make on the accuracy of their estimate. Participants were given eight options, ranging from wagering none of their potential bonus to all of it. Participants were arranged into three-person groups according to 1) their estimates on the initial survey, and 2) the divergence in their wagers. The range of wagers represents the disagreement in the model above. Possible outcomes included consensus, majority, and disagreement. Participants were not required to reach a group decision in order to receive the result of their wager. See Appendix A for the text of the fictional scenario.

To determine the varying levels of pre-discussion disagreement on group decision making, participants were grouped based on their responses to the wager question. Those who responded \$0 on the initial survey were considered the most risk averse while those who stated a preference for wagering their entire bonus were considered the most risk acceptant, these users were coded as -2 and 2 on our scale. Participants who responded \$3-\$4 were coded as the midpoint of the scale, e.g. 0, while \$1-\$2 and \$5-\$6 were consider to be -1 and 1 respectively. Initial wager statements served as our best estimate for participant risk tolerance and the source of disagreement. Using this coding scheme participants were enrolled into high- and low-disagreement groups; each group required a '0' participant and the overall number of neutral respondents served as a limiting factor on the total number of groups possible. These groupings and groups sizes are shown in Table 1.

Table 1. Experimental Groups

		DISAGREEMENT LEVEL	
		Low	High
NETWORK STRUCTURE	Clique	29	31

Broker	30	33

The group assignment had two other salient features. First, the online discussion software allows manipulation of the size and topology of the network as an experimental condition. Clique groups are characterized by free communication among the members while broker groups limits the ‘end points’ to communicating through a central node. Approximately half of the participants were in broker groups and half in cliques (Figure 3). Second, users were divided into like groups based on their responses to a survey question stating that we, the researchers, had conducted a representative poll of Americans using our foreign policy scenario wherein we described a missile strike against a foreign country in response to cyber-terrorism. In this instance, rather than asking for an exact proportion, we asked the study participants to estimate whether more, or fewer, than 50% of those surveyed would support such a strike. As expected, responses fell fairly evenly, with slightly more than half of our participants estimating the proportion to be higher than 50%.

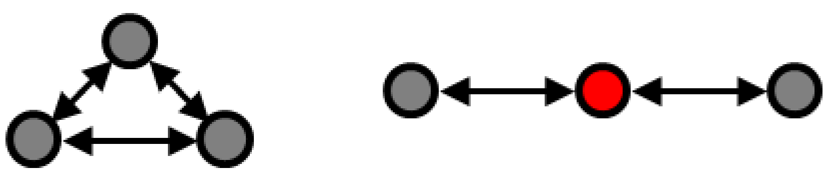

Figure 3. Clique (l) and broker (r) networks used in experiment. Communication in the broker network must move through the central node.

Participants received 20 minutes to discuss the topic before they were allowed to finalize their wager choice. Their choices mirrored the pre-survey, any amount from nothing to the full $7 bonus payment. Participants were told that if their estimates were correct they would double their money while an incorrect answer would result in a commensurate loss (e.g. a wager of $6 would result in $13 if correct, $1 if wrong). Because each group contained people who had given the same response of greater or less than 50% we anticipated that users at the more extreme ends

would be more likely to stick to their position and to try to persuade others that the risk of losing their bonus was (un)acceptable.

After participants finalized their decision they were directed to a post-survey page. The post-survey asked them to record their pre- and post-discussion opinions (for calibration) and assess the group discussion. They were asked whether their wager changed, the best arguments they heard, estimates of how they influenced others, as well as how seriously the group took the discussion, which was later used to filter out groups that did not earnestly discuss the issue.

Our scenario and individual estimates contain information that the group can assess through the process of discussion. Group members should be able to ascertain, by comparing their answers with the others, whether they were likely to be wrong or right in their estimate of public opinion. If the group members understand that they themselves are likely representative of “the public” then they should conclude that their best option is to maximize their gains by, essentially, betting on themselves. If, however, the group is susceptible to a risky shift then we should observe that their wagering behavior is higher, whether right or wrong. Below we describe the experimental results.

Results

The experiments resulted in useable data from 123 group discussions with a total of 369 unique participants. The distribution of groups was split roughly equally across the experiment’s four conditions: high-disagreement broker groups, high-disagreement clique groups, low-disagreement broker groups, and low-disagreement clique groups. Before presenting the results that bear upon our hypotheses, we discuss some other metrics that help characterize the group discussions.

We encountered an initial unexpected result. Participants’ personal support/opposition for the scenario appears to strongly influence their initial estimates of public opinion (right or wrong). Figure 4 shows the difference in personal support; those who were themselves opposed appear more ambiguous about public support while those in favor seem more likely to believe that others hold the same position (χ^2 = 25.75, p < .001). Initially those who expressed strongest support stated 2:1 that they would stake the maximum amount on their answer. However, the discussion mitigated those opinions: afterward on 14% chose the maximum wager. Figure 4 shows the distribution of vote results with respect to the experimental conditions. A log-linear

model of the vote results including the experimental conditions, as well as the groups' right/wrong answers (3*2*2*2 = 24 possibilities), indicated that the initial right/wrong answers to the estimate of public opinion should have a significant impact and that the answer term interacted with the broker condition.

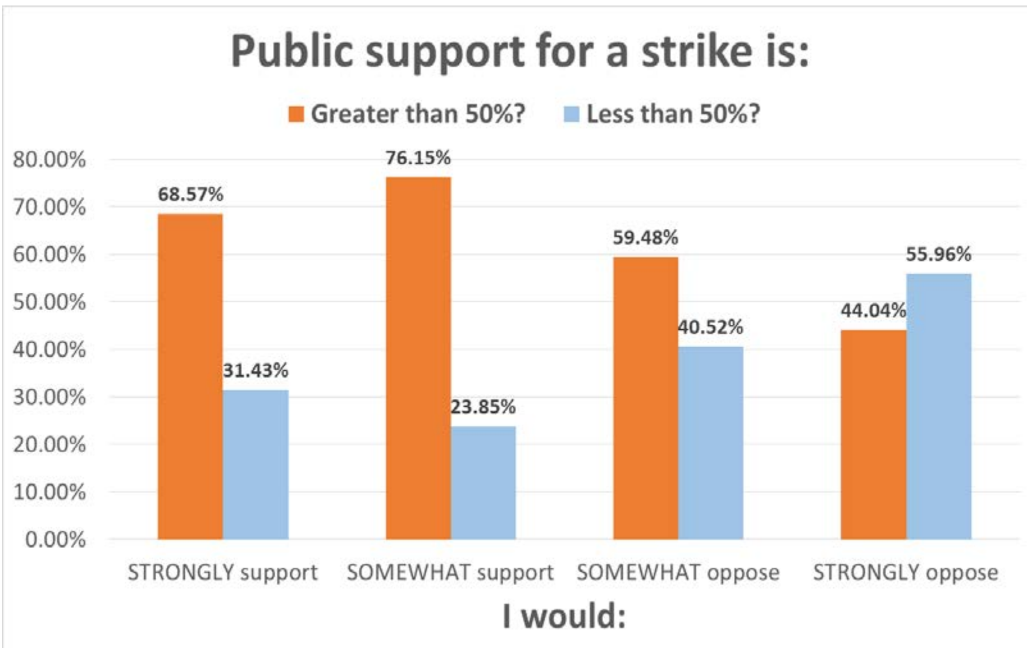

Figure 4. Participants' responses to the fictional scenario (4-point scale) and their estimates of public support (50/50).

To test our two hypotheses we compare the wagering behavior of broker and clique groups. An initial test indicates that low disagreement broker groups are more conservative. However, this overlooks the processes by which groups arrive at their judgments of an appropriate wager given both the scenario and their uncertainty. To address the process by which groups arrive at judgments and wagers we incorporate information-discovery by incorporating whether groups were initially right/wrong (recall that these answers were used to construct the groups, all three members made the same estimate up front). Figure 5 shows the improved performance of broker groups at moving in the correct direction depending on their initial answers to the public opinion question. Broker groups show a significant ability to discover, through the discussion, the accuracy of the group's estimate and move their wagers higher when correct and lower when wrong. However, recoding this variable to indicate binary movement

suggests that disagreement plays a strong role in the probability of correct movement. Assessing the individual-level data confirms that high disagreement increases the probability of an individual shifting his/her vote in the 'correct' direction. The average votes for broker groups (right v. wrong) are significantly different at $p = .002$ and the clique groups' means are different from at $p = .07$. This result supports our hypothesis, conditional on the group's right/wrong answers, that broker networks should adopt less risky positions. Moreover, broker groups are better at optimizing, betting high when correct and betting low when incorrect.

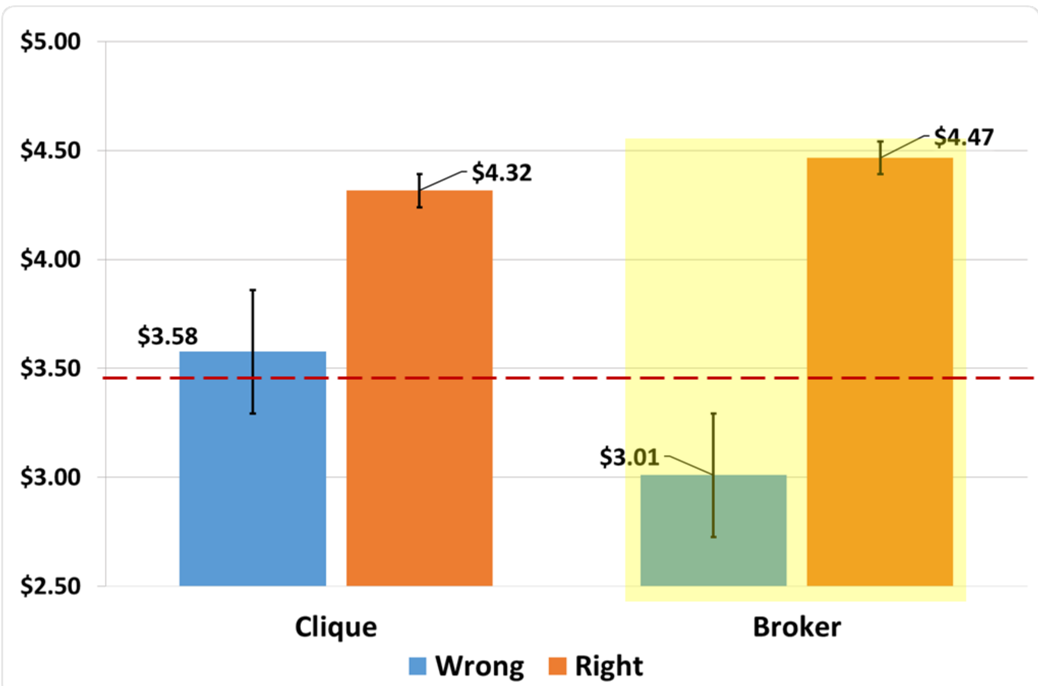

Figure 5. Broker networks are better at optimizing their performance conditional on whether group's answer was right or wrong. Black lines indicate standard errors. Initial pooled mean: $3.41 (dashed line); post-discussion mean $3.74.

Figure 6 shows the wagering results by disagreement levels across wrong/right answers. When faced with high disagreement, groups appear better able to discover whether they are correct about their estimates of public opinion and respond accordingly. High disagreement groups are significantly different ($p < .001$) but low disagreement groups have difficulty arriving at best outcomes, ultimately voting slightly higher than their initial averages. The low disagreement wrong/right groups are not significantly different from one another. Combining the

conditions and estimates yields Figure 7 below. Broker-High groups outperform the other conditions while Clique-Low groups tend toward

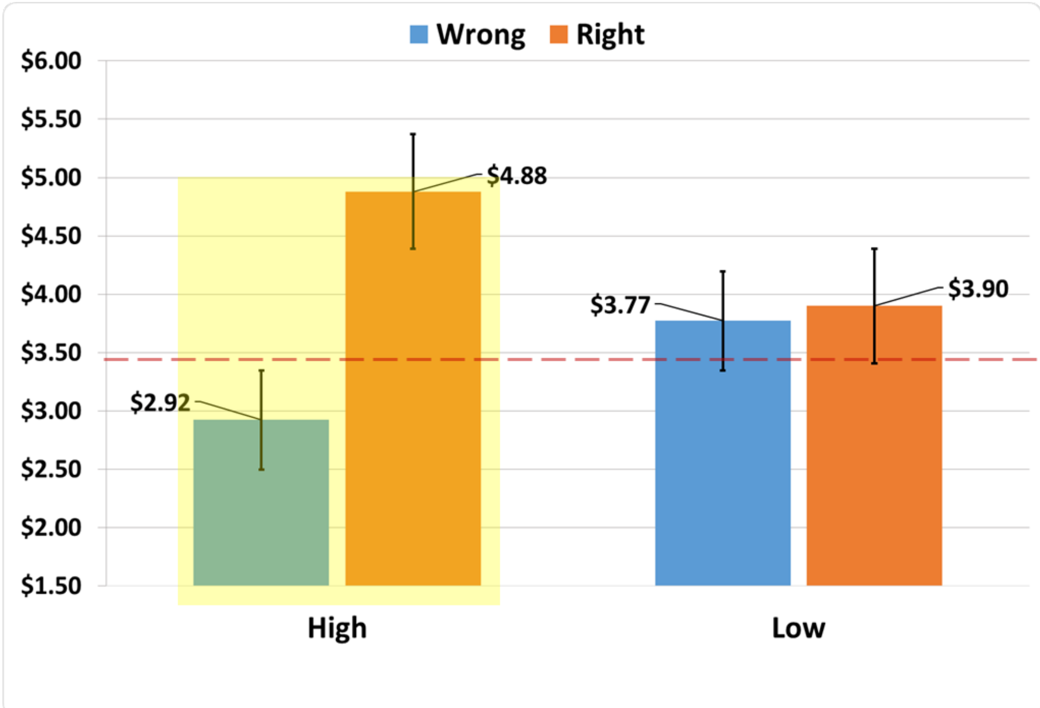

Figure 6. High disagreement groups are better able to arrive at a superior wager by discovering as a group whether their estimates were wrong or right.

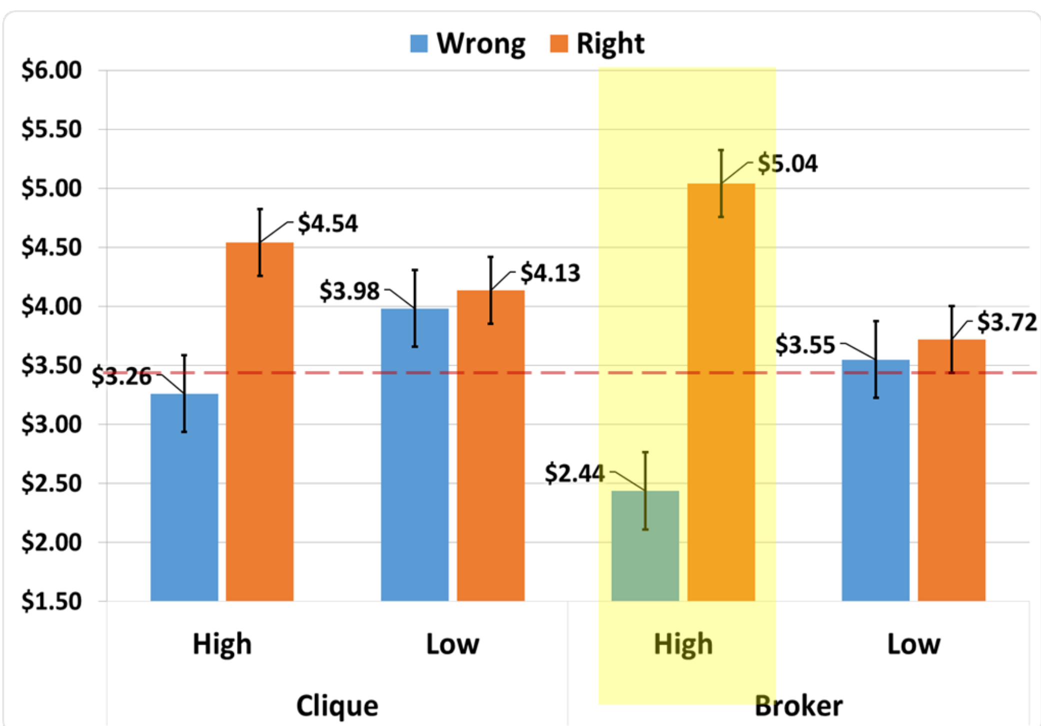

Figure 7. Combined network structure and disagreement levels.

Conclusion

The most interesting outcome of our experiment is the tendency of broker and high disagreement groups to move their votes in the ‘correct’ direction. We speculate that high disagreement exposes a greater range of opinions and ideas and discourages premature consensus. Brokers themselves are able to weigh statements from the other participants and shift their support in the direction that offers a more compelling argument. Rather than acting as leaders and uniting the opposing poles, drawing them toward the center, the broker appears to reach a decision about which of the other members offers better reasoning for his/her preferred vote. In almost all cases of majority outcomes, the central member is the one that moves in the direction of one of the extremes.

Pooling the results indicates that the average tendency is toward higher wagers compared to initial positions ($p = .05$). However, considering the results in view of the wrong/right *a priori* answers reveals that broker networks and high disagreement facilitate reaching better solutions. This result also relates to small group literature on task conflict. A recent meta-analysis of 116 empirical studies found that process conflict is associated with worse group outcomes but task conflict has a more ambiguous effect (de Wit et al. 2012). Our findings imply that task conflict, rather than creating dissatisfaction and stress and distracting the group, can facilitate discovery and produce better outcomes. A key element of our experimental design is that, unlike many other small group studies, the group faces no endogenous pressure to arrive at a consensus. A large body of literature on economic experiments is divided over whether individuals outperform groups. Our experiment implies that setting is an important element of this research. By embedding individuals in a network structure we can better understand how decision-making bodies reach decisions under different conditions of disagreement and uncertainty. This research also suggests that different types of groups could respond differently to outside disruption – strategies aimed at degrading or disrupting open communications may actually lead networks to re-form in ways that enhance their decision making process. Future experimental and observational research should consider such attempts to coercively restructure networks as well as the role of leadership within small groups insofar as the broker position can substitute for the leader position.

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Appendix A: Scenario used in experiment

(P1) Scenario: American companies and government agencies are attacked thousands of times a year in cyberspace. In many cases, these attacks are sponsored, or even directed by, foreign governments. Cyber-attacks and data breaches cost billions of dollars to American companies and taxpayers. Official policy has been to develop defenses but not to launch counter- or preemptive attacks.

Intelligence officials have newly discovered evidence of a plan to launch a major attack against industrial control systems in the Midwestern U.S. A group of cyber terrorists has already gained access to multiple computers. The attack will attempt to disrupt and destroy a large oil refinery; at best the refinery will be offline for several weeks for costly repairs, at worst there will be a major industrial accident and many fatalities.

The terrorists planning the attack are operating from a hostile country in the Middle East and are state-sponsored. Our proposed response is a preemptive missile strike targeting the building where we believe the cyber-terror cell operates. This will eliminate the terrorists and prevent the attack but an unknown number of civilians are in the same facility and the surrounding area. This response will send a message that we will not allow continued cyber-attacks on the U.S. but it may invite retaliation. As a key member of the President's national security team you must make a recommendation as to this course of action.

(P2) Polling Question: "We took a poll of Americans through an online survey company to find out how people responded to this hypothetical scenario. After reading the scenario, what percentage of Americans do you think expressed support for launching the missile strike? Less than or more than 50%?"

Appendix 9

Social Network Analysis in the Study of Terrorism and Insurgency: From Organization to Politics

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ANALYTICAL ESSAY

Social Network Analysis in the Study of Terrorism and Insurgency: From Organization to Politics

Steven T. Zech
The University of Denver
The University of Washington

and

Michael Gabbay
The University of Washington

Research using social network analysis to study terrorism and insurgency has increased dramatically following the 9/11 attacks against the United States. This research emphasizes the importance of relational analysis and provides a variety of concepts, theories, and analytical tools to better understand questions related to militant group behavior and outcomes of terrorism and insurgent violence. This paper defines key network concepts, identifies important network metrics, and reviews theoretical and empirical research on network analysis and militant groups. We find that the main focus of existing research is on organizational analysis and its implications for militant group operational processes and performance. Few studies investigate how differences in network structure lead to divergent outcomes with respect to political processes such as militant group infighting, their strategic use of violence, or how politically salient variables affect the evolution of militant cooperative networks. Consequently, we propose a research agenda aimed at using network analysis to investigate the political interactions of militant groups within a single conflict and provide illustrations on how to pursue this agenda. We believe that such research will be of particular value in advancing the understanding of fragmented civil wars and insurgencies consisting of multiple, independent militant groups.

Keywords: social network analysis, terrorism, insurgency

Scholars have increasingly applied concepts and methods from social network analysis (SNA) to problems in political science and International Relations (IR) (Hafner-Burton, Kahler, and Montgomery 2009; Maoz 2010; Ward, Stovel, and Sacks 2011). A component of this methodological movement involves the application of SNA to terrorist and insurgent networks. This scholarly activity, along with the continued salience of terrorism and insurgency to contemporary conflicts, warrants a comprehensive review of how SNA can answer questions posed in research on these conflicts. In this essay, we survey how network analytic ideas and tools have been employed in the study of violent "militant groups"—a term we will use to refer to both

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terrorists and insurgents.[1] In our review, we find the existing body of research has been oriented predominantly toward understanding the internal structure of individual militant organizations. We propose that greater attention be devoted to the network of interactions between multiple militant groups within a single conflict.

Given its organizational emphasis, research on militant networks has been primarily concerned with questions involving group operations. The theoretical literature has engaged with how network structure affects operational effectiveness: are decentralized networks more capable and adaptive than centralized ones? How does network structure reflect the trade-off between the need for coordination between militants and the risk that such communications may be detected by state security services? Do militant networks have a "scale-free" structure in which highly connected hubs both improve information flow and enhance robustness to random targeting by counterterrorism operations? Empirical studies that use SNA data on militant groups have, for the most part, engaged with a different set of questions. For example, these studies predominantly identify leaders, key individuals, and roles; explain clustering patterns based on similar roles, backgrounds, or ideologies; and examine how the characteristics of individual nodes relate to outcomes such as lethality, recruitment, or the diffusion of technology.

The covert nature of militant groups makes them a difficult subject to study in general, a problem that becomes particularly acute when the objective is to map out the very internal structure that militants go to great lengths to conceal. This is perhaps the principal reason for the disjuncture between the theoretical and empirical militant network literatures. While their operational aspects are necessarily covert, militant groups have another side which, equally necessarily, is public—their political face. Militant groups are engaged in a political competition not just with the state but often also against their fellow militants, a competition that forces them to declare themselves as a group and reveal, to a considerable extent, their aims, allies, enemies, and targets of violence. This visibility at the group level facilitates network analysis because identifying the "nodes" in a network precedes mapping their ties. Assessing the number of individuals within a terrorist or insurgent group is notoriously difficult, whereas estimating the number of groups within a broader militant movement is a less severe problem.

A focus on groups rather than individual militants will be more amenable to empirical analysis for both historical and contemporary cases. In this essay, we note several recent empirical studies that do so and argue that this research should go beyond the current emphasis on operational questions to consider political outcomes such as alliances, inter-militant clashes, more extreme violence, and negotiations. The thrust of this proposed line of research—studying interactions between groups with an eye toward political behavior—dovetails nicely with the growing literature on fragmented insurgencies and civil wars in which multiple militant groups contend with the state and often each other.

We illustrate how important questions within the militant fragmentation literature can be approached using an SNA framework, opening up new avenues of empirical investigation. For example, a greater degree of institutionalized coordination is claimed to be associated with a lower likelihood of militant infighting (Bakke, Cunningham, and Seymour 2012). Within SNA, a commonly observed network structure consists of a "core" of well-connected nodes surrounded by a "periphery" of more isolated ones (Borgatti and Everett 1999). A core-periphery structure can indicate that powerful groups are cooperating with each other rather than primarily

[1] We use the term "militant group" throughout the paper except in the review section where we often use the terminology the author(s) of the piece employ. Terrorism is a tactic. Insurgency refers to the scale or type of conflict. Insurgents are usually capable of conducting sustained guerrilla warfare campaigns against regular or irregular armed rivals. Neither concept is mutually exclusive. An insurgent group might use terrorist violence and a designated terrorist group may or may not be involved in an insurgency.

seeking ties with weaker groups. One might then expect that fragmented militant movements exhibiting core-periphery structure will be less susceptible to infighting than movements in which powerful groups anchor separate constellations of weaker allies. Core-periphery militant movements might also be more resistant to "outbidding" dynamics in which competition for popular support causes groups to engage in ever more extreme forms of violence (Bloom 2004). Additional applications include the role of anarchy, ideology, and credible commitments in the formation of militant alliance networks and the use of tactical cooperation networks to gain insight into the composition of militant groups.

This paper begins with an overview of network concepts and tools in the militant context. We then review the theoretical and empirical literatures and provide an appraisal of their strengths and weaknesses, suggesting methodological improvements such as more precise characterization of ties and greater effort to investigate networks over time. Next, we present our argument for opening up a more politically oriented research agenda and offer some candidate approaches to specific research questions which comport with that agenda. A brief conclusion follows.

Network Theory and Methods

SNA examines patterns of relations, or social structure, among actors within a defined analytic boundary. Simply put, a network is a system of interconnected actors and, hence, network analysis is at heart structural analysis. This section summarizes basic aspects of SNA in the militant context: the nodes and ties that constitute a network; fundamental network metrics, methods, and concepts; and the challenges of missing data.

Nodes

Nodes in network models can represent individuals, groups, or other collections of actors such as states. While studies in international relations have commonly used states as the actors in network analysis, studies of militant violence usually focus on individual actors or groups. Krebs (2002) maps out the individual hijackers and the broader neighborhood of actors from the 9/11 attacks on the Twin Towers and the Pentagon. Koschade (2006) and Magouirk, Atran, and Sageman (2008) map individuals and subgroups from the Jemaah Islamiyah. Rodríguez (2005), Jordan, Mañas, and Horsburgh (2008), and Zech (2010) map out individual actors from the 2004 Madrid train bombings. Pedahzur and Perliger (2006) graph individuals from four different Palestinian suicide-bombing networks. Helfstein and Wright (2011a) use individual-level data to construct terror attack networks before drawing comparisons across cases.

Most individual-level node studies set the analytical boundary within a single group and so can be considered as intra-organizational analysis. These militant organizations have defined structure and processes for collective decision-making, their members occupy functionally different roles, some members exhibit greater influence, and members have collective goals they pursue as a unit (Crenshaw 1985). However, boundary specification, or deciding which actors to include in the network, is itself an analytical question—one for which there is no set practice and that should be guided both by the research objectives and empirical limitations. Setting the boundary to include individuals beyond a single organization may be justified by the analytical questions of concern. For example, a terrorist incident such as the 2004 Madrid train bombings would not have been possible without the Spanish nationals who supplied the explosives but were unaware of any plans to carry out a domestic attack on Spanish soil (Zech 2010).

The use of group-level nodes in militant network analysis is underdeveloped compared with scholarship on inter-organizational networks from the organizational studies and social movement theory literatures (Diani and McAdam 2003;

Brass et al. 2004). A network of independent militant groups lacks the defined structure and processes that characterize a single militant organization. Interorganizational networks can be viewed through the lens of a systems approach that emphasizes how patterns of relations enable or constrain action. Several papers that examine groups as network nodes set a transnational analytical boundary combining multiple conflicts (Asal and Rethemeyer 2008; Asal, Ackerman, and Rethemeyer 2012; Horowitz and Potter 2014). Gabbay (2008) sets the boundary within a movement engaged in the same conflict: Sunni insurgent groups in Iraq. The greater focus on militant factional politics, which we argue for later in this paper, centers on using groups as nodes.

Ties

Ties most often capture exchange and dependency relationships. Network research on militant groups can examine specific social processes or more general ties between actors in a system. Ties can represent personal relationships of family, friendship, or acquaintance (Pedahzur and Perliger 2006; Magouirk and Atran 2008). Some studies focus on communication between actors in a network and ties represent an exchange of information through telecommunications, the Internet, letters, or face-to-face and indirect contact (Krebs 2002; Koschade 2006). Other studies lump ties into an all-encompassing category that can include kinship, friendship, personal contact, interaction, shared experiences, or other forms of relations (Rodríguez 2005; Magouirk, Atran, and Sageman 2008; Zech 2010). Ties can also represent alliances or cooperation between militant organizations (Gabbay 2008; Asal, Ackerman, and Rethemeyer 2012; Horowitz and Potter 2014).

Graph theory, the mathematical discipline that studies the topology of networks, defines networks as sets of node pairs and ties between them. In SNA, however, an "adjacency matrix" or "sociomatrix" is usually constructed in which the actors are arrayed along both the rows and the columns and numerical tie values are the matrix entries (Wasserman and Faust 1994). In directed networks, separate ties are used to capture incoming or outgoing social processes, whereas undirected networks represent symmetric mutual influence. Ties between actors can be binary, that is, present or not, or valued to represent gradations in tie strength. The strength of a tie or frequency of interaction between actors might affect future social dynamics (Granovetter 1973). Differentiating between strong kinship ties and general ties may help explain bridges within and across organizations (Magouirk, Atran, and Sageman 2008). Ties represent a wide range of social processes that can serve as a fertile testing ground for competing theories on social structures and behavior.

Network Metrics, Methods, and Concepts

After identifying the nodes and ties, SNA contains a variety of quantitative tools and qualitative concepts to capture the importance and roles of individual nodes, the processes occurring within the network, and the structure of the network as a whole. Table 1 lists selected metrics, processes, and methods useful in studying militant groups.

Studies most commonly reference concepts that can use centrality, clustering, or density metrics. The degree of a node refers to the number of ties it has to other nodes in the system and the degree centrality metric can be used to capture the relative power, influence, or prominence of that actor. Betweenness centrality, however, captures a different type of influence that stems from an actor's position as a bridge between distinct subsets of nodes or communities in the network. A network density metric is a ratio of the total number of observed ties given the

Table 1. Selected SNA metrics, processes, and methods

	Meaning	Concepts/behaviors
Node metrics		
Degree centrality	How connected a node is in terms of its number of ties	Leadership, prominence, influence, power
Betweenness centrality	How important a node is as a bridge for connecting other nodes	Brokerage, controlling information flow, structural holes
Closeness centrality	How close a node is on average via the shortest paths to the other nodes	Ability to send/receive information to/from other nodes
Transitivity	Tendency of nodes who are tied to a common other to also be connected	Triad closure, clustering, cohesion, role adoption
Network metrics		
Network density	Ratio of number of actual ties to maximum possible number of ties in network	Connectivity, cohesion, effectiveness, coordination, information sharing, resilience
Degree distribution	Probability of finding a node with a given degree	Scale-free networks, hubs, information flow efficiency, resilience
Degree assortativity	Extent to which nodes with similar degrees are preferentially tied	Core-periphery structure, alliance or rivalry of powerful actors
Mean path length	Average number of ties traversed in shortest paths between all node pairs	Information or resource flow, small-world effects ("six degrees of separation")
Processes		
Homophily	Tendency of nodes with common attributes to develop ties	"Birds of a feather," opportunity or choice-driven ties
Preferential attachment	Greater probability of new nodes to form ties with high-degree nodes than with low-degree ones	Scale-free networks, hubs, "rich get richer"
Methods		
Community detection	Finding distinct communities using patterns of ties within and across network subsets	Factions, cells, compartmentalization
Visualization	Depicting information about network structure and individual nodes	Topology, communities, homophily, core-periphery, leaders, brokers
Exponential random graph models (ERGMs)	Simulation of random tie formation conditioned on specified network properties	Endogenous structural effects, closure, reciprocity, homophily

total number of potential ties in the system. Network density provides indication as to the connectivity of a network, the ease with which actors might communicate or coordinate with other actors in the system, or how resilient a network might be to disruption through node elimination or severing of ties. Homophily predicts tie formation based on shared nodal attributes such as religion, region of origin, sex, or age.[2]

As is common SNA practice, the tools and concepts listed in Table 1 are applied to networks of individuals and networks of groups alike. As Hafner-Burton, Kahler, and Montgomery (2009) point out, however, this practice often results in insufficient attention being paid to grounding the application in processes specific to the level of analysis. For militant networks, a particular metric or concept may have different significance or interpretation at the individual and group levels. Such differences can arise from the greater degree to which intra-organizational network structure can be deliberately engineered compared with inter-organizational networks. The structure of ties within a militant organization may reflect a fusion of the social processes between individuals at a local level and the global decision by its leaders to structure the organization in a particular way for operational purposes; militant groups often restructure themselves in response to changes in the level of repression (Shapiro 2013, 16–17). There is no such global authority in a collection of independent groups, however, so inter-group ties predominantly arise from the system of social and political processes operating between groups.[3]

We now provide several illustrations of how the same metric can have different implications at the individual and group levels of analysis. At the individual level, a high degree of centrality in a network of communication ties could signify that a node is a formally designated operational leader, whereas at the group level, it could signify that the node has access to material or media resources (Diani 2003). An individual with high betweenness may serve as a leader or courier between different operational or functional cells; a group with high betweenness may have a national presence that bridges geographically distant and localized groups or a centrist ideology bridging divergent wings of an insurgent movement (Diani 2003; Gabbay and Thirkill-Mackelprang 2010). Within a network of militant groups, the density of ties may be taken as an indicator of the movement's cohesiveness. This need not be the case for the network of individuals within an organization, where the density of ties may reflect a choice by the leadership—high density favoring information flow and redundancy and low density favoring covertness—with no implications for the willingness and likelihood of those individuals to cooperate with an absent central authority or to cohere in the face of adversity.

In general, using SNA tools to identify patterns in social relations and to then draw conclusions about how those structural properties relate to outcomes requires assumptions about actors and their behavior. The nodes that represent network actors are not black boxes compelled to behave in some way based solely on their structural position. What research on militant groups may need most is an approach that recognizes and accounts for agency given structural facilitation or constraint.[4] For example, Carley (2006) and Carley, Lee, and Krackhardt (2002) suggest how policymakers might use network tools to learn how to better destabilize covert networks. While these authors demonstrate the importance of identifying influential actors, subgroups, and the structural factors that make a network

[2]For further details on the metrics and methods in Table 1, the reader can consult a number of textbooks on social network analysis including Wasserman and Faust (1994), Carrington, Scott, and Wasserman (2005), Jackson (2008), and Newman (2010).

[3]External state sponsors often seek to force cooperation among militant groups, but the mixed results of such attempts indicate that state sponsors cannot be construed as global authorities of militant movements.

[4]See Stevenson and Greenberg (2000) for a brief review on how strategies of action may relate to social structure.

Table 2. Selected theoretical and qualitative work

Paper	Cases (conflict type)	Concepts	Selected analytical claims	Node type	Tie type
Arquilla and Ronfeldt (2001)	Numerous illustrative cases (insurgency, terrorism, social movements)	Leadership, authority, typology	The network form of organization is on the rise, networked organization provides advantages in terms of adaptability and resilience	Individual actor or group	Communication, broad relations
Eilstrup-Sangiovanni and Jones (2008)	Al-Qaeda, other examples (insurgency and terrorism)	Scale-free, centralized versus decentralized	Decentralized networks have serious deficiencies: poor information sharing, poor strategic decision-making, excessive risk-taking, and obstacles to collective action	Individual actor or group	Organizational membership
Enders and Su (2007)	Al-Qaeda, other examples (insurgency and terrorism)	Hubs, density	There is an information flow versus security trade-off, higher network density increases vulnerability of the group	Individual actor	Communication
Gutiérrez Sanín and Giustozzi (2010)	Colombia and Afghanistan (insurgency)	Leadership, homophily, resource flow, centralized versus decentralized	Insurgents can be classified on an army-network spectrum, the FARC is centralized army and the Taliban is decentralized network	Individual actor	General interaction
Jackson (2006)	Al-Qaeda (terrorism)	Leadership, authority	Groups can be better understood by classifying ties with respect to authority relationships, command, and control structure affects strategy and efficacy	Individual actor	Communication, information exchange, convey authority
Kirby (2007)	London bombing (terrorism)	Leadership, cohesion, clique	"Self-starter" cases lack a wider network of support and experience heightened small-group dynamics	Individual actor	Friendship, communication, joint activities
Matthew and Shambaugh (2005)	Al-Qaeda (terrorism)	Scale-free	Networks face collective action problems which limit ability to form consensus on goals	Individual actor or group	General interaction
Parkinson (2013)	Palestinian groups in Lebanon (insurgency)	Brokerage	Informal social ties allowed women to bridge cells and to help groups withstand and recover from repression	Individual actor	Friendship, kinship, communication

(*continued*)

Table 2. Continued

Paper	Cases (conflict type)	Concepts	Selected analytical claims	Node type	Tie type
Sageman (2004)	Global Salafi Jihad (terrorism)	Cohesion, homophily, small-world, hubs, cliques	Global Salafi Jihad is a scale-free, small-world network	Individual actor	Friendship, kinship, communication
Staniland (2014)	Kashmir, Afghanistan, Sri Lanka (insurgency)	Leadership, cohesion, network typologies	Pre-war social ties constrain insurgent forms of organization; groups with strong horizontal and vertical ties most resilient	Individual actor and local communities	General interaction
Stohl and Stohl (2007)	Numerous illustrative cases (insurgency and terrorism)	Information transmission, social capital, scale-free, homophily, small-world	Ethnic terrorist groups like Hamas or ETA are scale-free, small-worlds which are harder to penetrate, but once known easier to monitor and destroy	Individual actor or group	General interaction
Vidino (2007)	Hofstad group in the Netherlands (terrorism)	Leadership, cohesion, infomation transmission, sub-groups	Network structure informs target choice, recruitment and leadership less important for homegrown terrorism	Individual actor	Friendship, shared ideology

more adaptive, they readily admit the problems that may emerge resulting from unforeseen effects and the "culture free" nature of this approach.

Data Challenges for Covert Networks

Existing research frequently highlights the difficulties of collecting and analyzing relational data for covert networks. The two primary sources of information have incentives to distort the truth, confounding efforts to construct an accurate account of militant networks: law enforcement, government officials, intelligence officers, and the justice system may exaggerate the extent of an individual's participation while militant actors themselves may deny or downplay their own involvement. Many militant groups take precautions to minimize their visibility to security forces and the outside world. Attributes and associations are often "missing by design," forcing scholars to recognize the problematic assumption of treating missing links as random. Working with non-random, incomplete network data leads to significant underestimation and confounds other network metrics such as centrality (Gill and Freeman 2013).

Some scholars offer strategies to overcome data limitations when studying clandestine networks. For example, Clauset, Moore, and Newman (2008) provide a technique for inferring hierarchical structure and then use existing observations to predict missing data for partially known networks. Gerdes (2014) proposes new methods for data transformation when working with "dark network" data. Koskinen et al. (2013) describe additional methods for dealing with unobserved relational ties and partially observed attribute data so as to achieve better estimates with Exponential Random Graph Models (ERGMs).

Theoretical and Qualitative Work

In this section, we review theoretically oriented research on militants that directly engages with network theory, concepts, or formalism, advancing arguments with qualitative analysis or mathematical models. Due to space limitations, we have selected publications that are representative of major themes and debates within the literature. Table 2 summarizes these studies with respect to their contexts, concepts, analytical claims, and network characterization. The column that lists the cases in each study also notes whether the piece focuses on terrorism, insurgency, or some combination of the two. We proceed to describe the most salient areas of concern and debate for this literature: the capabilities of decentralized versus centralized forms of militant organizations; the relationships and trade-offs between network structure and operational security, efficiency, and resilience; and application of the scale-free network concept to these questions.

Decentralized Versus Centralized Networks

The frequent yet disparate use of the term "terror network" in news reports, books, and academic journal articles illustrates the need for conceptual clarity. In recent years, scholars and policymakers have shifted their attention from hierarchical militant organizations to looser organizational forms and labeled anything without strict top-down command structure as a "network." Given that the SNA definition of network is unconcerned with the presence or absence of hierarchy, this usage amounts to shorthand for a network in which authority is decentralized. Debates have arisen as to the relative merits of centralized and decentralized networks, their applicability to specific cases, and their impact on militant behavior.

Arquilla and Ronfeldt (2001) suggest that "networked" forms of organization are on the rise as many organizations and movements take advantage of new

information and communication technologies. These forms may include chain, star/hub, or all-channel topologies or some combination of these forms with elements of hierarchical organization. They argue that organizations find advantage in the increased adaptability of decentralized network structure. Decentralized decision-making processes allow groups to respond more quickly to changing conditions and give them greater flexibility in catering strategies to particular contexts. Eilstrup-Sangiovanni and Jones (2008) review many of the purported benefits of decentralized structure to militant organizations but note that, despite enjoying advantages in terms of flexibility and adaptability, these organizations may not process and share information as well as their advocates suggest. They also point out problems with the speed and coherence of decentralized decision-making and observe that, without central direction, organizations face coordination difficulties and may be prone to excessive risk-taking. The importance of trust in illicit networks may hinder scalability advantages found in their licit counterparts. Matthew and Shambaugh (2005) build on insights found in research on collective action and conclude that terrorists must evolve into more cohesive and hierarchical organizations to effectively achieve their goals, but they also argue that decentralized networks are more difficult to defeat. In-depth studies of the strengths and vulnerabilities associated with centralized and decentralized militant groups, informed by an organizational studies perspective, are found in books by Sinno (2008) and Shapiro (2013).

The focal case for this debate has been Al-Qaeda and transnational terrorism motivated by a global jihadist strain of Islamist ideology. Identifying Al-Qaeda as a hierarchical organization, loose network, or broader social movement affects assumptions and expectations about behavior that guide subsequent analysis (Jackson 2006). Two prominent terrorism experts, Marc Sageman and Bruce Hoffman, engaged in heated dispute about the nature of the threat posed by jihadist terrorism grounded in their respective characterizations of it as primarily a loose-knit ideological movement or as a more centrally led Al-Qaeda organization (Hoffman 2008; Sageman and Hoffman 2008). Neumann, Evans, and Pantucci (2011) recognize the amorphous nature of the Al-Qaeda movement and, using case studies, suggest that a set of middle managers play a critical role integrating grass-roots movements with top leadership. With respect to the broader global jihadist movement, early work by Sageman (2004) provides a description of the ideas, attributes, and relations of actors involved in the global jihad. Sageman (2008) also suggests that these Islamist terror networks have evolved into more fluid and independent entities, creating a “leaderless jihad.” Kirby’s (2007) article on the 2005 London bombings describes a decentralized, close-knit network of “self-starters” inspired by this broader movement with no direct ties to Al-Qaeda leadership. Instead, Mohammad Sidique Khan exerted influence over his three young collaborators in perpetrating suicide bombings that killed fifty-two civilians and injured over 700 more. Vidino (2007) describes another group within the movement and suggests that the Hofstad group’s amorphous structure and lack of ties to international Islamist groups will influence recruitment practices and target selection in future attacks in the Netherlands and elsewhere in Europe.

Carrying the debate to the insurgency context, Gutiérrez Sanín and Giustozzi (2010) claim that the degree of decentralization can explain differences in behaviors of the Afghan Taliban and the Revolutionary Armed Forces of Colombia (FARC) in their comparative study. The Taliban is characterized as a decentralized “network,” the FARC as a hierarchical “army.” The authors claim that these contrasting forms impose different demands for organizational survival and that there are trade-offs between them, though neither is objectively superior to the other: army-like insurgent organizations must emphasize internal cohesion while networked insurgents must actively broaden their social base, maximizing their integration with civilian populations.

Operational Security, Efficiency, and Resilience

The United States Army and Marine Corps (2007, 320) counterinsurgency field manual states that "while high network density groups are the most dangerous, they are also the easiest to defeat and disrupt." This is an assertion about the trade-off between a militant group's vulnerability to detection and its ability to effectively carry out attacks, one supported by early research on other types of illicit networks that prioritized secrecy over efficiency (Baker and Faulkner 1993). Many of the studies in our review address this trade-off between operational security and efficiency.

Eilstrup-Sangiovanni and Jones (2008) suggest that decentralized networks that compartmentalize nodes for security purposes make information transmission and complex, strategic decision-making more difficult. Enders and Su (2007) develop a formal model to address the security versus efficiency trade-off faced by groups that use terrorist violence. The authors use the model to show that minimizing network density provides greater security for a terrorist network by decreasing the efficacy of network disruption through infiltration. They suggest that there is a critical density below which it becomes more efficient for terrorists to work in disconnected subgroups. Furthermore, counterterrorism efforts may lead terrorists to switch to less complex attacks requiring lower density that ultimately have higher probabilities of success.

The relationship of the efficiency versus security trade-off to the resilience of a terrorist network was studied by Lindelauf, Borm, and Hamers (2011). As is commonly done, they treat resilience as a structural property of a network in terms of its ability to maintain information flow as nodes are removed. The authors claim that terrorist networks are able to operate near the optimal balance of efficiency and security even in the face of very high node attrition. Parkinson (2013) presents an interesting aspect of operational security and resilience—the use of women by Palestinian militants in Lebanon to circumvent and recover from repression. Informal social ties helped women serve as bridges between the subunits of a group and between different groups. She describes the role played by these informal social networks in helping the groups recover from severe losses in the wake of Israel's invasion of Lebanon in 1982, illuminating resilience as a process and not just as a structural property. Staniland (2014) argues that insurgent leaders do not have a free hand in the structuring of their groups, rather they are constrained by pre-war social ties. He studies how the resilience of a rebel group depends on two sets of ties: the horizontal ties connecting the group's leadership and the vertical ties between leaders and local communities. "Integrated" groups with both strong horizontal and vertical ties are most robust to counterinsurgent strategies of leadership decapitation and disembedding local communities from the group.

Scale-Free Networks

"Scale-free" networks provide one example of a system that may face an operational security and efficiency trade-off. Scale-free networks contain a small number of prominent nodes (hubs) that possess a greatly disproportionate number of ties to other nodes in the system, while the vast majority of nodes have relatively few ties. More precisely, the probability distribution for node degree falls off as a power law so that the degree distribution has a fat tail. Barabási and Albert (1999) proposed a model in which scale-free networks arise from a simple preferential attachment effect in which the ties formed by a new node with existing nodes are made with a probability proportional to the degrees of the existing nodes. So, in effect, "the rich get richer." Power-law degree distributions are empirically observed in a wide range of biological, social, informational, and technological

networks such as the Internet (Newman 2010, chapter 8). It should be borne in mind, however, that an important characteristic of observed scale-free social networks—for example, movie actor co-appearances, calls of a telephone company's customers, sexual partners—is that they represent ties in a one-off nature and not as repeated bidirectional exchanges intrinsic to richer forms of social interaction, such as traditional friendships which do not follow power-law degree distributions (Amaral et al. 2000). A key property of scale-free networks is that they are robust to attacks against random nodes, since the vast majority of nodes have only a few connections and their removal does not appreciably affect the global connectivity of the network. Conversely, however, they are very vulnerable to the targeted removal of key hubs that transmit information or resources through a network (Albert, Jeong, and Barabási 2000).

The scale-free network concept has been applied to militant groups primarily with respect to the importance of hubs and robustness to counterterrorism attacks and infiltration efforts. Matthew and Shambaugh (2005) refer to Al-Qaeda as a hub in a scale-free terrorist network but highlight the fragility of its position, noting that hubs primarily serve instrumental purposes and do not engender loyalty. Motivated by the horizontal, not hierarchical, processes by which scale-free networks evolve, Pedahzur and Perliger (2006) draw a distinction between individuals who become informal hubs as opposed to formally designated leaders in Palestinian suicide bombing networks. Stohl and Stohl (2007) describe ethnicity-based, "inward looking" terrorist organizations operating at the local level as reflecting a scale-free, "small world" network with short degrees of separation, strong ties, and powerful hubs. In contrast, they suggest that ideologically oriented movements do not exhibit these tendencies and remain more vulnerable to infiltration and random attacks. They cite the historical record to support their claim, stating that the scale-free, ethnic-based organizations have shown greater resilience in the face of counterterrorism or counterinsurgency efforts than ideologically oriented ones. Based on their case studies of three militant groups, Bakker, Raab, and Milward (2012) claim support for the scale-free network structural property. Networks that are more centralized (the Tamil separatist group LTTE), in the sense of having a few well-connected nodes, are less robust to attacks against central nodes than groups the authors characterize as having a more decentralized structure (the FARC in Colombia).

Empirical Work

We now turn our attention to research that quantitatively applies SNA techniques to empirical data on militant groups either for the purpose of description and characterization or to test propositions about social structure. Table 3 summarizes selected empirical pieces on networks and militant violence. These represent a wide breadth of SNA tools such as centrality, network density, and ERGMs. The research also illustrates diverse sources for data and evaluates individual- and group-level relational data as dependent and independent variables. A salient feature of the empirical literature as seen in Table 3 is the much greater attention given to cases of terrorism compared with insurgency. In this section, we highlight empirical research findings involving centrality, attribute-based clustering, network density, and the use of groups as nodes.

Centrality

Research on militant networks, specifically terrorist networks, often emphasizes the importance of prominent actors on outcomes. Central nodes may have more influence on other actors in the network. Some studies provide measures for an actor's status using centrality indices. Pedahzur and Perliger (2006) stress the importance

Table 3. Selected empirical work

Paper	Cases (conflict type)	Metrics/ methods	Concepts	Selected analytical claims	Node type	Tie type	Data sources
Asal, Ackerman, and Rethemeyer (2012)	Global set of terrorist organizations (terrorism)	Centrality	Alliance	Connectivity measures positively related to use or attempted use of CBRN weapons	Terrorist group	Alliance	Memorial Institute for the Prevention of Terrorism (MIPT)
Asal and Rethemeyer (2008a)	Global set of terrorist organizations (terrorism)	Centrality	Alliance	More alliances related to higher likelihood of using lethal violence	Terrorist group	Alliance	MIPT
Gabbay (2008b)	Iraq (insurgency)	Centrality, visualization	Homophily, influence, alliance	Cooperation related to targeting policy proximity; leadership and tactical networks show different structure	Sunni insurgent group	Number of joint operations, joint statements, and references between groups	Translated statements from Jihadist websites and interviews on U.S. Open Source Center
Gill et al. (2014)	The PIRA in Northern Ireland (insurgency and terrorism)	ERGMs	Homophily, role adoption	Homophily based on gender, age, proximity, other involvement measures like geography and role	Individual actor	Coinvolvement friendship, blood relative, marriage ties	Network and attribute data collected for the study
Helfstein and Wright (2011a)	11-M Madrid train bombings, Southeast Asia incidents (terrorism)	Density, ERGMs, cross-case comparison	Effectiveness scale-free, trust	Greater density and cohesion as attack approaches, scale-free network structure not as prevalent as thought	Individual actor	Communication, personal interaction, kinship	Global Transnational Terrorism (GTT) database (also called JJATT)
Horowitz and Potter (2014)	Global set of terrorist organizations (insurgency and terrorism)	Eigenvector centrality	Alliance, core, periphery	Terrorist alliances have core-periphery structure with preferential attachment to stronger groups	Terrorist group	Alliance	Terrorism Knowledge Base (TKB)

(*continued*)

Table 3. Continued

Paper	Cases (conflict type)	Metrics/ methods	Concepts	Selected analytical claims	Node type	Tie type	Data sources
Jordan, Mañas, and Horsburgh (2008)	11-M Madrid train bombings (terrorism)	Centrality, degree distribution, clustering	Leadership, cohesion, sub-groups, homophily	Advantage in autonomy, weakness in need for contact with social environment, centrality metrics do not always indicate importance, network clusters by role	Individual actor	Communication, personal interaction, family	Court indictment, testimony, newspapers
Koschade (2006)	Jemaah Islamiyah (terrorism)	Centrality, clustering, density	Leadership, role adoption, cohesion, bridges	Low density more stable but less efficient, centrality captures importance but not always power	Individual actor	Communication interactions, frequency and duration	Case study through secondary sources
Krebs (2002)	9/11 attacks (terrorism)	Centrality, density, mean-path	Info. and resource transmission, leadership, cohesion, effectiveness, size	Efficiency for secrecy trade-off, use caution in drawing conclusions from centrality metrics, covert networks distinct from normal social networks	Individual actor	General contact, meeting	Newspapers, Internet, Moussaoui indictment
Magouirk and Atran (2008)	Jemaah Islamiyah (terrorism)	Clustering	Cohesion, homophily	"Bottom-up" network development and not just "top-down" indoctrination important in extremist violence	Individual actor	Acquaintance, friendship, family, common madrassah or training camp	GTT database
Magouirk, Atran, and Sageman (2008)	Jemaah Islamiyah (terrorism)	Clustering, degree distribution, visualization	Leadership, hubs	Shifting group strategy based on structure and leadership (moderates sidelined in org.)	Individual actor	Communication, personal interaction, kinship, reliability	Media accounts, other open-source primary documents
Metternich et al. (2013)	Thailand, 2001–2010 (insurgency or "anti-governm ent actors")	Lowest eigenvalue, latent space analysis	Alliance, cohesion	Network structure improves conflict level prediction; less cohesive movements produce more conflict	Antiregime group	Cooperative acts	Automated processing of event data from news stories

(*continued*)

Table 3. Continued

Paper	Cases (conflict type)	Metrics/ methods	Concepts	Selected analytical claims	Node type	Tie type	Data sources
Pedahzur and Perliger (2006)	Palestinian suicide networks (terrorism)	Centrality, density	Hubs, scale-free, size	Cohesive subgroups increases efficacy, more hubs equals more attacks	Individual actor	Kinship, friendship, acquaintance, reliability	Press, org. Internet sites and published documents, studies and articles
Rodríguez (2005)	11-M Madrid train bombings (terrorism)	Degree distribution, centrality	Cohesion, hubs, sub-groups, core-periphery	Resiliency of sparse networks with an efficiency trade-off, removal of key nodes would weaken or destroy network	Individual actor	Kinship, friendship, co-habitation, meeting attendance, contact, reliability	Press accounts from two Spanish newspapers
Zech (2010)	11-M Madrid train bombings (terrorism)	Centrality, clustering, visualization, ERGMs	Leadership, bridges, sub-groups, homophily, dynamic network	Leadership bridges sub-groups, actors cluster by role, structure influences outcome	Individual actor	Communication, personal interaction, kinship	GTT database, court indictment, testimony, newspapers

of central figures in their study of Palestinian suicide networks. The authors include degree, closeness, and betweenness centrality scores for hubs, general members, and suicide bombers for each of their four networks. They conclude that networks with a greater number of hubs will carry out more attacks and that peripheral actors usually carry out suicide attacks to limit network vulnerability in case of capture. Jordan, Mañas, and Horsburgh (2008) and Zech (2010) use centrality metrics to identify central actors in the Madrid train bombing network. Horne and Horgan (2012) use three distinct centrality measures to identify network elites in their study of radical Muslims in the United Kingdom. Koschade (2006) identifies two key hubs in the Bali bombing network using centrality measures.

Krebs (2002) also calculates centrality metrics in his work on the 9/11 network. He finds the scores to generally reflect "common knowledge" about individuals such as Mohamed Atta concerning leadership and influence, but urges caution in drawing conclusions based on these measures. Studies on terrorist networks are especially susceptible to missing nodes and ties and centrality measures are sensitive to minor changes in nodes and links (Krebs 2002, 47). Krebs also finds the network to have an unusually long mean path length for such a small network. Krebs suggests that this metric reflects a concern with secrecy, but a network must create shortcuts and indirect connections to aid in information flow necessary for the minimum efficiency levels to carry out a complex operation. Borgatti (2006) uses the same network to illustrate the potential inadequacy of centrality measures to identify "key players" in subsequent counterterrorism operations that aim to influence or disrupt militant networks and suggests alternative measures for finding optimal sets of key players.

Clustering by Attributes

Another important concept to analyze in militant networks is clustering by shared attributes. Militant actors may tend to form ties on the basis of similar roles, backgrounds, and preferences, among other factors. Scholars can use network graphs to display relational data and observe whether or not actors cluster based on theorized common attributes. Koschade (2006), Zech (2010), and Gill et al. (2014) demonstrate clustering based on role adoption in their network graphs. Jordan, Mañas, and Horsburgh (2008) suggest some degree of homophily, or the tendency for nodes to associate with like-nodes, based on country of origin in their graph of the Madrid bombing network and Spanish Al-Qaeda. Gabbay and Thirkill-Mackelprang (2011) find Sunni insurgent groups to cluster based on nationalist and jihadist ideologies. Harris-Hogan (2012) identifies cliques based on shared experiences within the Melbourne cell of the Australian neo-jihadist network. In addition to visualization techniques, scholars can use K-core analyses, which search for the largest subset where every node is connected to at least K members of the subset, to locate subgroups in networks (see Pedahzur and Perliger 2006). ERGMs can also be used to evaluate clustering based on shared attributes (see Zech 2010; Gill et al. 2014).

Density

Density scores for a complete graph may also provide useful information when comparing two or more networks. Density is the proportion of observed edges in a network to the total number of potential edges. Krebs (2002) provides a density score of 19% for the 9/11 network of trusted prior networks and meeting ties. Koschade (2006) provides descriptive structural characteristics that include a density score of 43% for the Jemaah Islamiyah 2002 Bali bombing cell. These metrics may provide some indication as to how covert particular terrorist networks are,

but without a baseline for comparison, any conclusions concerning the meaning of a single density score is purely speculation.

Helfstein and Wright (2011b) evaluate relational data for three categories of Al-Qaeda attack networks to test propositions about density derived from network theory. The authors differentiate these attack networks into core, affiliate, and periphery cells based on their strength of ties to Al-Qaeda's core leadership. Helfstein and Wright (2011b) use these data to test the hypothesis that core and affiliate cells should demonstrate greater capacity than periphery ones, and thus will have greater network density scores. Contrary to expectation, the authors find that core cells had lower density scores than affiliate and periphery groups. They speculate that attack cells across these different levels may form differently. Small cells on the periphery may tend to be groups of pre-existing friends and acquaintances drawn to the ideology of the broader movement. These cells may be less experienced than the core in the design of secure operational structures.

Militant Groups as Nodes

While most network analysis research on militant groups focuses on individual actors as nodes, a few studies have used group-level nodes. Asal and Rethemeyer (2008) analyze group-level data in their study of organizational lethality. They evaluate the number of casualties attributed to terrorist organizations from 1998 through 2005 and find that an organization's size, ideology, territorial control, and connectedness affect lethality levels. The authors evaluate organizational connectedness using a count of relationships to other organizations intended to capture an SNA degree metric. In related work, Asal, Ackerman, and Rethemeyer (2012) use quantitative data at the organizational level to evaluate why certain groups decide to pursue chemical, biological, radiological, or nuclear (CBRN) weapons. They measure an organization's "embeddedness" with other well-connected actors using eigenvector centrality, a metric that accounts for the extent to which a node is connected with high-degree nodes. They observe that transnational terrorist organizations with more alliances and greater embeddedness are more likely to pursue CBRN materials.

In their empirical analysis of an international set of militant organizations, Horowitz and Potter (2014) use an alliance count as well as an eigenvector centrality metric and find that groups with a broader network of intergroup relationships increase their lethal capacity. The eigenvector metric captures an "alliance depth" concept that indicates relationships to core groups involved in international terrorism. These relationships may provide greater knowledge and capabilities that increase their lethal capacity. The authors suggest a core-periphery structure in the broader network of alliances, with groups prefering to link to stronger groups.

A recent piece by Metternich et al. (2013) also examines group-level data and provides an example of the empirical application of network-based formal modeling. Their pioneering and methodologically sophisticated analysis integrates game theory, statistical network analysis, and automated coding of event data. The authors investigate networks of opposition parties and militant groups in Thailand and find that the inclusion of network structure—specifically, the lowest eigenvalue of the adjacency matrix—improves prediction of conflict intensity beyond contextual factors such as GDP, the nature of the government, and proximity to elections. They find that more negative values of the lowest eigenvalue are associated with greater levels of conflict directed against the government, which leads them to the conclusion that less cohesive opposition movements are more effective in fighting against the government—a claim at odds with the expectation that cohesive movements typically will be more effective since, among other reasons, they can curb

counterproductive behaviors such as infighting and outbidding. Such behaviors are not accounted for in the public goods model described by Metternich et al. (2013), which provides the basis for their use of the adjacency matrix lowest eigenvalue as a metric of network structure, one that has not yet been employed extensively in SNA.

Appraisal of Theoretical and Empirical Research

In this section, we remark upon the strengths and shortcomings of SNA research on militants. Addressing the weaknesses we identify—such as the imprecise usage of network concepts, the lack of hypothesis testing, poorly-defined ties, and the neglect of temporal evolution and political processes—will strengthen and expand the application of SNA techniques to militants.

Perhaps the most striking feature of SNA-based work on militants is the lack of overlap between the theoretical and empirical research. Empirical studies have mostly not engaged the central concerns of the theoretical literature—the relative advantages of centralized versus decentralized networks, the relationship between operational capability and network density, and the relevance of scale-free degree distributions—with a dedicated hypothesis testing program. A notable exception, however, is the work of Helfstein and Wright (2011a) that endeavors to test propositions about minimal network density and scale-free structure and finds support for neither. This gap can be chiefly attributed to the covert nature of militant networks that greatly hampers empirical assessment of their internal structure, although the often loose usage of network concepts in the theoretical literature is a contributing factor. Ultimately, the value of these theoretical debates is limited if a path to their empirical resolution is not provided.

Among the major components of the theoretical literature, the efficiency versus security trade-off is most concrete, centering on well-defined SNA metrics such as network density. However, the centralization–decentralization and scale-free networks components have been less precise. While decentralization is consistently used in the sense of distributed authority, the domains of that authority—for example, target selection, personnel selection, operating procedures—are not sufficiently elaborated; each can have different implications for behavior and network structure. Nor does the literature adequately distinguish between lines of authority and lines of communication. The structure of the former need not be the same as the latter: a reclusive terrorist leader may communicate with his deputy via a chain of several couriers who, however, would not appear interposed between leader and deputy in the group's hierarchy. The ambiguity of the decentralization concept in the theoretical literature inhibits its consistent empirical application. For instance, the FARC has been contrastingly depicted as a decentralized "network" (Bakker, Raab, and Milward 2012) and as a hierarchical "army" (Gutiérrez Sanín and Giustozzi 2010).

The concept of scale-free networks has been used primarily on a metaphorical level and not distinguished from other degree distributions in which high-degree hubs are present. A scale-free network assumes a "complex network" that continually expands with preferential attachment to hubs. Many militant groups may not expand or even seek to do so and are often too small to be considered as candidates for scale-free structure. The looser equation of scale-free structure to networks with hubs is also problematic even if one assumes that the theoretical properties of the former transfer to the latter. In covert networks, the most important actors may not actually act as hubs linking up a significant number of actors. For example, a recruiter in a terrorist organization may attract preferential attachment without exerting influence over network behavior. In addition, claims that scale-free militant networks are resilient due to their robustness against random attacks on nodes have not adequately reckoned with the countervailing effect that high-degree nodes are

more visible. Countermilitant forces do not simply delete nodes as do simulations of scale-free networks (Albert, Jeong, and Barabási 2000)—they can capture and interrogate them—and such a process would rapidly point to the very hubs that provide the network's information flow advantages.

Most of the quantitative empirical research is descriptive in nature, not aimed at directly testing propositions using network metrics. This descriptive work provides valuable observations that can inform theory development and more rigorous proposition testing. A number of studies have found that centrality metrics can be used to identify influential individuals within militant groups, although they do not always align with power or formal status. Metrics that identify important people can help scholars understand militant group decision-making processes and social influence, especially when combined with contextual knowledge about group leadership. Observing that terrorist networks have relatively long mean paths provides evidence that network data can shed light upon the efficiency versus security trade-off. Network visualizations that show nodes clustering by common attributes such as functional roles, shared experiences, or ideology suggest the types of homophily that are important in driving network structure.

In a sign of progress, recent empirical work has placed greater emphasis on proposition testing. ERGMs have proven useful in this endeavor. Using individual-level nodes, Gill et al. (2014) investigate the factors that shaped network structure in the PIRA during the "Troubles" in Northern Ireland; Helfstein and Wright (2011a) investigate how network structure affects attack severity. Structure has also been used as an independent variable in networks of groups to investigate outcomes such as CBRN activities and lethality as discussed above (Asal, Ackerman, and Rethemeyer 2012; Horowitz and Potter 2014).

The dearth of proposition testing is one shortcoming of the empirical body of research on militant networks. Another is the use of implicit, poorly characterized relational ties in many studies. Some analyses use relational data coded to cover a wide range of social exchange. Studies by Rodríguez (2005), Magouirk, Atran, and Sageman (2008), and Zech (2010) operationalize social ties as an all-encompassing category that can include kinship, friendship, personal contact, interaction, shared experiences, or other forms of relations. While such a measurement certainly helps in providing a preliminary visualization to begin studying a network, this approach is less suited for studying specific militant group behaviors and testing hypotheses derived from existing theories. In Tables 2 and 3, we explicitly identify the various social processes that ties represent in select qualitative and quantitative research. When scholars collect relational data for network analysis, disaggregating actor ties in terms of communication networks, exchange of material resources, or specific types of cooperation will better allow investigators to theorize about and generate predictions concerning actor behavior. Greater attention to specifying functions and tie types will also allow for clearer interpretation of centrality metrics, the significance of which depends upon assumptions about the underlying flow process, such as information transmission or attitude change, occurring over the network (Borgatti 2005).

An additional limitation of the empirical studies is that they largely examine static aggregate networks, ignoring how networks change over time. Researchers who collect relational and attribute data across multiple time periods for the same network can begin to substantiate claims concerning how network density, shifts in leadership, or subgroup task specialization lead to outcomes of militant violence or how these structural factors may influence decision-making, recruitment, or other behaviors. For example, Zech (2010) maps out the Madrid train bombing network for the ten years leading up to the attacks. He finds that important actors bridged the necessary subgroups in the network during the three years leading up to the attack. The change in network structure resulted from post-9/11 Spanish counterterrorism efforts and social ties that developed when two

actors shared a prison cell in the years leading up to the attacks. Magouirk and Atran (2008) demonstrate the importance of Jemaah Islamiyah leadership structure using network data across two time periods; one representing ties and leadership influence between 1993 and 1995 and the other in the year 2000. The social ties of central actors differed between periods and these connections influenced leadership decisions to use violence. Gill et al. (2014) examine the PIRA across four time periods. Dynamic network models can aid in exploring a group's life cycle, growth, and decay. Research that demonstrates a change in networks after the introduction of some type of intervention can begin to evaluate the effects of countermilitant policies on observed behaviors and the resulting outcomes.

Both the theoretical and empirical literatures on militant networks exhibit an overwhelming focus on operational and organizational characteristics. The debate about the structure of these networks centers around issues of efficiency, resilience, robustness, and information sharing—properties that share much in common with technological systems like the Internet and communications networks. Nodes are usually taken to be individuals rather than groups and network analytic concepts and metrics are employed to identify leaders, roles, and functional communities. These are certainly important properties and features of militant groups, particularly given their covert and often informal nature, but they are primarily issues of organizational analysis. In this perspective, the network structure of a militant group influences the dimensions of a group's effectiveness such as lethality or survivability, analogously to how the structure of employee relationships influences the success of a business in terms of its sales or innovativeness. The existing research largely consists of applying concepts from other fields such as network science, management and organizational studies, and sociology. This focus underdeploys theories and insights from political science itself. The considerable attention devoted within the terrorist network literature to scale-free networks, an idea from physics, is an example of this reliance upon other disciplines for research guidance and priorities.

Beyond the Organization: Militant Networks as Political Networks

While not denying the relevance of the organizational emphasis predominant in the first wave of militant network studies, we contend that a second track emphasizing the application of network analytic methods to intrinsically political questions needs to be opened. Terrorist and insurgent groups are rooted in political goals, yet quintessential matters of politics—for example, alliance dynamics, factional rifts and infighting, splintering, the legitimate targets of violence, rhetoric, and conflict resolution—have received scant attention from a network perspective. A growing body of research within the field of insurgency and civil war studies considers such behaviors for conflicts with multiple groups. Examples include: alliance formation (Bapat and Bond 2012; Christia 2012); inter-militant clashes (Fjelde and Nilsson 2012); the effects of repression on cooperation (McLauchlin and Pearlman 2012); competitive "outbidding" toward more extreme violence (Bloom 2004); the effect of fragmentation on conflict duration (Findley and Rudloff 2012); and spoilers in negotiations (Kydd and Walter 2003). By providing a quantitative framework for representing and analyzing fragmented conflicts, a network approach could bring both greater insight and precision to the study of militant interactions.

A research agenda for connecting network analysis with political dynamics among militants calls for the following elements: (i) defining nodes at the group level rather than at the level of individuals; (ii) the use of precise and well-defined tie indicators of cooperative and/or competitive relationships between groups; (iii) the integration of politically-relevant node attributes—power, ideology, targeting practices, territorial presence, state sponsors, etc.—with the network tie

data; and (iv) a focus on political questions within a single conflict such as alliance formation, militant infighting, ideological or policy repositioning, and conflict resolution. The first element stems from the fact that the group is the locus of greatest political visibility and salience—a public face not a covert one. The second element echoes our call above for more well-defined tie measures. The third element brings in key political variables, which can be directly related to network structure as causes or outcomes and also provide competing or complementary explanations of the behaviors under investigation. The fourth element restricts our concern to groups involved in the same conflict rather than setting a more expansive boundary, which includes transnational interactions between militant groups involved in different conflicts (Asal and Rethemeyer 2008; Asal, Ackerman, and Rethemeyer 2012; Horowitz and Potter 2014). Groups involved in a single conflict have the potential for much more intense competition, which can greatly alter their patterns of interaction relative to behaviors found in transnational networks.

Strides toward the above approach have been made. The paper by Metternich et al. (2013) on anti-government groups in Thailand represents one example. Another example is Gabbay's (2008) analysis of Sunni insurgent groups in Iraq using militant group rhetoric as data. Cooperative tie networks at the leadership and tactical levels are constructed using joint statements and claims of joint operations, respectively. A "targeting policy" variable based on the portfolio of target classes—for example, U.S. forces, government security forces, government civilians, Sunni and Shiite civilians—that insurgent groups claim to attack is used as a policy measure. He constructs a measure of a group's overall influence using the prominence of the group within the rhetoric of all the other groups. These measures are integrated in "factional map" diagrams, which are used to address political questions involving insurgent constituencies, goals, cohesion, and negotiations. More generally, this work shows the value of rhetoric as a source of data on militant networks.

The remainder of this section illustrates how to apply network theory and methods to questions involving militant group infighting, outbidding, alliance formation, and constituencies. Consistent with the call to focus on single conflicts, we assume each network consists of militant groups—terrorists or insurgents—participating in a violent, non-state opposition movement aimed at a regime, foreign occupation, or rival ethnic or religious group. We present approaches for using network analysis to address open questions in the literature on militant fragmentation such as how patterns of fragmentation affect the likelihood of inter-militant clashes or their use of extreme violence and the role of anarchy and ideology in shaping militant alliances.

Militant Infighting

Fragmented militant movements are prone to infighting (Fjelde and Nilsson 2012). A network approach would investigate: (i) how the susceptibility to infighting at a systemic level relates to network structure; and (ii) whether the network positions or roles of individual groups affect their likelihood of clashing with other militants. We suggest approaches to the first question using several ways of assessing network structure and our example approach to the second question considers the mismatch between a group's centrality and its size.

Bakke, Cunningham, and Seymour (2012) propose characterizing militant fragmentation by three key variables: the number of groups, the level of institutionalization, and the distribution of power. The authors combine these independent variables into hypotheses concerning the probability and pattern of infighting. Given a conflict consisting of many groups, a network framework can be constructed, which complements this theory of fragmentation enabling a more

precise and direct way of testing its implications. Network density can be used to quantify institutionalization. Core-periphery structure and assortativity can visually and quantitatively account for how institutionalization and the distribution of power intersect. Community detection algorithms can provide a potentially more telling alternative to the number of groups.

Bakke, Cunningham, and Seymour (2012) state that institutionalization improves the overall cohesion of a movement, thereby reducing the odds that groups will turn their weapons against each other. They define a movement as strongly institutionalized if it has an overarching institution, such as a popular front, central committee, or government-in-exile, which ties groups together whereas informal coalitions and alliances are considered indicators of more tenuous institutionalization. This suggests that a network of cooperative ties between groups, particularly at the leadership level such as joint statements, can be used to quantify institutionalization on a continuous scale. One approach would employ network density as a measure of institutionalization. Social network theory commonly associates the number of ties within a network with its cohesion (Friedkin 2004). Making the connection between network density and institutionalization leads to the expectation that increasing network density decreases the probability of militant infighting.

Network density, however, may be a problematic measure for cross-case comparison since there might be considerable variation in the rate at which tie indicators are generated and observed across conflicts, which need not correspond to differences in movement cohesion. A more refined approach integrates the distribution of power and institutionalization variables by using centrality measures as a proxy for group power. If one takes a group's degree centrality as a reflection of its overall power in terms of its size, material resources, and popular support, then the distribution of group degree centralities should correspond with the distribution of group power. Doing so allows us to account for how different patterns of network structure may affect the cohesion of a militant movement beyond simple tie density. We consider a multipolar situation in which there are a relatively small number of powerful groups and a much larger number of weaker ones. A movement in which the powerful groups are preferentially tied to one another will be more cohesive than one in which powerful groups are tied to weaker ones and not each other. In the former case, the groups whose infighting would be most debilitating to the movement have good cooperative relationships; in the latter case, those same groups do not cooperate, instead they head rival alliances. Network visualizations and metrics can capture these distinct patterns.

In visualizations, the better institutionalized situation above will exhibit a core-periphery network structure in which nodes with high-degree centralities form a dense core of ties surrounded by a periphery of low centrality nodes. In contrast, the more poorly institutionalized case will be characterized by a multiplicity of star-like network patterns, each with a powerful node at its center linked to weaker nodes. This correspondence leads to the expectation that networks with a core-periphery structure will be less prone to infighting than networks in which powerful actors are dispersed from each other.

Turning to a metrical representation, the tendency of a network to preferentially connect high centrality nodes with each other is quantified by the degree assortativity (Newman 2010, 230). Networks in which that tendency is present have positive degree assortativity and are said to exhibit assortative degree mixing, whereas networks in which high centrality nodes preferentially connect with low centrality ones have negative degree assortativity and are said to exhibit disassortative degree mixing.[5] Assortative networks tend toward a core-periphery

[5]Although assortativity can be used as a synonym for homophily, we use it specifically in reference to degree mixing in this paper.

appearance, whereas in disassortative networks, high-degree nodes form the nuclei of the dispersed star-like clusters noted above. Therefore, analogously to the visualization case, assortative networks should exhibit a lower probability of infighting than disassortative ones.

Network analysis also provides a potential alternative to the number of groups as an indicator of fragmentation, namely the number of factions. Algorithms for detecting community structure use the pattern of ties between nodes to assign them to distinct clusters (Girvan and Newman 2002). These algorithms could be used to identify factions of militant groups. Scholars could then test whether the number of militant factions is more strongly associated with infighting than the number of groups. Furthermore, the number of factions could be related to the stability of a militant movement similar to how the polarity of the international system affects its stability (bearing in mind the existence of the regime as a separate pole).

We now address the question of what network position may indicate about an individual group's propensity for fighting its fellow militants. The assumption made above that degree centrality faithfully reflects group power need not always be fulfilled. Two groups of the same size may have different degree centralities because one is more insular than the other, preferring to work alone or warier of infiltration. Of particular concern is the case when a large group does not cooperate with others commensurately with its size so that there is a shortfall between the group's degree centrality and its non-network power as gauged by its military capabilities, resources, and territorial control. Powerful groups that abstain from participation in the cooperative mechanisms of the movement harm its overall level of institutionalization, thereby raising the potential for infighting (Bakke, Cunningham, and Seymour 2012). Consequently, a group whose relative degree centrality is much lower than its relative size should be more likely to clash with other groups (assuming it is not geographically remote from them). This expectation is analogous to that of the status inconsistency theory of international conflict, which Maoz (2010) formulates using network theory, finding empirical evidence that states whose centralities fall short of their hard power capabilities tend to act more belligerently.

Outbidding

We consider how network structure may play a role in militant policy dynamics, specifically, the use of violence against civilians. Policy dynamics can be viewed as a network process in which a group's policy evolves as a function of the policies of its network neighbors. In particular, policy dynamics can be treated as a "social influence" process, which holds that connected nodes experience convergent forces toward greater similarity. Formal models of social influence networks have been developed for behaviors such as opinion change (Friedkin and Johnsen 2011). Here, we discuss the potential implications of social influence for outbidding.

Outbidding is an important route toward extremism arising from competition among multiple militant groups (Bloom 2004; Nemeth 2014). Outbidding can lead to more extreme types of violence, such as the indiscriminate mass targeting of civilians, because rival groups competing for popular support strive to outdo each other by conducting increasingly spectacular attacks. However, the same institutional mechanisms that dampen the potential for conflict between militants can also moderate pressures toward outbidding. Powerful groups interconnected by a web of strong cooperative ties, as in a core-periphery network, will have mechanisms to help manage the competitive pressures that could trigger spirals of extreme violence. The powerful core can provide a brake against a movement-wide slide toward extremism when isolated, weaker groups try to increase their popular

support via more extreme violence. This resistance toward outbidding in core-periphery networks arises from a social influence process. Assuming the core prefers a relatively moderate use of violence, the social influence within the core will make it more difficult for a group which shifts to an extreme policy to drag the others with it by its direct influence alone.[6] Conversely, a disassortative network structure indicates competition among powerful groups thereby increasing susceptibility to outbidding dynamics. This assumes the operation of social influence forces from other levels, in particular a group's rank-and-file or the population at large: if a given group's rank-and-file start defecting to a more extreme group, then the group will feel pressure to become more extreme itself. Such a dynamic speaks to the utility of a multilevel network conceptual framework as noted below.

Alliance Formation

Whereas the preceding examples concerned the significance of network structure to militant behavioral outcomes, the question of militant cooperation and alliance formation concerns the processes which influence that structure. In organizational studies, networks of strategic alliances between corporations have been found to exhibit a core-periphery structure indicating a preference for high-status firms to ally; they also display an endogenous mechanism whereby existing ties and structural positions are reinforced over time (Gulati and Gargiulo 1999). Whether or not networks among militant groups also exhibit these tendencies is an unresolved question. One fundamental difference between the corporate and militant contexts is the severe cooperation under anarchy problem characterizing the latter. Accordingly, one line of research would address how the anarchical environment shapes militant network structure. Anarchy is a core concern of international relations theories and here we focus on two issues surrounding its potential effects on militant network structure: the insignificance of ideology or social identity and credible commitment problems.

In neorealist theory concerning alliance politics among states, the distribution of power and the associated balancing and bandwagoning dynamics are afforded nearly all causal power, whereas ideology and social identity are of slight reckoning (Walt 1987). Christia (2012) adapts this neorealist paradigm to alliance formation during insurgencies and civil wars, arguing that power calculations drive alliance shifts as groups seek minimal winning coalitions; ideology and social identity are claimed to play no sustained causal role. In this view, one would not expect homophily in these soft variables to be a significant factor shaping militant cooperative networks, at odds with homophily's status as a fundamental process within social network theory. However, the neorealist account of alliance formation between states is disputed by those who argue for an important role for ideology and identity (Barnett 1996). For example, a network analysis of interstate alliances has empirically observed the presence of homophily involving democratic regime types and cultural attributes (Maoz 2010, 367). Similarly, network analysis could identify whether homophily is a significant process in the formation of cooperative ties and alliances among militant groups in a conflict. Beyond testing blanket, time-independent propositions about homophily's presence or absence, a more highly resolved approach would consider the conditions and phases in a conflict which affect the balance between ideologically and power-driven alliance choices. Ideological differences may at first be obscured in an

[6]An indirect influence may be possible, however, if the extreme violence changes the very nature of the conflict that could force a movement-wide shift. For instance, if an extremist group is successful in triggering an escalation of communal violence, fellow militants, who would prefer to focus their struggle against the regime, may have no choice but to fight rival ethnic or sectarian forces as well.

insurgency, which arises suddenly in response to a foreign invasion or a violent regime crackdown on protests but may reemerge as the conflict matures.

Irrespective of ideology, the level of threat from counter-militant forces should be important to the evolution of militant network structure. Initial intuition would expect that a high level of threat would encourage cooperation among powerful groups thereby fostering a core-periphery structure among militants, whereas more disassortative structures would evolve at lower threat levels. However, the credible commitment problems generated by anarchy—there is no authority to enforce militant alliance agreements—may counteract this intuition. Using game theory, Bapat and Bond (2012) claim that militant dyads are more likely to overcome commitment problems when opposed by a weakly repressive regime. When taken to the network level, this dynamic leads to the expectation that core-periphery structures would arise at lower, not higher, threat levels.

Another potential manifestation of the credible commitment problem concerns the stability of militant network structure. An argument in favor of stable network ties would contend that militant groups that overcome the commitment problem to cooperate successfully would be more likely to collaborate again. Repeat interactions among trustworthy partners are one mechanism that leads to stable networks in the corporate context (Gulati and Gargiulo 1999). In contrast, Christia (2012, 32–34) argues that the commitment problem faced by militants is so intense that it nullifies cooperation histories and trustworthiness, leading to the conclusion that militant alliance networks will be plagued by instability.

The contrasting predictions above concerning anarchy's impact on network structure can be empirically tested. Care should be taken as to how alliance is operationalized. Various definitions that encompass general forms of cooperation have been used (Bapat and Bond 2012; Christia 2012; Horowitz and Potter 2014), but it is also possible to more precisely employ formal alliance declarations among militant groups as indicators of alliance ties. For instance, groups have declared numerous mergers and fronts in the Iraqi and Syrian insurgencies (Siegel 2010; Szybala 2013). Ideology or social identity could be coded via manual or automated content analysis of militant rhetoric, for example by examining the frames they employ (Gabbay and Thirkill-Mackelprang 2011; Johnston and Alimi 2012). The level of threat faced by militants could be gauged by the size of opposition forces from the regime, paramilitaries, or foreign forces or by the intensity of violence.

Constituencies

The ability to assess group composition is important with respect to political behavior because militant field commanders and fighters often shift their affiliation between groups. Hence, militants at the local level can be regarded as constituents whom groups attempt to win over, not merely subordinates to be commanded. And, as with domestic party politics, a group's constituents constrain its freedom to maneuver in pursuing political ends and means. Divisions between constituencies within a group's rank-and-file can ultimately lead to its splintering, and so, methods for inferring group composition can aid in the investigation of the causes of fragmentation, an underdeveloped and challenging area of research (Bakke, Cunningham, and Seymour 2012). Gleaning insight into a group's constituencies among its active supporters is possible even when given only group-level node data. In particular, we discuss how inferences about the social identity composition of groups can be made from community structure and betweenness in the network of tactical cooperation among groups.

Militant group field units often have considerable freedom in cooperating with the field units of other groups without explicit guidance from their leadership; such tactical cooperation occurred during the anti-Soviet insurgency in Afghanistan (Rubin 2002, 229–30). In this horizontally driven process, homophily

among units with similar social identities would likely play a significant role in the initiation of contacts and subsequent collaboration between field units. In terms of network structure, homophily would cause the network of tactical cooperation to exhibit a community structure in which groups cluster according to the salient social identities—religious, ethnic, tribal, nationalist, etc.—among the militants. Groups of hybrid composition, however, could cross the divide between social identities. Given its use in measuring the extent to which a node acts as a bridge linking distinct node clusters, betweenness could, therefore, serve as a surrogate for the extent to which a group is of mixed social identities. Evidence for clustering by social identity comes from Iraq where the joint operations network showed clear jihadist and nationalist divisions (Gabbay 2008; Gabbay and Thirkill-Mackelprang 2010). Furthermore, the group with the highest betweenness, the Islamic Army in Iraq, bridged the jihadist and nationalist factions leading, along with other indicators, to the inference that it was a hybrid jihadist-nationalist group. Politically relevant observations are that the hybrid composition of the Islamic Army in Iraq may have helped it become the largest group and, on the flip side, this mixed constituency meant that its leaders had to appeal to constituencies whose ultimate goals were in opposition, a constraint that had serious implications for its cohesion.

While the group-level tactical network can be used to make inferences about group composition, a fuller representation would use a multilevel network framework that would include: (i) organizational ties between group leaderships; (ii) membership ties affiliating local units or individuals with group leaderships; and (iii) cooperative ties between local units or individuals. Within the broad SNA field, however, multilevel networks have proven easier to conceptualize (Brass et al. 2004; Moliterno and Mahony 2011) than to implement empirically. There is a paucity of true multilevel network data sets given their higher data demands and standard SNA tools such as ERGMs have only recently been extended to the multilevel context (Wang et al. 2013). Assembling a multilevel data set for militants is likely to be especially difficult given their covert nature at the individual level. Shy of empirical implementation, multilevel networks are still likely to have value as conceptual frameworks; for instance, in the consideration of multiple levels of social influence as in outbidding dynamics.

Conclusion

SNA complements conventional approaches to terrorism and insurgency studies (Perliger and Pedahzur 2011). SNA has made significant contributions to understanding militant operations regarding questions such as the relative merits of centralized and decentralized structures, the relationship between efficiency and security, the network signatures of key individuals, and the factors that shape network structure. However, the alignment between the theoretical and empirical fronts must be improved. Theoretical studies would benefit by more tightly invoking network concepts. Empirical analysis would profit from the more precise use of relational data and a greater emphasis on temporal factors. Overall, scholars must endeavor to move beyond description toward explanation, comparing a sample of similar networks or observations of the same network over time.

Existing research gives short shrift to political processes. It is largely focused on operations and organizational theory and primarily evaluates network structure using individual-level nodes. Consequently, we have outlined a research agenda that applies network analysis to intrinsically political questions, entailing a focus on group-level nodes engaged in the same conflict. We illustrated this agenda with subjects relevant to research on militant fragmentation: infighting, outbidding, alliance formation, and group constituencies. Violent conflict and alliance dynamics are matters with which international relations scholars are well

acquainted, but constituencies is a term more frequently associated with those who study domestic party politics. This highlights the hybrid nature of fragmented militant movements in which the anarchical aspects of the international system are mixed with elements of political party competition: like a state, a militant group may need to decide upon a balancing or bandwagoning response to a rival who could turn its guns against the group; like a political party, the group may also worry about that same rival winning over the group's constituents. This complexity poses a great challenge to theory development, one which we believe will be more readily met by the application of network analysis, through both hypothesis testing using network theory and metrics and the development of formal models, which employ a network framework.

Finally, in characterizing the study of militant networks, the distinction between analogy and model drawn by Snidal (1985) is helpful. Analogies transfer propositions from a different system to the system of interest and, hence, their logic is external and inductive in nature, whereas models have an internal and deductive formal logic. Much use of analogy has been made in militant network analysis, understandable given the opacity of its subjects. The simplest use of analogy is the generic application of SNA concepts and metrics without attention to a particular process or context. A better approach argues for the relevance of an analogy with a more developed context such as corporations, social movements, political parties, or states. However, the hybrid nature of the militant context implies that no single analogy will serve well; models with the unique logics of terrorism and insurgency will have to be built. Progress from analogies to models will be best achieved when SNA practitioners pay close attention to the specific processes identified by conventional analytical approaches to violent militancy. This synthesis will, in turn, lead to both new theoretical insights and more solid footing for empirical results.

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Appendix 10

Leadership Network Structure and Social Influence

Leadership Network Structure and Social Influence

Michael Gabbay
Applied Physics Laboratory
University of Washington, Seattle
gabbay@uw.edu

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Introduction

Pivotal policy decisions in states or organizations like militant movements are often made by a small group of top leaders (Hermann, 2001). This speaks to the importance of developing systematic methods for improving the ability to understand and anticipate the dynamics of leadership groups. This chapter describes a quantitative methodology for the analysis and modeling of leadership networks which leverages research in complex systems, in particular nonlinear dynamical systems theory (Strogatz, 1994) and network science (Newman, 2010). The nonlinear systems element is the model of small group decision making which can exhibit complex phenomena such as large, discontinuous transitions (bifurcations) as a parameter is varied and non-trivial interactions with network structure. Factional and other divisions within leadership networks can induce meaningful structure in them. Algorithms developed in complex networks research for analyzing community structure can probe this factional structure and, crucially, relate that structure to policy divisions. Investigation of both the network and issue space, as well as their integration, is a core focus of the methodology and is accomplished statically via structural analysis and dynamically via the nonlinear group decision making model which evolves leader positions on issues in response to their mutual influence over their network of ties.

This chapter introduces a recently developed prototype software package, PORTEND, that provides a user interface for the analysis and simulation methods. PORTEND's analytical capabilities are illustrated for an application to Iranian leadership elites regarding seven major issues with a particular focus on whether their nuclear technology capabilities should or should not be constrained and subject to international monitoring. Previous applications of the methodology to Russian and Afghan leadership networks have been reported elsewhere (Gabbay, 2007a, Gabbay, 2013). The analyst survey which provides the input for empirical application of PORTEND is briefly described here. The factional structure of the Iranian leadership group is analyzed first based on their positions on the issues, then with respect to the network of inter-actor influence relationships, and finally by a synthesis of the issue and network data. Moving from structural analysis to simulation, a qualitative description of the nonlinear group decision making model is presented followed by application of the simulation to the nuclear issue and discussion of its implications with respect to Iranian decision making concerning the nuclear negotiations that took place from 2013-2015.

PORTEND Software

PORTEND (Political Outcomes Research Tool for Elite Network Dynamics) integrates quantitative techniques from nonlinear systems theory and network science to aid the analysis of policy and factional outcomes with respect to the internal dynamics of a system of political actors. The political actors may be individual leaders or organizations within a government or movement. The outcomes of concern may be policy decisions, winning and losing factions, the positions of individuals, or the potential for issues to cause dissension or factional realignment. Political actors are represented mathematically with respect to their preferences on one or more issues, the saliences of those issues, the network of inter-actor influence, and actor power and susceptibility to influence. The data from which these quantities are calculated is obtained from

surveys given to expert analysts. PORTEND imports these surveys and aggregates them to form a composite analyst if desired. It then allows for structural analysis regarding issues and the inter-actor network and for the simulation of group decision making. The analyses can be performed for the composite analyst or separately for the individual analysts. An overview of the methodology is shown in Figure 1. PORTEND is currently in a prototype stage of development and is implemented in Matlab.

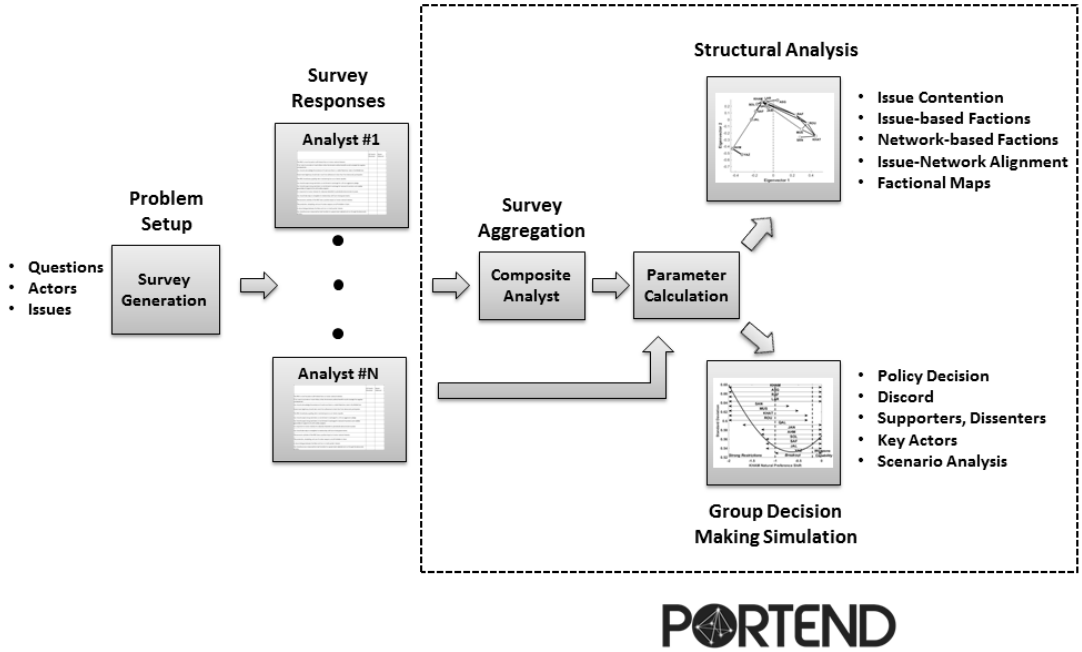


Figure 1. Methodology overview.

Iran Application

This section introduces the Iranian leadership case study which will be used to illustrate the capabilities of the methodology implemented within PORTEND in this chapter. The case study, which was initiated in 2013, considered fifteen top members of the Iranian leadership, as identified by analysts of Iranian politics (Table 1). A survey was developed and then completed by two Iran experts in the autumn of 2013. The elements of the survey will be discussed in the next section. While a major concern of the study involved the Iranian nuclear program, the broader context of Iranian elite politics was also of interest and so the survey included the seven issues below (abbreviations in parentheses):

- Liberalism (LIB): The proper role for Western culture, Islam, media sources, and democratic institutions.
- Economic Reform (ECON): Whether economic policies should benefit the current elites or a wider set of interests.

- Arab States (ARAB): Whether Iran's peers in the Arab world are potential allies or enemies.
- Syrian Regime (SYR): Whether the Assad regime in Syria should be supported.
- US/Israel (USISR): The extent to which Iran should confront the U.S. and Israel.
- Nuclear Issues (NUKE): The extent to which Iran should develop nuclear technology.
- IRGC Influence (IRGC): The appropriate role for the Islamic Revolutionary Guard Corps (IRGC).

Actor (Abbr.)	Role/Notes
Ali Hoseini Khamenei (KHAM)	The supreme leader, the highest political and religious authority in the Islamic Republic of Iran.
Qasem Soleimani (SOL)	Commander of the Quds Force, a unit of the Islamic Revolutionary Guard Corps (IRGC).
Mir Hossein Musavi (MUS)	Prime Minister of Iran from 1981 to 1989. In 2009 he was the reform candidate for president, around whom the Green Movement coalesced. He has been under house arrest since February 2011.
Mohammad Taqi Mesbah Yazdi (YAZ)	A hardline cleric and politician. He is a member of Iran's Assembly of Experts and is seen as the most conservative cleric in Iran.
Ahmad Janati (JAN)	A hardline cleric and chairman of the Guardian Council.
Asadollah Asgaroladi (ASG)	An important businessman with interests in exports, banking, real estate and healthcare. President of several of Iran's international Chambers of Commerce.
Ali Akbar Hashemi-Rafsanjani (RAF)	Served as president of Iran from 1989 to 1997 and chairman of the Expediency Council.
Ali Ardeshir Larijani (LAR)	Current chairman of the Iranian Parliament and former secretary of Iran’s Supreme National Security Council.
Yousef Sanei (SAN)	An Iranian scholar and Islamic theologian and philosopher. He serves as a Grand Marja of Shia Islam.
Mohammad Baqr Qalibaf (QAL)	The current mayor of Tehran.
Yahya Rahim Safavi (SAF)	An Iranian military commander and former Chief Commander of the IRGC.
Mahmud Ahmadinejad (AHM)	The former president of Iran.
Seyyed Mohammad Khatami (KHAT)	President of Iran from 1997 to 2005. One of Iran’s most prominent reformers.
Saeed Jalili (JAL)	Secretary of Iran’s Supreme National Security Council, the equivalent of the U.S. National Security Council.
Hassan Rouhani (ROU)	The current president of Iran.

Table 1. Iranian elites in case study. The abbreviations used in plots are shown in parentheses. Information on roles is as of late 2013.

The analytical questions of interest included:

- Will Iran agree to a nuclear deal that places strong restrictions on enrichment?
- Who might dissent from a nuclear deal and who are possible swing players?
- What are the most controversial issues? Which actor inter-relationships do they stress?
- What issues have the potential to lead to factional realignments?

In November 2013, after the survey had been developed, an interim nuclear deal was announced between Iran and its negotiating counterpart, the P5+1 countries, consisting of the five permanent members of the UN Security Council (China, France, Russia, US, UK) and Germany. This spawned an additional question as to what may have caused the shift in Iran's posture toward nuclear negotiations which will be discussed in the section on simulation results. Space does not allow background on Iranian politics to be provided here – a good discussion of Iranian factional politics can be found in Rieffer-Flanagan (2013).

Analyst Survey

The analyst survey elicits expert judgment on the leadership group under study. The use of a survey methodology allows analysts to complete the survey at their convenience and avoids potential groupthink effects associated with oral elicitation of a group of analysts at one sitting. Only the Actor Opinions and Influence Network components of the survey are discussed here as they are the ones most essential for understanding the results presented below (other components are described in Gabbay (2013)). The surveys can be averaged to form a composite assessment or analyzed individually in order to bring out differences in analyst perspectives.

The Actor Opinions survey section contains a list of statements designed to assess the attitudes of the group members relevant to the policy issues of concern. For each member, analysts are asked to estimate the member's level of agreement/disagreement with a series of statements covering a range of issues, goals, identities, and specific policies. Examples include "The production, stockpiling, and use of nuclear weapons are all forbidden in Islam" and "The IRGC should play a guiding role in maintaining Iran as an Islamic republic." The instructions direct analysts to score the statements on the basis of the private beliefs of the members if thought to be at odds with their public rhetoric. The Actor Opinions section is used to calculate member issue positions known as "natural preferences," a key parameter in both structural analysis and the simulation.

The Influence Network section contains a matrix in which analysts estimate the strength of each actor's direct influence upon each of the other members in the group and vice versa. This (directional) dyadic influence strength depends on factors such as the frequency of communications, status within the group, common or rival factional membership, and personal relationships of friendship or animosity. The influence network is used directly in structural analysis and to calculate the "coupling strengths" which scale the persuasive force of one member on another in the group decision making simulation.

Structural Analysis

Structural analysis involves quantitatively and visually probing the factional composition of the group as a whole and how individuals are situated within the group. Analyst judgments on discrete elements concerning individual actors and actor dyads are synthesized to enable the discovery of broader features and patterns in the group. In addition to being illuminating in its own right, structural analysis can help focus the simulation effort on particular issues such as those which are most polarizing or have the potential to result in new alignments of actor subgroups distinct from the dominant factional configuration. It also allows for insight into dynamics not encompassed by the simulation such as interactions between multiple issues, alliance formation, and succession considerations.

Issue Analysis

The methods for issue analysis utilize only the group member issue positions (natural preferences) calculated from the Actor Opinions. The analyses can address how contentious an issue is, how similar actor positions are for any given pair of issues, and patterns of actor alignment across the whole set of issues. This section presents examples of these analyses for the Iran case.

The most fundamental element of issue analysis is simply the actor natural preferences themselves as is shown in the plots of Figure 2. The positive end of the scale indicates support or a favorable attitude with respect to the issue and has a maximum value of 2. Similarly, the negative axis signifies opposition or an unfavorable attitude. These plots are useful for visual inspection of individual actor positions and their distribution within an issue as well as examining clustering across issues. To better highlight clustering patterns and deviations from them, conservatives are identified as those actors having negative scores on the Liberalism plot and marked by solid gray circles; reformers have positive Liberalism scores and are marked by open squares. The Liberalism plot shows a bloc consisting of KHAM (the Supreme Leader), SOL, SAF, JAN, YAZ, and JAL at the far negative end of the axis indicating strong opposition to political and cultural liberalization whereas ROU (the president), KHAT, MUS, and SAN are found oppositely at the pro-liberalization side. This pattern of opposed clustering is repeated for other issues as well thereby leading to the interpretation of the former subgroup as a core conservative or hardline faction and the latter one as a core reformist or moderate (from a US/Western viewpoint) faction. Note that RAF is usually aligned with the reformists except on the Economic Reform issue towards which he is most opposed. A subgroup composed of LAR, QAL, and ASG typically forms a conservative-leaning centrist bloc with Economic Reform again a notable exception. The level of disagreement over an issue is indicated by the amount of spread in the actor positions as can be quantified by standard deviation (see Table 2 below). Nuclear Weapons, in which the actor positions are most compressed, is the least contentious issue by this measure.

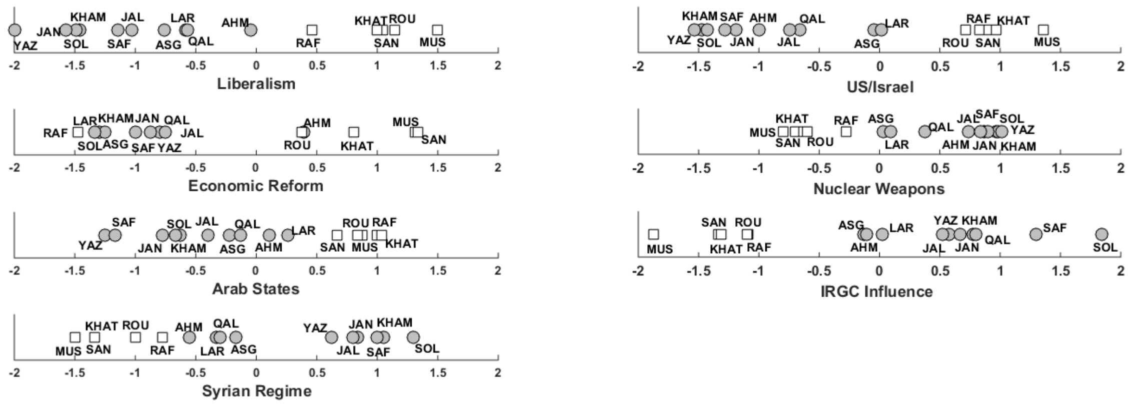


Figure 2. Actor natural preferences for the seven issues. Conservatives are gray circles, reformers are open squares.

To get a sense of the relationship between issues, Figure 3 shows two-dimensional plots of actor natural preferences. Observe in Figure 3(a) that the actor positions in the joint US/Israel and Nuclear Weapons space fall essentially on a line as is indicated by the almost perfect (anti-) correlation of -0.98. This implies that, although the issues are plotted on a two-dimensional plane, the system is essentially one-dimensional in the sense that if given the actor positions on one issue, then their positions on the second can be inferred to a high degree of accuracy. In Figure 3(b), Economic Reform appears on the vertical axis: there is now more scatter of actor positions and the correlation is lower in magnitude (although still highly statistically significant) indicating a less one-dimensional aspect. The core conservative and reform factions are still effectively at the opposite ends of the main axis (PC 1) but RAF and AHM are significantly off axis as are, to a lesser extent, ASG and LAR. The two plots have different implications with respect to potential coalitions if the two issues interact so that changing position on one issue affects an actor's position on the other. In Figure 3(b), RAF is nearer the conservatives and could side with them increasing his support for a more robust nuclear capability and bolstering their opposition to economic reforms. An analogous implication holds for AHM with respect to the reformist faction. Such realignment would not be possible if the two issues in play were US/Israel and Nuclear Weapons as in Figure 3(a): RAF would remain close to the reformers and AHM to the conservatives. However, it could be possible for ASG and LAR to be forced to side with one of the main factions if maintaining their centrist positions were to become untenable.

(a) **(b)**

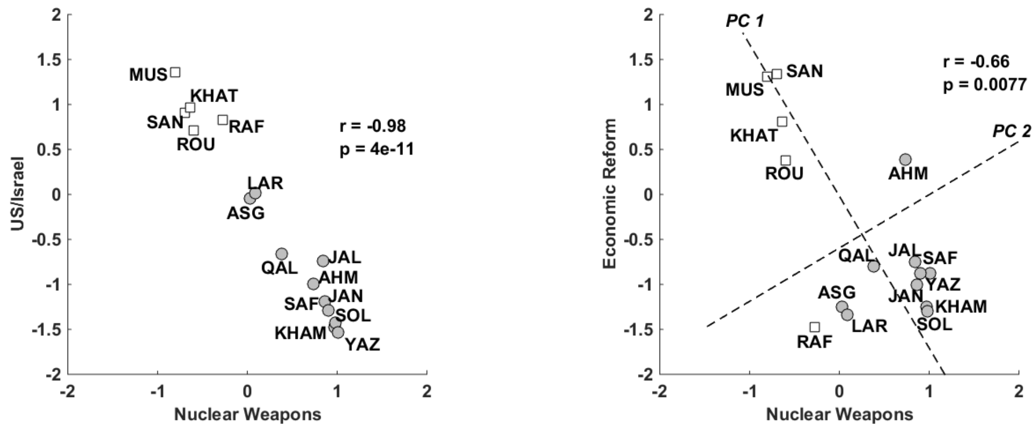
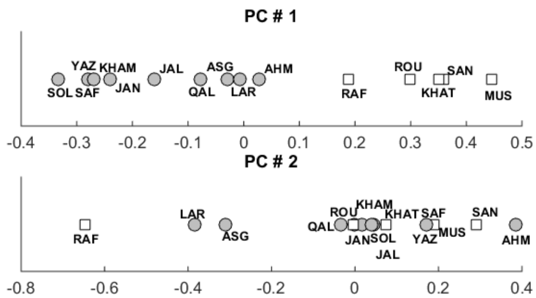


Figure 3. Two-dimensional issue plots: (a) US/Israel and (b) Economic Reform plotted vs Nuclear Weapons. The numbers in the upper right-hand corner are: the cross-correlation between the actor positions on the two issues (*r*); and the p-value measure of statistical significance (*p*) which indicates the probability that the observed correlation could have occurred by chance and are really unrelated – lower p-values imply stronger statistical significance. The dashed lines in (b) correspond to principal component axes.

While the discussion of factional alignments so far has involved visual inspection across issues, numerical methods exist for automatically revealing patterns of alignment. One such technique is principal components analysis (PCA) which seeks to represent a data matrix by a series of coordinate vectors, known as principal components (PCs), each of which corresponds to a pattern of covariation in the data (Webb and Copsey, 2011). The PCs are ranked in descending order of importance as determined by how much of the variance (the data scatter around the mean) they carry which is given by their "eigenvalues." Each PC is uncorrelated with the others so that they run as perpendicular directions through the data and, in fact, they correspond to an alternative set of coordinate axes.

For example, we can interpret Figure 3(b) as measurements of the two issue variables, Nuclear Weapons and Economic Reform, with each actor's natural preference pair as a data point. The first PC then points in a direction along the dashed line running from upper left to lower right and the second is the line perpendicular to that. In essence, PCA has rotated the standard coordinate system, wherein each axis corresponds to one issue, to the dashed system where each PC is a weighted combination of the two issues (the weights can be negative). The origin is the intersection of the two PCs located at the point given by the mean of each issue. An actor's coordinate on each PC is the (signed) distance between this origin and where he falls on the PC axis (the nearest point on the axis to him). The variance in the actor coordinates on PC 1 is given by its eigenvalue of 4.22 whereas PC 2's eigenvalue is 1.75 so we see that PC 1's share of the total variance (71%) is much larger than that of PC 2 (29%) indicating that PC1 is more important in approximating the data. (The disparity between the two PCs would be even greater for Figure 3(a) given that it is much more one-dimensional.)



PC 1 Eigen-value	PC 1 Issue	PC 1 Issue Value	PC 2 Eigen-value	PC 2 Issue	PC 2 Issue Value
0.57	LIB	0.46	0.17	ECON	0.89
	IRGC	-0.44		USISR	-0.34
	USISR	0.41		ARAB	-0.26
	SYR	-0.39		NUKE	0.16
	ECON	0.32		IRGC	0.09
	ARAB	0.30		LIB	0.06
	NUKE	-0.28		SYR	0.03

Figure 4. First two principal components of actor natural preferences. Left: Actor coordinates. Right: Eigenvalues and issue values. Eigenvalues are expressed as the fraction of the total sum of eigenvalues. Issue values are listed in descending magnitude.

Turning to the complete set of issues, Figure 4 shows the first two (out of seven) principal components obtained from the data matrix formed by the natural preferences of each of the fifteen actors on all seven issues. The top plot on the left side shows the actor coordinates for the first principal component. This corresponds well to the dominant factional alignment identified in our discussion of the issue plots of Figure 2. The core conservative bloc of KHAM, SOL, SAF, JAN, YAZ, and JAL is on the extreme negative end; the conservative-leaning centrists QAL, LAR, and ASG are just left of zero and the core reform bloc of ROU, KHAT, MUS, and SAN is on the far positive side. Rafsanjani is aligned with the reformers on PC 1 as is the case on six of the issue plots. Ahmadinejad's location as a centrist may be surprising given his international reputation as a hardliner during his presidency but is supported by his position near the center or on the reform side for four of the issues. The eigenvalues in the corresponding table show that PC 1 carries 57% of the total variance, much larger than PC 2's 17% share. This supports the interpretation of PC 1 as the dominant factional alignment. The PC 1 Issue Value column shows that there is no single primary issue whose magnitude is much larger than the others, again suggesting that PC 1 represents the most common pattern across the set of issues. This is not the case, however, for PC 2 where the Economic Reform component of 0.89 is by far the strongest. The plot of the PC 2 coordinates shows RAF and AHM at opposite ends reflecting the fact that, while the majority of the actors preserve the standard factional composition for Economic Reform, RAF and AHM make large against-the-grain shifts in the conservative and reformist directions respectively as observed in Figure 2 (RAF and AHM also appear at opposite ends of the second PC for the two-issue example of Figure 3(b)).

Network Analysis

Parallel to the investigation of issue-based factions described above, the factional structure which arises from the network of inter-actor influence relationships is also of concern. Network science has developed many algorithms for detecting community structure in networks. Intuitively, the goal is to find subgroups of nodes which have more links among them than they do with other subgroups. Community structure may reflect similarities in preferences among network members via: the homophily principle (also known as assortative mixing), a formal construct for the commonplace that "birds of a feather flock together" (Newman, 2010); or the mechanism of

social influence which assumes that people who interact more often tend to become more similar (Friedkin and Johnsen, 2011). This section presents the application of a community structure algorithm which is then extended to illustrate how community structure and actor natural preferences can be integrated to address joint issue-network alignment.

The algorithm employed in PORTEND seeks to divide a network into two communities so that the network “modularity” is maximized (Newman, 2006). The contribution to the total modularity from a given pair of nodes is proportional to the difference between their observed tie strength and that which would be expected if their interactions were solely due to chance; these contributions for all the dyads form the elements of the modularity matrix. The total modularity expresses the extent to which a putative division of the network into two communities exhibits a level of intra-community linking exceeding the level expected if the division were, in fact, arbitrary with no correspondence to behaviorally meaningful subgroups. The maximization is done in an approximate but efficient way by calculating the first eigenvector of the modularity matrix (eigenvectors are ranked in order of descending eigenvalue) and then assigning all nodes whose components in the first eigenvector are positive to one community and the nodes with negative components to the other. As an example, Newman (2006) presents an application to a network of 62 dolphins and finds that the two communities identified by the first eigenvector matched, to a high degree of accuracy, the two groups into which the network actually split after a key dolphin died (only three dolphins were misclassified).

The application of the community detection algorithm to the Iranian influence network is shown in Figure 5 which plots the actor coordinates obtained from the first two eigenvectors of the modularity matrix (using the symmetrized network in which tie strengths are the same in both directions in a dyad). The initial discussion of Figure 5 will center on the meaning of Eigenvector 1 but, as will be seen below, Eigenvector 2 also has a significant interpretation regarding the Economic Reform issue. The dashed line corresponds to the division formed by separately grouping nodes with positive and negative signs in Eigenvector 1. The left and right sides correspond to conservative and reformer classifications respectively. The correspondence with the issue-based factions is immediately apparent because, as in PC 1 in Figure 4, all the gray circles are on one side and the white squares on the other. All members of the core conservative and reform blocs as identified by the issue analysis above are correctly classified. Only ASG can be considered to be misclassified as a reformer, perhaps understandable given that he is more of a centrist than a hardline conservative (and in fact he appears in the middle of Eigenvector 1).

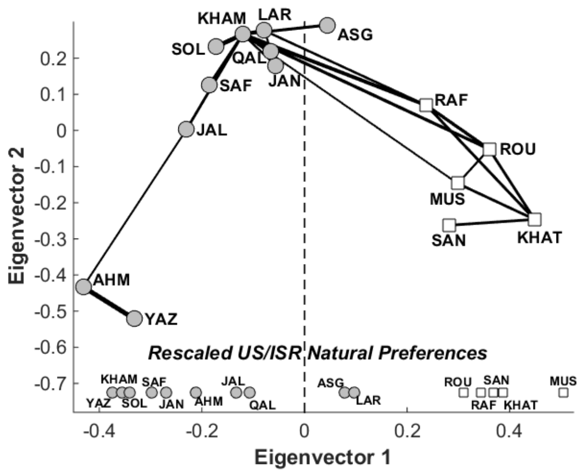


Figure 5. Community structure in the Iran influence network. Dashed line partitions network into conservative (left) and reformist (right) communities. Link thickness between actors is proportional to relationship strength (weak links have been thresholded). Points at bottom of plots are actor natural preferences for the US/Israel issue rescaled to fit inside x-axis.

It is also possible to combine issue and network data for purposes of addressing polarization and factional realignment. As used here, polarization refers to the extent to which disagreement over an issue reinforces divisions present in the network. Hence, polarization is not simply the level of disagreement over an issue as might be gauged from the standard deviation of actor issue positions. Quantitatively, the contribution of an actor dyad to the polarization for a given issue is found by multiplying the corresponding modularity matrix element – network data – by the product of the two actor natural preferences – issue data (which makes the polarization equivalent to the covariance between natural preferences over all the ties in the network (Newman, 2010)). The polarization value for each issue is shown in Table 2. For comparison purposes, the standard deviation of actor natural preferences is shown in the last column. The US/Israel issue is most polarizing even though Liberalism has the highest standard deviation. Nuclear Weapons and Economic Reform have very nearly the same polarization whereas the latter has a substantially larger standard deviation. Consequently, we see that the integration of network and issue data gives a different and perhaps more significant picture of issue divisiveness than issue data alone.

The Aligned Eigenvector column in Table 2 is the number of the eigenvector with which the issue has the highest magnitude correlation. The larger value of polarization of US/Israel as compared with Liberalism, despite the latter's higher standard deviation, is a reflection of the greater alignment that US/Israel has with Eigenvector 1 as seen by its better correlation in Table 2. A visual sense of this alignment can be gleaned from Figure 5 by comparing the actor network positions along the horizontal axis with their (rescaled) natural preferences at the bottom. Whether or not the correlation represents a genuine relationship between the eigenvector and the actor natural preferences can be assessed from the p-value column with lower values indicating greater significance. All of the correlations in Table 2 are highly significant. Six of the seven issues best correlate with Eigenvector 1 reinforcing the conclusion that it represents the dominant

factional division in the network. Consequently, these issues stress the major faultline in the group but are not likely to cause a fundamental factional realignment (although centrists may be forced to side with one camp or another as noted in connection with Figure 3(a)). However, Economic Reform is seen to align best with Eigenvector 2 (the vertical axis in Figure 5) and, therefore, if it were to become more salient, a factional realignment could be induced in which RAF allies more strongly with the conservatives and AHM does likewise with the reformers.

Issue	Polarization	Aligned Eigenvector	Correlation Magnitude	Standard Deviation
US/Israel	0.395	1	0.885***	1.036 (3)
IRGC Influence	0.393	1	0.779**	1.095 (2)
Liberalism	0.358	1	0.799**	1.142 (1)
Syrian Regime	0.248	1	0.731*	0.974 (5)
Economic Reform	0.198	2	0.645*	0.998 (4)
Nuclear Weapons	0.195	1	0.907***	0.701 (7)
Arab States	0.158	1	0.796**	0.786 (6)

Table 2. Integrated issue-network analysis metrics. Issues are listed in descending order of polarization. Statistical significance levels of correlations: * p<.01, ** p<.001, *** p<.0001. The last column shows the standard deviation of the actor natural preferences (rank in parentheses).

Group Decision Making Simulation

Model Description

The nonlinear model of small group decision making simulates the evolution of group member positions along the policy axis due to their mutual interactions. The social science underpinnings of the model derive primarily from social psychology theories of attitude change and small group dynamics and theories of foreign policy decision making (Eagly and Chaiken, 1993, Hermann et al., 2001). A brief summary of the model is presented in this section; fuller descriptions can be found in Gabbay (2007c, 2007b). It should be noted that since the model is focused on group dynamics, it does not involve a representation of the decision making calculus associated with particular policy choices (see Davis and O'Mahony (2013) for an example of a computational model that does so in the context of insurgent groups). With respect to other models of group dynamics, on a mathematical level, the nonlinear model is most similar to that of social influence network theory (Friedkin and Johnsen, 2011) to which the model can be made equivalent in the (linear) limit of low disagreement. The most prominent formal model of decision making applied to real-world political contexts is that of Bueno de Mesquita (1997, 2009) which, however, has received some criticism regarding lack of transparency (Scholz et al., 2011). While Bueno de Mesquita’s model uses analyst input and a one-dimensional issue axis as does the present model, it is based on expected utility theory whereas PORTEND is rooted in nonlinear dynamical systems theory and network science, cornerstones of complex systems research.

In the model, an actor’s position changes under the influence of two separate forces: the “self-bias force” and the “group influence force.” Considering the self-bias force first, each actor is assumed to come to the debate with an initial issue position given by his natural preference (also

called the natural bias) which reflects the actor's underlying beliefs, attitudes, and worldview pertinent to the issue. If an actor's position is shifted from his natural preference due to group pressures, he will experience a cognitive dissonance that resists this change and strives to move the actor's position back toward the natural preference.

The group influence force is the total force acting to change an actor's position due to the persuasive efforts of the other actors in the group. It is assumed to operate in a pairwise manner so that an actor – the message receiver – experiences a persuasive "coupling force" from another actor – the message sender – to whom he is connected (and vice versa). The functional form of the coupling force is nonlinear in the difference between the sender and receiver positions: if the difference is small, the force increases roughly linearly; the force then reaches a peak at a difference known as the "latitude of acceptance," beyond which it begins to wane towards zero. This form is motivated by social judgment theory which posits that the amount of opinion change in a person receiving a persuasive message follows an inverted U-curve as a function of the difference between the opinion advocated in the message and that of the receiver (Eagly and Chaiken, 1993) (however, the coupling force in the model has a long tail rather than ending abruptly as in an inverted-U). The coupling force that actor *j* exerts on member *i* also depends on the "coupling strength" from *j* to *i*, which is obtained from the influence network. The "coupling scale" is the mean of the incoming coupling strengths (in-degree).

The model description above governs how actors change their positions under their mutual influence but does not yield the decision itself. In order to do so, the appropriate decision rule – leader choice, weighted majority, or consensus – must be applied. Typically, this is done after the simulation reaches equilibrium, i.e., the actor positions reach steady-state values that no longer change perceptibly. For purposes of determining whether an actor supports or dissents from a policy decision, an actor is considered to support a policy if it lies within a specified maximum distance, usually taken to be the latitude of acceptance, from the actor's final position. Similarly, actors are taken to dissent from a policy if it lies beyond this distance.

Complexity enters into the model via the nonlinear form of the influence between actors and its interaction with the network formed by the inter-actor coupling strengths. The model can be considered to have two regimes of behavior: a "linear" one, in which the behaviors typically correspond to initial intuition, and a "nonlinear" regime corresponding to high disagreement (roughly, position differences exceeding twice the latitude of acceptance) in which behaviors can run counter to initial intuition. The linear regime is always characterized by gradual changes in outcomes as parameters such as the level of disagreement or coupling scale are varied whereas the nonlinear regime can exhibit discontinuous transitions, referred to as bifurcations, between states such as deadlock, majority rule, and consensus (Gabbay and Das, 2014). With respect to the interaction of nonlinearity and network structure, at high disagreement levels networks with lower tie density (e.g., a chain) can be more effective at reducing group discord and yielding consensus than ones with higher density (e.g., a complete network) in contrast with the "linear" expectation that a higher number of ties is better for consensus formation (Gabbay, 2007b).

Simulation Results

All seven issues were simulated. Here only the Nuclear Weapons results are discussed as that issue was of primary analytical concern. The simulation using the set of parameter values calculated directly from the composite analyst is shown in Figure 6(a). The latitude of acceptance is taken to be one unit along the issue axis as that corresponds to a step along the attitude survey scale, say from "neutral" to "weak agreement" or from "weak agreement" to "strong agreement." Actors start out at their natural preferences and, as remarked above, the time units are essentially arbitrary given that the equilibrium is of concern.

The policy labels and corresponding intervals in Figure 6(a) are calculated from the Actor Opinions section of the survey (they can also be set manually) and are intended to be rough guides to assist in interpretation of simulation results rather than hard and fast boundaries. The Weapons Capability policy corresponds to an actor believing that a nuclear weapons capability is critical to ensuring the survival of the Iranian regime. Breakout signifies that the actor prefers that Iran should have the ability to develop nuclear weapons without building or testing them. Strong Restrictions signifies that the actor is willing to accept more forceful constraints on Iran's nuclear enrichment program such as intrusive monitoring of nuclear facilities in exchange for the removal of economic sanctions (a fourth policy of No Enrichment was not preferred by any actor).

The decision rule is leader choice and the open diamond indicates the final policy, coincident with KHAM's final position. We see that the policy choice is located in the Weapons Capability zone, justly slightly less hardline than KHAM's initial natural preference. This is not surprising given the outsize influence that KHAM has on the group; his network out-degree – the sum of all his outgoing influence network values on the rest of the group is more than three times the second highest actor. Rafsanjani does move sufficiently towards a harder line so that he can support the policy. However, the core reformers, and most notably ROU, dissent as they end up greater than one unit (the latitude of acceptance) from the policy.

The above result, however, is inconsistent with the more conciliatory posture that Iran took in reaching the interim nuclear agreement in November 2013. It is not tenable that the Iranian president Rouhani, a savvy political insider, would have been vigorously pursuing a nuclear deal with the United States completely at odds with the Supreme Leader's policy, thereby setting himself up for failure. This leads to the inference that the Iranian policy may have shifted to a softer line than represented in the original analyst data. Possible scenarios underlying this shift can be investigated by changing the simulation parameters. Simulations of scenarios involving increased group cohesion or increased reformer status due to Rouhani's election in June 2013 could not produce a significant enough policy shift. Another potential explanation is that Khamenei himself softened his position, which can be modeled by shifting his natural preference in the negative direction of the Nuclear Weapons issue axis. This can indeed account for the softer line policy: given the leader choice decision rule and his great influence, the policy essentially follows his natural preference; a shift of -0.2 brings the policy into the Breakout range and a shift of -1 moves it into the Strong Restrictions range.

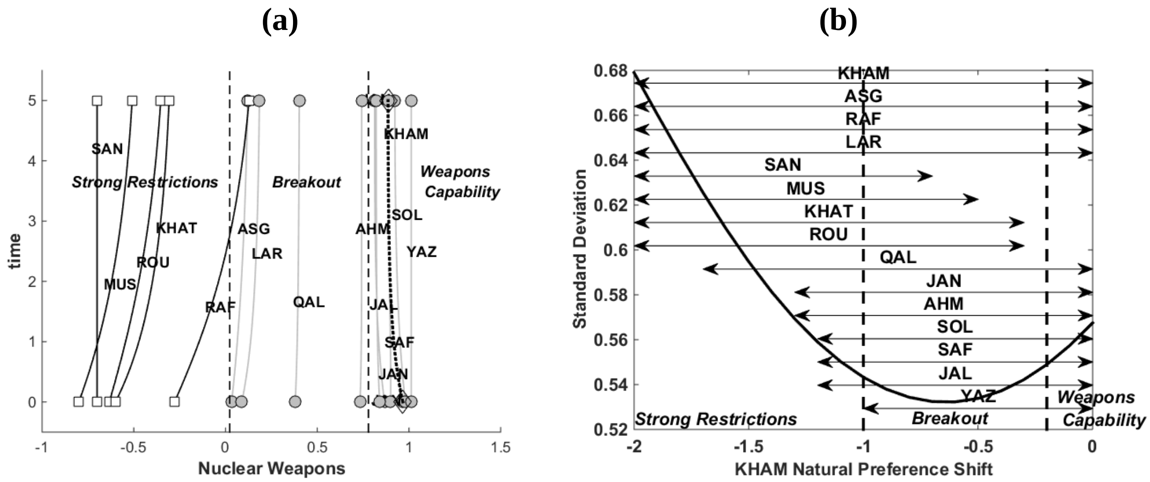


Figure 6. Nuclear Weapons issue simulation. (a) Actor trajectories using composite analyst values (first letter of actor abbreviation intersects with trajectory curve). Dashed lines demarcate boundaries between different policy labels. Dotted line is policy value (same as KHAM trajectory due to leader choice rule) and open diamond at top is policy decision. (b) Effect of Khamenei softening his natural preference: standard deviation of actor final positions (solid curve) and actor concurrence intervals (double-headed arrows, actor listed above). Horizontal axis is shift from KHAM original natural preference from composite analyst.

While it is clear that Khamenei can shift the policy if desired, considerations of minimizing discord within the leadership as a whole and, in particular, maintaining the support of key hardliners – the IRGC members, Soleimani and Safavi, and Janati, the chairman of the Guardian Council – are doubtlessly important in his decision making calculus. These factors can be assessed using Figure 6(b) which plots the standard deviation of the final positions and the concurrence interval for each actor – the range of the natural preference shift over which the actor supports the policy decision. Observe that there is a minimum in the standard deviation at a shift of about -0.6 approximately in the middle of the Breakout interval (the fact that the curve has a minimum rather than simply monotonically increasing or decreasing stems from the nonlinear nature of the model). Furthermore, there is a range from about -0.7 to -1 for which all actors concur with a policy in the Breakout zone. These observations imply that a Breakout policy would minimize discord within the group. Indeed, as KHAM moves into the Strong Restrictions zone, he rapidly begins to lose conservative support: first YAZ and then, crucially, at -1.2 the IRGC members SOL and SAF followed shortly thereafter by JAN.

The above analysis leads to the conclusion that the Khamenei softening scenario is a plausible explanation for Iran’s shift to a posture more amenable to reaching a deal on the nuclear issue; he can maintain consensus while pursuing a Breakout policy, which is consistent with trying to reach a nuclear agreement, albeit one which would be very weak from the perspective of the United States. The fact that there were secret meetings between US and Iranian officials on the nuclear issue starting in 2012, a year prior to Rouhani’s election (Associated Press, 2014), suggests that Khamenei may very well have shifted towards a more flexible position than the original hardline Weapons Capability ascribed to him from the analyst surveys. With respect to the prospects of reaching a final deal, his original analyst-derived position would imply that a deal would be extremely unlikely. The analysis of the softening scenario indicates that

Khamenei's room for maneuver is limited and he can only move a small amount into the Strong Restrictions zone before losing the support of key conservatives. This suggests that a deal which provides robust provisions against an Iranian breakout capability – in particular, the US stated that it sought a minimum breakout time of one year – would indeed be possible but very difficult to reach. A deal between Iran and the P5+1 was in fact announced in July 2015. An assessment as to the strength of the deal from the P5+1 perspective – whether the monitoring and other restrictions on Iranian nuclear activities are sufficiently robust as to prevent a rapid breakout capability or a covert program – cannot be made here. However, the fact that the negotiations took twenty months from when the interim deal was announced to reach a final agreement, including two six-month extensions of the interim deal, attests to the difficulty in consummating the negotiations.

Conclusion

Relationships among leadership elites and their preferences on important issues are essential elements in determining the outcomes of policy debates. This chapter has presented a methodology, implemented with the PORTEND software package, for the analysis of the factional structure of leadership elites and the simulation of their group decision making. Methods for investigating factional structure based on issue data alone range from simple standard deviations and plots of actor natural preferences to more sophisticated pattern extraction using principal components analysis, which revealed meaningful dominant and subordinate factional alignments among the set of Iranian leaders; the first corresponding to the primary conservative-reformer divide over most of the issues and the second reflecting key departures from this alignment with respect to economic reform. Complementary to the issue analysis, the application of a community structure algorithm to the inter-actor influence network also yielded similar dominant and subordinate structures via the first two eigenvectors. The uncovering of the parallel structure in the issue and network data illustrates the power of applying methods from research in complex networks. This research also forms the basis for the polarization metric which quantifies the extent to which differences in actor issue positions also stress network faultlines, thereby providing an integrated measure of how divisive an issue is.

The group decision making model entails complexity via its nonlinear coupling of actors over their influence network and was applied to Iranian nuclear decision making. Simulation of the original analyst values yielded a policy decision that was so hardline as to be inconsistent with apparent Iranian moves towards more negotiating flexibility in late 2013. The application of the model for scenario analysis was illustrated to address this inconsistency. Khamenei's shift towards a more moderate natural preference was found to be the most plausible explanation. Sweeping over his natural preference shift, the simulation indicated that he had sufficient room to maneuver before losing the support of key hardliners so as to make negotiations tenable. However, his ability to enter the "strong restrictions" zone, which presumably would have some overlap with the goals of the P5+1 countries, was found to be quite limited implying that achieving an agreement would be quite difficult – a conclusion perhaps supported by the long period of time required to reach a deal.

Finally, addressing further research, one area could involve the investigation of whether automated content analysis of actor rhetoric could be a viable input source for either the

structural analysis or the simulation. Another area could be extending the group decision making model to a multidimensional issue space in order to allow issues to trade off against each other. Additionally, complexity research on adaptive networks could be used to develop an issue-network coevolution model in which both issue positions and network ties would interact and change dynamically, thereby explicitly modeling alliance formation processes, a capability not present in the current model.

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